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PRACTICAL
DESCRIPTIVE GEOMETRY

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PRACTICAL DESCRIPTIVE GEOMETRY

BY

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PREFACE TO THE SECOND EDITION

The first edition of this book, published in 1912, represented the author's views on the presentation of Descriptive Geometry, so as to arouse the student's interest in the subject without sacrificing rigorous training in the principles.

In three years the text has been used in many schools, and the present, second edition, aims to embody the suggestions of many leading teachers of the subject, who have, at the request of the author and publishers, expressed their views as to this development.

The main idea of the first edition, that of "practicality," has been adhered to and amplified. The most important changes are:

1. The division into more chapters.
2. A more logical sequence for certain portions of the text.
3. The notation of points and planes.
4. Graphic layouts for exercises.
5. More exercises of a practical nature, embracing more lines of engineering endeavor.
6. A number of new solutions, utilizing auxiliary planes more frequently.
7. A better system of perspective drawing.
8. A chapter on the intersections of plane solids.

Although yielding to the request of several teachers for the inclusion of graphic layouts of exercises, the author still believes that dimensioned layouts are of great value despite the amount of time expended on the layout itself. With both of these, however, in practically unlimited quantity, the instructor may decide the question for himself. The layouts of both kinds have been made so flexible, by interchanging and substituting data, that the number of variations is almost incalculable. This removes the objection, expressed by some teachers, to a definite and limited number of exercises, the solutions of which are sometimes preserved by student organizations. No such collection of solutions will benefit future classes, as the exercises can be made entirely different each succeeding year.

As to the treatment of Perspective (how and how much), the author received a large amount of constructive though conflicting advice, mainly from teachers of architecture. The writer's choice is "Conical Projection," a process that is sane, in that it is a direct application of the principles of Descriptive Geometry, and is easy and swift in operation.

It has been previously stated that much interest was manifested in the revision of this work by prominent teachers in all parts of the country, and their combined opinions moulded it into its present form. The writer wishes to convey his thanks to all who assisted in this way. For special criticisms and material he acknowledges indebtedness to Professor Thomas E. French, Ohio State University; Professor Howard Moore, Colorado College; Professor Frederick C. Biggin, Alabama Polytechnic Institute; Professor Robert H. McNeilly, Vanderbilt University; Professor Chace Newman, Michigan Agricultural College; Professor H. B. Monges, University of California; Mr. Charles R. Swineford, Armour Institute of Technology.

W. G. S.

Armour Institute of Technology,
Chicago, April, 1916.

PREFACE TO FIRST EDITION

In presenting a new text-book on Descriptive Geometry, the writer is fully aware of the excellence of existing treatises, and appreciates what they have contributed toward a higher standard of technical training. The authors of these treatises, pursuing their several courses, have carried the development of the subject to a high plane.

However, in spite of the indisputable excellence of many of the text-books, they seem to have failed to arouse the interest of the student, partly by ignoring the practical applications, and partly by making only a slight attempt to present the subject attractively. Some of the books are valuable only for reference and are useless in the class room; others are incomplete in the essentials; some are faddish, emphasizing certain features and treating the rest inadequately, while even the best convey to the student only a very slight idea of the practical value of the subject. This has given rise to the belief, prevailing almost universally in the student body, that Descriptive Geometry is merely a disciplinary study, having little or no relation to the life work of the individual.

The aim of the writer has been, therefore, to present the subject to the student in a simple manner, as progressively as possible, reminding him constantly of the relation which exists between Descriptive Geometry and Practical Drafting; and to avoid needless difficulty by using language and directions of the greatest possible clarity. The writer believes that a thorough knowledge of the subject is achieved not through much study of the text, but by working exercises. To this end he has provided a large number of exercises, scattered through the text, of considerable variety and capable of infinite multiplication by the clever instructor.

An examination of the subject will reveal the following features and innovations:

1. A thorough drill in fundamentals.
2. Repetitions of statements for the sake of emphasis.
3. Notation comprehensive, yet reduced to its lowest terms.
4. Analyses separated from proofs.
5. Tabulated order of the operations in each analysis and construction.

6. Exercises to the number of 860, of which about one-fourth are such as may be met in actual practice. Most of these are dimensioned for a space of standard size.

7. Notes on exceptions, checks on solutions, special cases, hints, and rules presented wherever they seem necessary or advisable.

8. Brief treatment only of such special subjects as Shades and Shadows, Perspective, etc.

9. The original seventeen "Point, Line, and Plane" problems expanded to forty-three.

10. New treatment of surfaces, less importance being attached to passing tangent planes, and more importance to plane sections, intersections, developments, and practical applications. For example, there are about thirty exercises which are based on practical illustrations of the various warped surfaces, showing actual uses for all the listed varieties.

The expansion of the problems noted in feature (9) is justified, the writer believes, in order to fully cover the ground, and to provide the necessary steps in the development of the subject. Some of them are special cases and some require little or no change in the analysis from those of preceding problems.

Students undertaking the study of Descriptive Geometry should have a fair knowledge of Mechanical Drawing, at least as much as is usually included in High School courses. The minimum prerequisite should amount to the work contained in the first eight chapters of French's "Engineering Drawing," or in Reid's "Mechanical Drawing" entire.

In preparing this text-book the writer has consulted most of the standard works on the subject, and desires to make special acknowledgment for ideas and information to the excellent books by MacCord, Church and Bartlett, Phillips and Millar, and Randall. Most of the exercises are original, but many have been adapted from such admirable books of exercises as those by Professors Hood, Marshall, and Fishleigh, as well as the problem files of Armour Institute of Technology, and the books of the American School of Correspondence.

The writer wishes to thank his colleagues, Robert V. Perry, Henry L. Nachman, John S. Reid, and Charles R. Swineford, of the Faculty of Armour Institute of Technology, for valuable suggestions and encouragement.

William Griswold Smith.

Armour Institute of Technology,
Chicago, June 20, 1912.

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PRACTICAL DESCRIPTIVE GEOMETRY

CHAPTER I

DEFINITIONS, NOTATIONS, GEOMETRICAL THEOREMS, PRELIMINARY PRINCIPLES AND EXERCISES

1. Definition.—**Descriptive Geometry** is the **Science** of **Drafting.**

Drafting is the **Art** of representing objects on paper, or other flat surface, by means of **projections** of the object on imaginary planes.

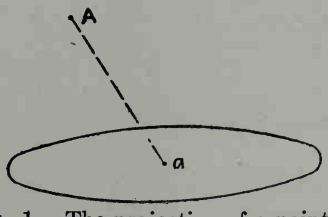


FIG. 1.—The projection of a point.

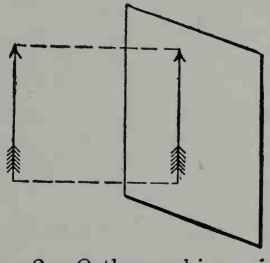


FIG. 2.—Orthographic projection.

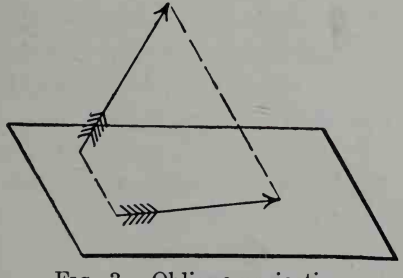


FIG. 3.—Oblique projection.

Projections are representations of objects made by imaginary lines, called **projectors,** running (or projected) from the important points of the object to the plane on which the image is to be made.

Classification of Projections

Orthographic Projection: representation of objects projected perpendicularly.

Perspective or **Scenographic Projection:** representation of objects projected by converging lines.

Oblique Projection: representation of objects projected by parallel oblique lines.

Single Plane Projections: representation of objects in various ways for pictorial purposes. Several styles are in use and are treated in Chap. X. They are mechanical variants on perspective and are effected through various devices of orthographic and oblique projection, and by artificial foreshortening.

2. Discussion.—Orthographic Projection is almost universally used in "working drawings" and "machine sketches;" hence

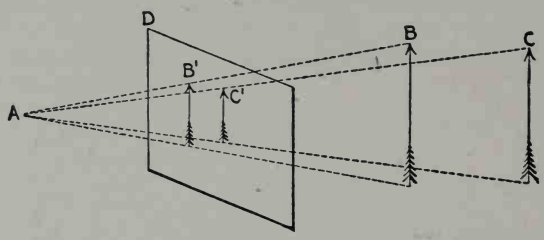


Fig. 4.—Perspective.

it is the most important class of projection. To thoroughly understand its significance, let us consider the difference between a picture and a working drawing.

A picture, or perspective, is a representation of an object, group, or scene, as it appears to the eye. Rays of light from the object reach the eye in converging lines. See Fig. 4. A is the eye of the observer; B and C are objects of equal size, but at different distances from A; D is a plane of projection between the objects and the eye. Converging rays (shown by the dotted lines) project the objects on D (piercing it at certain points). These points determine the size of the pictures of B and C, and the images B′ and C′ are the pictures. As B is the nearer, B′ is larger than C′. Thus distant objects appear smaller than nearer ones of equal and often smaller size. By means of this fact and the knowledge of the observer of the comparative sizes of the objects, a picture shows the entire composition of a scene.

The working drawing takes no such account of distances. See Fig. 5. Here the rays are all parallel, and the projections are of equal size to the objects, be the latter near or remote. In orthographic projection, therefore, the eye must be considered to be at an **infinite** distance. If the effect of a finite viewpoint should be desired, the eye would have to be shifted (as A, Fig. 5) to the line of each ray. This, of course, is an artificial condition, and shows that a working drawing is not, and cannot be, a correct visual image of the object.

This fact would seem to be a disadvantage against the use of orthographic projection, but for manufacturing and construction work, the orthographic is much the more useful and universal

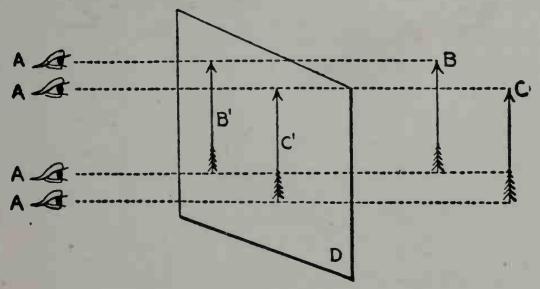


FIG. 5.

in its adoption than perspective, and much easier for the draftsman to use.

On account of the complexity of many machines and structures, it is necessary to show many dimensions in many directions, interior dimensions, thicknesses, and irregularities, which are all but impossible in a picture. This is accomplished by projecting the object orthographically on two or more imaginary planes, called the **Planes of Projection.**

THE PLANES OF PROJECTION

3. The two principal planes are the **Vertical** and **Horizontal.** The Vertical Plane (which we shall hereafter write *V*) is a plane normal to the earth's surface; that is, perpendicular to the apparent surface of the earth. The Horizontal Plane (hereafter written *H*) is a plane perpendicular to any vertical line; that is, it is parallel to the apparent surface of the earth. *H* and *V* are perpendicular to each other. The projections on these two planes determine the relative position of any two or more points in space.

There are conditions arising often, which render a third plane desirable, and indeed necessary, in the solution of certain problems. This third plane is perpendicular to both H and V, and is called the **Profile Plane,** which will always be written P.

Any number of **auxiliary planes,** which may be parallel, perpendicular, or oblique to H or V, may be used, according to the

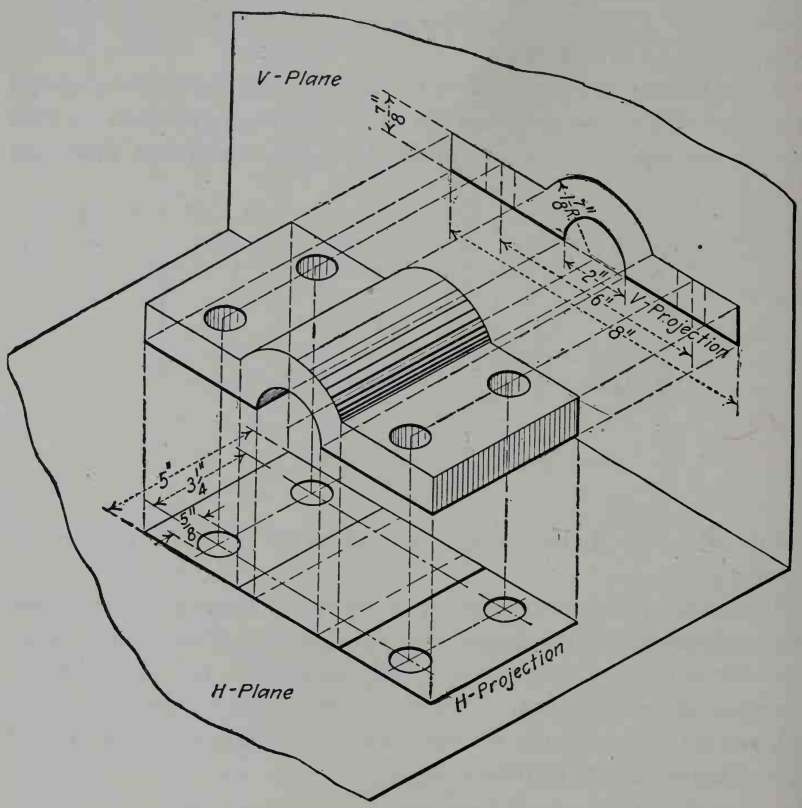


FIG. 6.—Machine part projected orthographically.

complexities of the problem. (In the architect's plans of a big building, two different vertical projections may be needed, two profiles, a horizontal projection for every floor, basement, and roof, auxiliary projections of certain oblique features, sectional interior projections, and a hundred "detail" projections of doors, windows, staircases, elevators, etc., all in orthographic projection. Besides these, it is customary to make a perspective of the projected building with colors, trees, grass, walks, etc. The average

purchaser cannot comprehend working drawings, while the picture is of little use to the workman. Perspectives are for the customers, orthographic projections for the workmen.)

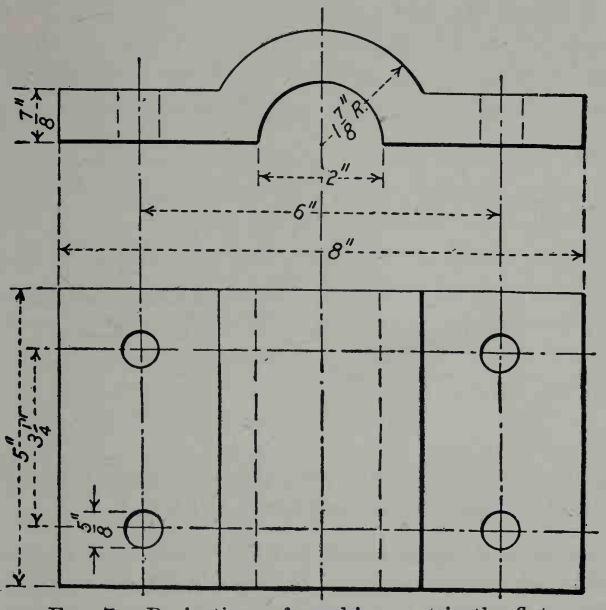


FIG. 7.—Projections of machine part in the flat.

4. It is obvious that it would be awkward and impracticable to have a drawing board made with horizontal, vertical, and profile sides, and the respective projections made on them. Not only would such an arrangement be unwieldy, but it is unnecessary. Fig. 8 shows two planes, H and V, intersecting, as they must, in a line, called the Ground Line (designated hereafter GL).

By revolving V about GL as an axis or hinge, it will be put into coincidence with H, and the H- and V-projections can be drawn on one surface. Similarly, P can be rotated into the same surface about its intersection either with H or V. For mere solution of problems, it is immaterial whether the P-projection is revolved into H or V, but in practical work it is customary to revolve it into V; therefore in this work the profile will always be rotated about its V-trace into V.

THE FOUR DIHEDRAL ANGLES

5. It will be seen from Fig. 8 that the planes of projection do not end at GL. In fact, they are unlimited in extent, and divide

space into four dihedral angles, with *GL* as their common intersection. In folding into the flat, therefore, no portion of the drawing board is monopolized by either *H* or *V*. There is a portion of *H* above *GL*, and a portion below *GL*. There is a portion of *V* above *GL*, and a portion below *GL*. (This circumstance will be found essential to the solution of certain problems of abstract character. In practical work the different projections, or **views,**

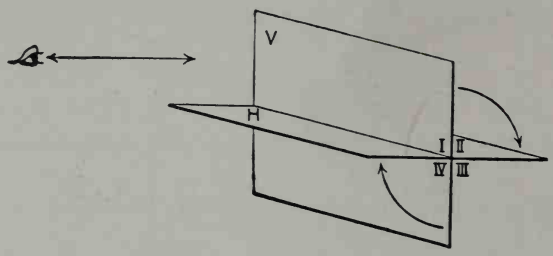


FIG. 8.

as they are often called, are entirely separated from each other, and definite spaces are reserved for *H*, *V*, and *P*.)

The four dihedral angles, in any of which objects may be imagined, are numbered, as shown by the Roman numerals in Fig. 8.

First Angle, in front of *V*, above *H;*
Second Angle, behind *V*, above *H;*
Third Angle, behind *V*, below *H;*
Fourth Angle, in front of *V*, below *H*.

The first angle (designated in this work as *I*) is largely used in our abstract problems, on account of its accessibility and the ease of presentation by the teacher to the class, when giving a solution of a problem in space.

The third angle (*III*) is commonly used in practical work, for the reason that the *H*-projection is usually made **looking down** on the object, and besides for this angle the profile comes out **right-handed** and detached, whereas the usual method of swinging a profile from a first-angle-projection will either bring the object out left-handed, or mixed up with the other views. Many of our problems are given in *III*.

6. Commercial Terminology of Projections.

As the terms *V*-projection, *H*-projection, etc., are not descriptive to non-professionals, certain terms are used by draftsmen.

The *V*-projection is often called the **Elevation, Front View, Rear View,** or **Sectional Elevation,** as the case may be.

The *H*-projection is called the **Plan, Top View, Bottom View,** or **Sectional Plan,** as the case may be.

The *P*-projection is called the **Profile, End View, Right** or **Left End View,** or **Profile Section,** as the case may be.

The Representation of Points, Lines, and Planes

7. Points.—It is obvious that the projection of a point on one plane does not determine that point. For example, let a point on the floor be the projection of some point in the room. How high is that point? It may be any distance. The point may be the projection itself, or the point may be on the ceiling above, or anywhere in between. To locate the point in space by one projection, it is necessary to add its distance, positive or negative, from the projection. A better way (the draftsman's way) is to locate the point graphically. By projecting that point on the wall, its height is determined and shown.

This definitely locates a point, for, if perpendicular lines be erected from each projection of a point, these perpendiculars will pass through the point. As there can be but one intersection of the perpendiculars, the point is absolutely located in space by its projection on two planes. The perpendicular lines from the point to the planes of projection are called **projectors.**

The Relation of Plane and Solid Geometry to Descriptive Geometry

8. All theorems in Plane and Solid Geometry are applied to Descriptive Geometry to a greater or less extent. There are, however, a number of theorems of outstanding importance, to which reference must be made with considerable frequency. A list of these is herewith submitted. The student need not be required to prove them, but should be able to give the conclusion to every hypothesis, and to quote his references on occasion.

Plane and Solid Geometry Theorems

1. The opposite sides of a parallelogram are equal.
2. Triangles are equal:
   (1) If three sides of one are equal to three sides of the other, each to each.

(2) If two sides and the included angle of one are equal to two sides and the included angle of the other.

(3) If one side and the adjacent angles of one are equal to one side and the adjacent angles of the other.

3. Right triangles are equal if any side and an acute angle of one are equal to the corresponding side and angle of the other.

4. If the two points of a line lie in a plane, the line lies entirely in the plane.

5. If a line is perpendicular to a plane, it is perpendicular to all the lines in the plane that intersect it.

6. Two straight lines perpendicular to the same plane are parallel to each other.

7. If two parallel planes are cut by a third plane, the intersections are parallel.

8. If a line external to a plane is parallel to any line in that plane it is parallel to the plane.

9. If a line is parallel to two intersecting planes, it is parallel to their intersection.

10. A line perpendicular to one of two perpendicular planes is parallel to the other.

11. A plane perpendicular to two intersecting planes is perpendicular to their intersection.

12. The dihedral angle between two planes is measured by the angle of the lines cut from them by a plane perpendicular to their intersection.

13. If a line is perpendicular to a plane, every plane containing the line is perpendicular to the plane.

14. If two planes are perpendicular, a line in one, perpendicular to their intersection, is perpendicular to the other.

15. The angle that a line makes with a plane is the angle that it makes with its projection on that plane.

16. Two lines parallel in space are projected on any plane in parallel lines.

Theorems Relating to Points

9. Theorem I.—The H- and V-projections of a point must lie on a common perpendicular to GL.

Proof.—The H- and V-projectors of the point determine a plane that is perpendicular to both planes, and therefore to GL. (Why?) The intersections of this plane with H and V will then

be perpendicular to *GL* and at the **same point,** which proves the theorem.

In Fig. 9, the point A, in space, is projected on *H* and *V*, in a^H and a^V respectively, so that the projectors and their projections a^Vx and a^Hx form a rectangle. This fact also proves the following theorem:

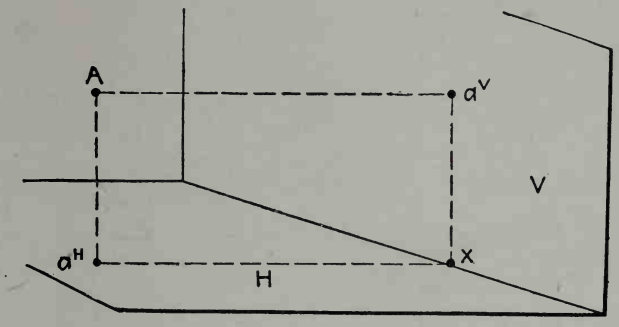


Fig. 9.

Theorem II.—The distance that any point is from H is equal to the distance from its V-projection to GL; the distance that any point is from V is equal to the distance from its H-projection to GL.

PROJECTIONS OF POINTS IN VARIOUS ANGLES

By folding *H* and *V* from their natural positions "into the flat," we find that the projections of points in the different space angles are differently located relatively to *GL*. The first and third angles are opened out, so that the projections of their points lie on opposite sides of *GL*, whereas the second and fourth angles are closed, locating their projections on one side of *GL*.

A study of Fig. 10 will show that the first and second angles are above *H* and on opposite sides of *V*, and that the third and fourth angles are below *H* and on opposite sides of *V*. The first and fourth angles are in front of *V* and the second and third are behind it. In this figure a point is shown in each of the four angles with arrows pointing the rotation of *H* into the flat.

Fig. 10*a* shows the representation of these points on the flat, and the following theorem is the direct deduction from the arrangement of the various projections after their revolution.

Theorem III.—Points located in front of V have their H-projections below GL. Points behind V have their H-projections

above GL. Points above H have their V-projections above GL. Points below H have their V-projections below GL.

Theorem IV.—Points in H have their V-projections in GL, and points in V have their H-projections in GL.

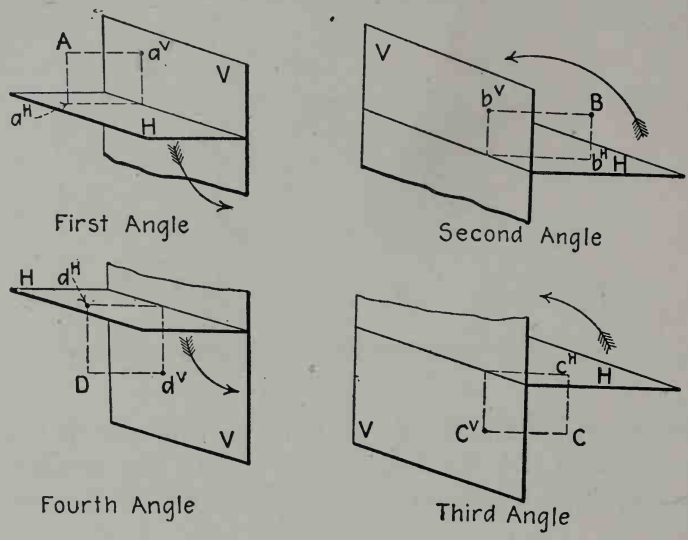


FIG. 10.—The four dihedral angles.

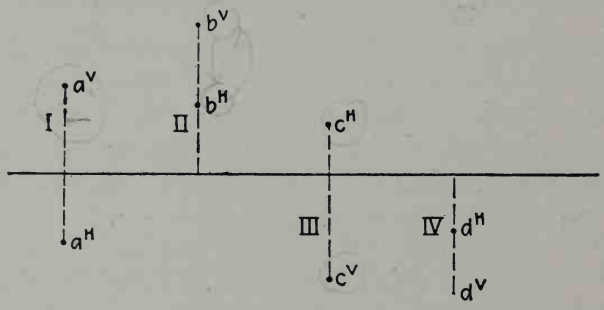


FIG. 10*a*.

Proof.—A point in *H* is zero distance from *H*, hence its *V*-projection is zero distance from *GL*. (Theorem II.)

DESIGNATION AND NOTATION OF POINTS

10. The actual point in space is designated by a capital, A, B, C, etc.

The *V*-projection is designated a^v, b^v, c^v, etc.

The *H*-projection is designated a^H, b^H, c^H, etc.

The *P*-projection is designed a^P, b^P, c^P, etc.

Revolved Positions.—If a point is taken from its original position, and placed in a new one, it is designated by a subscript, as a_1^V, or a_1^H. If there be a second revolution, it becomes a_{11}^V, or a_{11}^H, etc.

NOTATION OF A POINT

There are three items in the notation of a point:

1. The distance from the margin, measured along *GL* to the right.
2. The distance above or below *GL* (+ or −) of its *V*-projection.
3. The distance above or below *GL* (+ or −) of its *H*-projection.

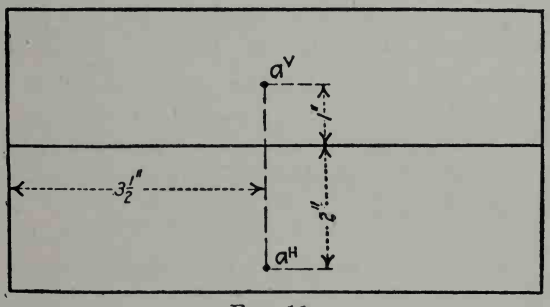


FIG. 11.

Example.—Draw on the flat the projections of A($3\frac{1}{2}$ + 1 − 2).

1. Lay off a point on *GL*, $3\frac{1}{2}$ in. to the right of the margin.
2. Locate a^V 1 in. above (+ 1) this point on *GL*.
3. Locate a^H 2 in. (− 2) below this point on *GL*.

Notes.—1. Plus always means **above GL, Minus** always means **below GL.**

2. First locate the projections of the point on paper, and then try to imagine it in space.

3. The unit for the drawing board is 1 in., and for blackboard work 5 in. is a good scale for the exercises in this book.

LAYOUT FOR DRAWING BOARD

11. A convenient layout is shown in Fig. 12. A sheet 15 × 20 in. is given a $1\frac{1}{2}$-in. margin on the left for binding, and a $\frac{1}{2}$-in. margin on the other three sides, leaving a working space of 14 ×

18 in. This space is divided into six rectangles, each 6 × 7 in., with *GL* drawn through the middle of each.

Inking Plates.—It is not advisable to ink plates, except the border lines, ground lines, and lettering. Good pencil work and neat, legible lettering should be insisted on.

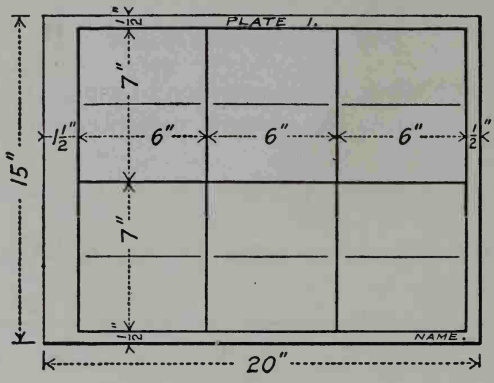


FIG. 12.

Projectors.—The perpendicular lines running from the projections to *GL* are called **projectors** and should be light dotted lines. See Figs. 10 and 11.

12. EXERCISES IN POINT REPRESENTATION

Locate on the flat and designate in space (see the Roman Numerals, etc., in Fig. 10*a*) the following points:

1. A(¾ − 1 + 2), B(1½ + ½ + 2), C(2¼, O + 1), D(3 + 1 − 1), E (3¾ − 2 − ½), F(4½, 0, 0), and G(5¼ + 1½, 0).
2. K(¾ + ½ − 2), L(1½ − 1 − 2), M(2¼ − 1½, 0), N(3 − 1 +2), O(3¾ + 2 + 1), P(4½ + 1, 0), Q(5¼, 0 + 1).
3. A(¾, 1 behind *V*, 2 below *H*), B(1½, in *H*, 2 before *V*), C(2¼, 1 before *V*, 1 above *H*), D(3, 1 behind *V*, 2 above *H*), E(3¾, in *GL*), F(4½, 1 before *V*, 2 above *H*), G(5¼, in *V*, 2 below *H*).
4. K(¾, in *H*, 2 behind *V*), L(1½, 1 before *V*, 2 below *H*), M(2¼, 1 behind *V*, 1½ above *H*), N(3, 2 before *V*, 1 above *H*), O(3¾ in *GL*), P(4½, 1 behind *V*, 2 below *H*), Q(5¼, 1½ above *H*, in *V*).

Locate in the flat the following points:

5. A(in *IV*, 1 to right, 1 from *V*, 2 from *H*),
   B(in *I*, 2 to right, 2 from *V*, 1 from *H*),
   C(in *II*, 3 to right, 1½ from *V*, 1½ from *H*).
6. D(in *H*, 1 to right, 1 behind *V*),
   E(in *III*, 2 to right, 2 from *V*, 1 from *H*),
   F(in *I*, 3 to right, 1 from *V*, 1½ from *H*).
7. G(in *II*, 1 to right, 1½ from *H*, 1½ from *V*).

K(in *V*, between *III* and *IV*, 2 to right, 2 from *H*),
L(in *H*, between *I* and *IV*, 3 to right, 1 from *V*).

8. M(in *IV*, 1 to right, 2 from *H*, 2 from *V*),
N(in *III*, 2 to right, 1 from *V*, 1½ from *H*),
O(in *H*, between *II* and *III*, 3 to right, 2 from *V*).

LINES

13. A straight line is the path of a point that moves constantly in one direction. Any two points therefore of a straight line are sufficient to determine it. Also, the projection of that line will contain the projections of the points. Until the subject of curved lines is taken up the term "line" will be understood to mean a straight line.

Theorem V.—Two projections of a line will, in general, determine the position of that line in space.

Proof.—Pass a plane through the *V*-projection of some line, and make it perpendicular to *V*. Also pass a plane through the *H*-projection of the same line and make the plane perpendicular to *H*. These planes will intersect in a line, and that line must be the given line.

Theorem VI.—If a line is parallel to H, its H-projection is parallel and equal to the line, and its V-projection is parallel to GL.

Prove This.—What would be the statement if the line were parallel to *V*?

Theorem VII.—If a line is parallel to H and V, both of its projections will be parallel to GL.

Theorem VIII.—If a line is perpendicular to H, its H-projection will be a point, and its V-projection will be perpendicular to GL.

What would be the statement for a line perpendicular to *V*?

Theorem IX.—If a line is oblique to both H and V, both projections will be oblique to GL, except when the line lies in a profile plane.

14. PROJECTIONS ON THE PROFILE PLANE

The projections of the solid shown in Fig. 13 do not completely identify the object, because the ends (bases) lie in a profile plane.

The end view (sometimes called the right section) might be one of many forms and in different positions, as the shapes given in Fig. 14. Thus the profile is sometimes necessary to complete the drawing.

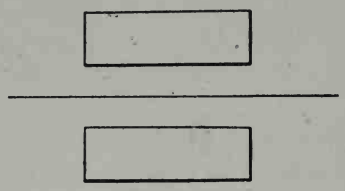


FIG. 13.

In drawing views in profile, the right end view is usually taken, although sometimes both profiles are necessary, and some cases arise where only the left view is needed. Taking the view at the right, we see the four dihedral angles as shown in Fig. 15. Looking along *GL* we see *H* and *V* in their true relation (perpendicular), and the four space angles as indicated by the Roman Numerals.

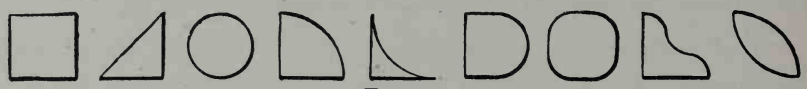


FIG. 14.

Assume *P* at any convenient location on the right. In the regular views, m^V gives the elevation and m^H the distance in front of *V*. In the profile view it will be seen that the same distances obtain. If the point lies in any other angle, the pro-

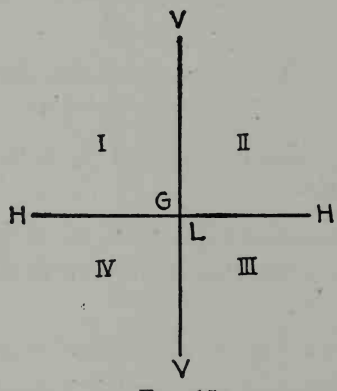


FIG. 15.

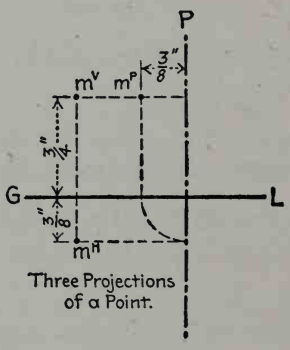


FIG. 15*a*.

cedure is the same. The rotation of the *H*-projection will determine whether the profile of the point is to lie to the right or left of *P*. If the *H*-projection is below *GL*, the profile will be on the left, as in Fig. 15*a*, and if it be above *GL*, the profile will be on the right. The elevation is always the same on both *V* and *P*.

Rule.—Project from the V-projection horizontally; project from the H-projection to P, and rotate to GL, thence up or down until the projectors intersect.

To obtain the profile of a line or plane figure, project the defining points and join them with lines. Fig. 16 gives a pictorial view of a line and its projections. Fig. 16*a* shows the same line in the flat.

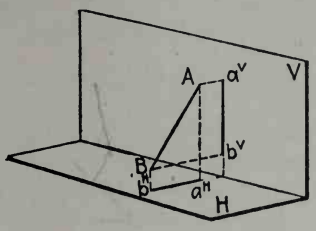


FIG. 16.

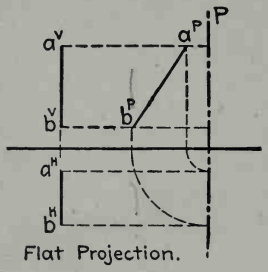


FIG. 16*a*.

Theorem X.—If a line lies in V, its V-projection is the line itself, and its H-projection is in GL.

Why? What would be the statement for a line in *H*?

Theorem XI.—If a point, line, or plane figure lies in P, its V-projection is in the V-trace of P, and its H-projection is in the H-trace of P.

15. EXERCISES

9. Draw the line A(1 + 2 − 2½) B(4 − 2 + ½), and tell where it travels.
10. Draw the line C(1½ − 2 + 1½) D(4½ + 3 − 1), and tell where it travels.
11. Draw and designate the lines: E(1 + 1 − 1) F(3 + 2 − 3), G(3½ + 1 + 1½) K(4½ + 2 − ½), L(5 − 1 − 2) M(5¾ − 3 − 1).
12. Draw and designate the lines: A(1 − 2 + 2) B(2½ − ½ + 1), C(3 + 1 + 1) D(4 + 2 + 2), E(4½ − 1 − 3) F(5½ + 1 + 1).

Draw and designate the lines in the following:

13. K(1 + 1 − 1) L(3 + 3 − 1), M(2½ − 2 + 2) N(4 − 2 + 1), and O(4 + 2 − 1) P(5½ + 3 + 3).
14. A(1 + 3 + ½) B(3 + 1 + ½), C(2 − 3 − 2) D(4 − 3 − 1), E(4½ + 2 − 1) F(4½ + 2 − 3), and G(5½ − 1 + 1) K(5½ − 1 + 3).
15. K(½ − 1 + 1) L(2½ − 3 + 1), M(2 − ½ − 1) N(4 − ½ − 1), and O(3 + 2 + 1) P(5 + 2 + 3).

Tell what angles these lines traverse:

16. A(1 + 2 − 1) B(3 − 1 − 3), C(2 + 3 + 1) D(4 − 2 − 2), and E(4 − 1 + 2) F(5½ + 2 − 2).
17. G(1 − 1 − 3) K(2 + 2 + 3), M(2 − 3 + 1) N(3½ − 1 − 1), and O(4 + 2 − 1) P(5½ + 1 + 3).

18. From A(1 − 1 + 2) draw a line 2 in. long, parallel to *V*, and inclined 60° to *H*, downward. Does it touch *H*? *V*? Draw one 60° upward, 2½ in. long. Where does it terminate?
19. From B(2 − 2 − 1) draw a line ‖ to *H*, inclining 30° to *V*. How far is it on this line from B to *V*?
20. Draw the *P*-projections of the lines A(1 + 1 − 2) B(1 + 3 − 1) and C(2 − 1 + 1) D(2 − 2 + 1). Locate the profile at 4½ on *GL*.
21. Draw the *P*-projections of E(1 + 1 + 1) F(1 − 1 − 2) and G(2 + 2 + 1) K(2 − 1 + 1). Profile 4½.
22. Draw the triangle L(2 + 1 − 1) M(2 + 2 + ½) N(2 − 1 − 1½) in its true size. Profile 4½.
23. What are the sizes of the angles and sides of the triangle K(2 + 1 + 1) O(2 − 1 + 1½) P(2 − ½ − 2)?
24. Draw the projections of a prism parallel to *H* and *V*, of 2-in. altitude, and its base a pentagon inscribed in a 1½-in. circle in *P*.
25. From M(1 − 2 − 1) draw a line ‖ to *H*, inclined 45° to *V*, 3 in. long, toward *V* to the right. Also from M draw a line ‖ to *P*, 30° to *V*, 3 in. long. Where do they terminate?

REPRESENTATION OF PLANES

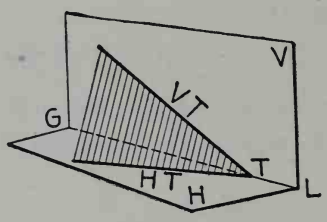


FIG. 17.

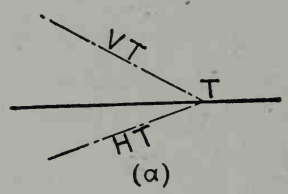


FIG. 17*a*.

16. A plane other than *H*, *V*, or *P* is shown by its **traces,** as its intersections with the three planes are called. In most cases the *H*- and *V*-traces are sufficient, and the *P*-trace is chiefly used as an auxiliary to assist in the solving of problems. Fig. 17 shows an oblique plane, T. The plane T cuts *V* in the line VT, called its **vertical trace.** It cuts *H* in the line HT, called its **horizontal trace.** Fig. 17*a* shows the flat representation of the plane T. VT is one line of T and HT is an entirely different one.

Theorem XII.—The two traces of a plane completely determine the plane.

Note.—The student must on no account think that VT and HT are projections of the same line. VT lies in *V*, hence its *H*-projection is in *GL*. Therefore, if a point is assumed in the plane with its *V*-projection in VT, its *H*-projection is in *GL*. What is the case then with HT?

Theorem XIII.—The two traces of any plane must both be parallel to GL, or they must intersect GL and in one point.

Let the student prove this.

Note.—Planes are unlimited in extent, and the traces, therefore, do not end in *GL*, but continue indefinitely. Every plane, therefore, that is oblique to *GL* occupies every space angle.

Theorem XIV.—If a plane is parallel to V, it has but one trace, the H-trace, parallel to GL. What would be true of a plane parallel to *H*?

Theorem XV.—The traces of P are both perpendicular to GL.

Theorem XVI.—A plane perpendicular to H, but oblique to V, has its V-trace perpendicular to GL. For example, see a swinging door as it takes various positions. Such a plane is called an *H*-projecting plane, for the reason that all points in it are projected on *H* in its *H*-trace. What would be true of a plane perpendicular to *V*? See a box lid.

Theorem XVII.—If a plane passes through GL, it can only be satisfactorily shown by its P-trace. Both *H*- and *V*-traces are in *GL*, therefore it requires a point, or the *P*-trace, to determine it. It is a *P*-projecting plane. Why?

Theorem XVIII.—Two parallel planes have parallel traces on any plane of projection. Give a reason for this from Solid Geometry. Is the converse always true? Take two intersecting planes parallel to *GL*.

Theorem XIX.—Two planes perpendicular to each other do not in general have their traces respectively perpendicular. The only exception to be found is the case of a profile plane and one parallel to *GL*. Two perpendicular planes that are perpendicular to *H* will have their *H*-traces perpendicular, but their *V*-traces will be parallel.

NOTATION OF PLANES

17. Planes are represented by their traces and are completely determined by them.

The conventional dot-and-dash line is sufficient to identify them in a drawing.

They are lettered VT, HT, PT, VS, HS, PS, etc., to distinguish the *V*-, *H*-, and *P*-traces of T, S, etc.

NOTATION

When the traces intersect in the problem limits, three points are necessary:

1. In the *V*-trace.
2. The *GL* intersection.
3. In the *H*-trace.

All these points are in *H* or *V*, hence there are zeros in the complete notation. These zeros are understood and omitted in our notation, as being cumbersome.

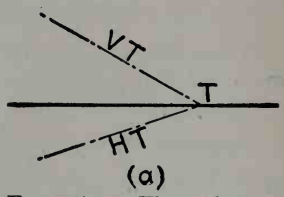


FIG. 17*a*.—The plane T (1 + 2) 5(1 − 1).

1. (1 + 2) means a point in VT, 1 in. to right, 2 in. up.
2. 5 means the *GL* intersection, 5 in. to right.
3. (1 − 1) means a point in HT, 1 in. to right, 1 in. down.

Rule.—*First.*—A point in the *V*-trace. *Second.*—The *GL* intersection. *Third.*—A point in the *H-trace.*

SPECIAL CASES

Planes parallel to GL have no *GL* intersection and their traces are parallel to *GL*, and their notation is shown in Fig. 18.

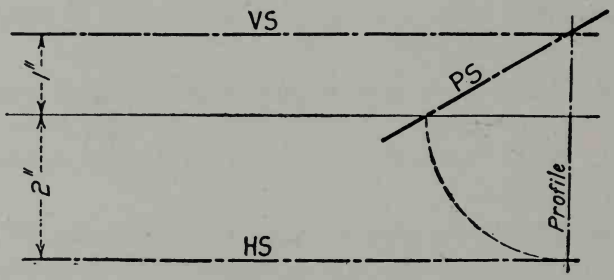


FIG. 18.—The plane S(+ 1) ∞ (− 2).

Fig. 18 shows the plane S and its profile trace, which is frequently necessary with such planes.

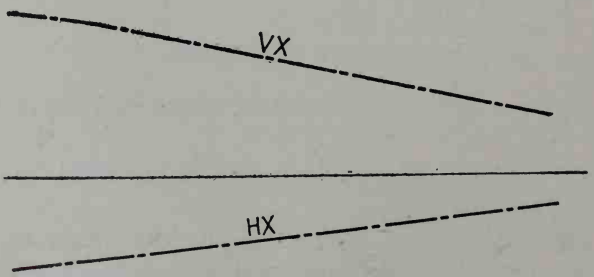


FIG. 19.—The plane X(1 + 3)(5 + 2)(1 − 1½)(5 − 1).

Planes parallel to H or V have but one trace, and that parallel to *GL*. They require special designation, but are seldom used except as auxiliary planes, and so it is not necessary to devise a special notation for them.

OTHER PLANES

A plane oblique to GL, but not crossing it within the problem limits, must have four points to determine the two traces.

In this plane the first two points are in the *V*-trace and the second pair are in the *H*-trace. See Fig. 19.

A plane through *GL* has its *V*- and *H*-traces in *GL*, therefore its *P*-trace is its best exposition. It can be described also as a plane passing through *GL* making a certain angle with the horizontal, or containing some point.

EXERCISES IN REPRESENTATION OF PLANES

18. Draw the traces of the following planes, and state their relations (parallel, perpendicular, oblique) to *H*, *V* and *P*.

Draw the traces with a dot-and-dash line. In Ex. 28 and 29, place *P* at 4½, and draw the *P*-traces.

26. T(3 + 2) 1(1 − 3), S(5 + 3) 3(3 − 3).
27. X(3 + 3) 1(3 + 1), Y(3½ + 2) 5(4 − 3).
28. R(+ 1) ∞ (+ 2), Q(− 1) ∞ (+ 2). Draw the *P*-traces.
29. S(− 2) ∞ (− 1), T(+ 2) ∞ (− 1). Draw the *P*-traces.
30. X(1 + 1)(4 + 2) (1 + 1½) (4 + 3), U(2 − 3) 4(5½ + 2¼).
31. Draw the traces of the *V*-projecting plane of the line M(1 + 2 − ½) N(2½ + ½ − 1), and of the *H*-projecting plane of the line A(3½ − ½ − 2) B(5 + 2 + ½).
32. Draw the traces of the *V*-projecting plane of C(1 + 3 − 1) D(3 − 1 + 3) and of the *H*-projecting plane of E(3½ + 1 − 2) F(5 + 1 − 1).
33. Draw the traces of the *H*-projecting plane of G(3 + 3 − 1) K(1 − 1 + 1), and of the *V*-projecting plane of M(3½ + 1 − 2) N(5½ + 3 + ½).
34. Draw the *P*-trace of S, which contains *GL* and O(2 + 2 − 1), and of R, which contains *GL* and A(4 + 1 + 1½).
35. Draw the *P*-traces of two planes, T and X, each containing *GL*, and each inclined 60° from the horizontal, T passing through *I* and *III*, X passing through *II* and *IV*.

Note.—To find the *P*-trace of any plane, take the plane *P* at any convenient place, where it will intersect the *H*- and *V*-traces. Then treat the line connecting these intersections (the *P*-trace) as you would any other profile line, and revolve it as in Art. 14.

THEOREMS RELATING TO LINES AND PLANES

19. Theorem XX.—Two lines parallel in space have parallel projections on any plane.

Theorem XXI.—Two lines perpendicular in space do not ordinarily have their projections perpendicular.

Special Case.—If one of these lines is parallel to H, the H-projections of the two lines will be perpendicular.

Fig. 20 shows a line AB, parallel to H, and BC (any one of four different lines), whose H-projection is perpendicular to a^Hb^H. Any one of the lines BC will be perpendicular to AB.

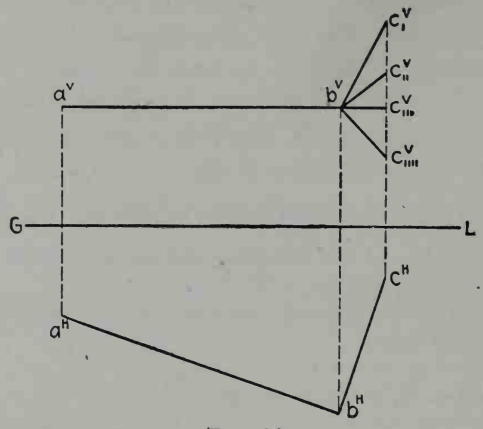


FIG. 20.

Proof.—If AB is parallel to H, any plane perpendicular to AB will be perpendicular to H, and will therefore be a projecting plane on H of all lines contained in it. Let the student finish the reasoning to show that all lines, whose H-projections are b^Hc^H, will be perpendicular to AB.

Theorem XXII.—Two intersecting lines in space must have their H- and V-projections intersect in points in a line perpendicular to GL. Prove.

Theorem XXIII.—Two intersecting lines, both parallel to H, are projected on H in lines parallel and equal to themselves, and the included angle will be equal in projection to the angle in space. Prove.

Theorem XXIV.—A line, that is parallel to an oblique plane, is not, in general, projected parallel to the traces. The exception is only possible when the traces are both parallel or both perpendicular to *GL*.

Theorem XXV.—A line, that is perpendicular to a plane, always has its projections perpendicular to the respective traces of the plane.

In Fig. 21 let BC be perpendicular to T To prove that bC is perpendicular to tt, the *H*-trace of T.

Let fall a perpendicular, Bb, from B to *H*. The lines BC and Bb determine a plane perpendicular to both *H* and *T*, and therefore to their intersection, tt. As the plane is perpendicular to tt, its intersection bC will be; that is, the *H*-projection is

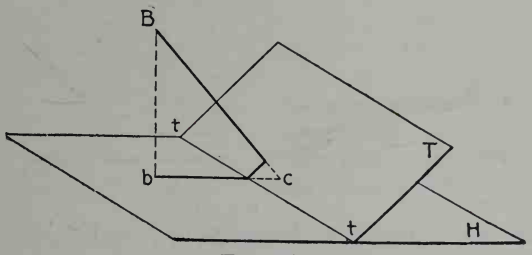


Fig. 21.

perpendicular to the *H*-trace. The same could be proved for the *V*-projection and *V*-trace.

Theorem XXVI.—Conversely, a line, whose projections are perpendicular to the respective traces of a plane, is perpendicular to the plane, with one exception.

Exception.—If the plane is parallel to *GL*, the line, whose projections are perpendicular to its traces, may not be perpendicular to the plane. Prove this by the *P*-trace of such a plane, and the *P*-projection of such a line.

Theorem XXVII.—A line in any plane pierces H in the H-trace of the plane, unless the line is parallel to H. Similarly, it will pierce *V* in the *V*-trace of the plane in which it lies. Prove.

Theorem XXVIII.—If a line is parallel to H, it is parallel to the H-trace of any plane in which it lies. Similarly for *V*. Prove.

Theorem XXIX.—Any plane that is passed through one of two parallel lines either contains or is parallel to the other. Let the student prove this with a figure. Refer to Plane and Solid Geometry theorems.

EXERCISES IN REVIEW

20. Directions.—All the exercises here given can be drawn in a space 6 in. × 7 in., with *GL* drawn horizontally across the middle.

Letter all points carefully, using small letters for all projections, a^V, b^V, etc., for *V*-projections, a^H, b^H, etc., for *H*-projections, a^P, b^P, for *P*-projections.

Use the following conventions for all lines etc.

Given lines—solid, medium,	1/64 in.
Required lines—solid, heavy	1/32 in.
Auxiliary lines—solid, light	very fine.
Invisible lines—dotted, medium	1/64 in.
Projectors—dotted, light	very fine.
Paths of Revolution—the same.	
Traces of Given planes—medium	1/64 in.
Traces of Required planes—heavy	1/32 in.
Traces of Auxiliary planes—light	very fine.

All planes are considered transparent, and there will be no invisible lines until problems involving solids are given. ∥ and ⊥ are abbreviations for parallel and perpendicular.

36. Through A(5 − 1 + 2) draw AB ∥ to C(4 + 2 − 2) D(2 + 1 − ½).
37. Through M(4 + 2 + 1) draw MN ∥ and equal to O(3 − 1 − 2) P(1 + ½ − ½).
38. From A(4½ − 2 + 1) draw two lines intersecting B(1 + 1 − 1) C(3 + 3 − 2) in points other than B and C.
39. Draw two lines EF and KL intersecting in the point D(3 − 1 + 1). Connect these two lines with a line ∥ to *H*.
40. Draw two lines MN and OP intersecting in the point K(2½ + 3 − 1). Connect these two lines with a line ∥ to *V*.
41. Draw a line ⊥ to *V* intersecting A(1 + ½ − 2) B(4 + 3 − 1) and C(1½ + 2 − ½) D(3½ + 1 − 1).
42. Draw a line ⊥ to *H* intersecting E(1½ − 3 + 2) F(5 − 1 + 3) and G(1½ − 1 + 2½) K(4 − 2 + 1).
43. Draw four lines at right angles to L(1 + 2 − 1) M(3 + ½ − 1) through the point M.
44. Through N(2 − 1 + 2) draw a line ⊥ to T(1 − 3) 5(3 + 3).
45. Through O(5 + 1 − 1) draw a line ⊥ to S(5 + 3) 3(1 − 3).
46. Through P(2, 0, 0) draw the traces of a plane R that is ⊥ to A(2 − 2 + 2) B(4 − ½ + 1).
47. Is C(2 + 2 − 1) D(2 + ½ − 2) ⊥ to the plane Q(+ 3) ∞(− 1)? If not, draw the traces of T, that is ⊥ to CD.

Draw the *P*-projections of the following points and lines, locating *P* 4½ in. to the right:

48. A(1 + 2 + 1), B(1½ + 2 − 1), C(2 − 1 − 2), D(2½ − 2 + 1), and E(3, 0 + 1).

49. The lines F(1 + 3 + 1) G(1 − 1 + 1) and K(2 + 1 − 1) G(2 − 2 − 1). What relation are these lines to H and V?
50. M(1 + 2 − 2) N(1, 0 − 1). What angles does this line make with H and V?
51. Through O(2 − 3 + 1) draw a 3-in. line ‖ to P, 45° to H. In what space angle does the line terminate?
52. Let A(3, 0 + 1½) B(4½, 0 + 1) be the diagonal of a square in H. With this square as the upper base of a cube, draw the projections of a cube. Draw the traces of the six planes bounding the cube.

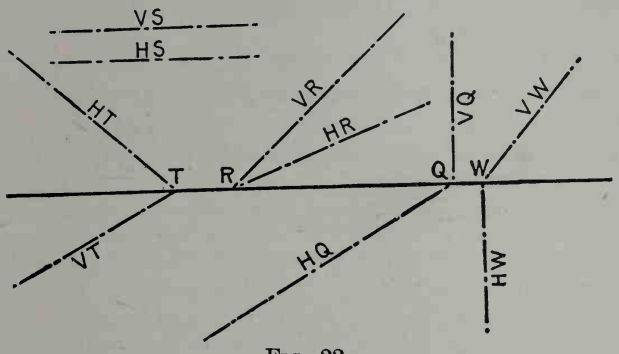


FIG. 22.

53. Draw the three projections of a hexagonal prism in I, its center line ‖ to H and V, with a 1¼-in. base and 2-in. altitude. Draw the H- and V-traces of the plane of one of its oblique faces.
54. Draw the prism given in Ex. 53 in *III*, and draw the H- and V-traces of the plane of one of its oblique faces.
55. Let C(2, 0 − 1) D(3½, 0 − 2) be the base-diagonal of a hexagonal prism of 2-in. altitude, standing on H. Draw the traces of the six face planes of the prism.
56. Write the notation of the planes in Fig. 22, and describe them. Use your own dimensions.

CHAPTER II

PROBLEMS RELATING TO POINTS, LINES, AND PLANES

Introductory

21. In the solution of problems that compose this chapter, we (1) analyze the operation in space, (2) offer proof of the correctness of the analysis, when it seems necessary, and (3) show the construction of a typical example of the problem "on the flat." The analysis should be learned, not by memorizing, but by visualizing the situation and process in space.

Descriptive Geometry, as the science of drafting, is intended to solve drafting difficulties. Drafting presents very few difficulties when objects are in their "natural position;" that is, when the lines, plane figures, and solids are parallel to H or V. It is only when the objects are in oblique positions, that we find their relations difficult. These difficulties show themselves in distortions, called "foreshortening."

The obvious general solution, therefore, is to bring the objects into their simplest relations (parallelism), and this is done by one of two methods. The first method might be called the "draftsman's method," and is equivalent to the changing of position by the observer in order to bring the objects into their desired relation. The second might be termed the "mathematician's method," and consists in rotating the objects, or in securing the results by means of a series of operations, such as passing planes, making intersections, rotating, and the like, which finally achieve the solution. Both methods have their advantages, and both are presented in many cases, but where the superiority of either is conspicuous, that method only is offered.

It has been shown that certain situations can only be represented by the use of a third plane of projection, the Profile, perpendicular to both H and V. In a like manner, there are certain solutions of this *type*, but different in situation, that can best be represented by projecting on a plane that is perpendicular to H or V, but not to both. This method, the draftsman's method, is merely a variant on profile projection, and is performed in the

same way. It requires no analysis, hence only the construction is given, when it is employed.

FUNDAMENTAL PROBLEMS

22. There are six problems that depend on no other previous problems for their solution and are themselves necessary to the solution of succeeding problems. To the complete mastery of these, then, it is evident that success is due. These fundamentals are:

Problem I. To project a line on an auxiliary plane perpendicular to *H* or *V*.
Problem II. To find the trace of an oblique plane on an auxiliary plane.
Problem III. To find the *H*- and *V*-piercing points of a line.
Problem IX. To revolve a point about an axis perpendicular to *H* or *V*.
Problem XII. To revolve a point about a line in *H* or *V* into that plane.
Problem XVII. To find the intersection of two planes.

AUXILIARY PLANES OF PROJECTION

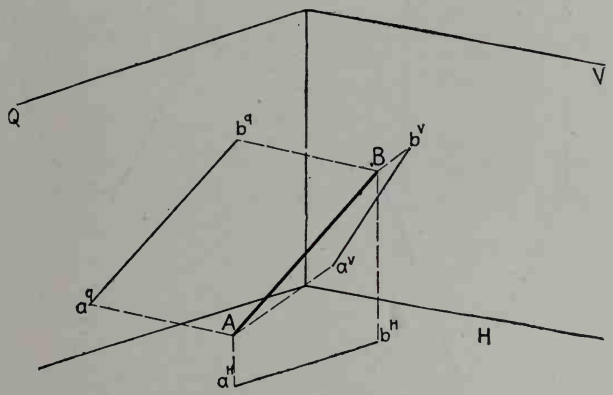


FIG. 23.—Showing *H*, *V*, and *Q* planes.

23. By projecting an oblique line on a plane parallel to it and perpendicular to *H*, the true length of the line and its true angle with *H* are shown by this projection, or if the plane were perpendicular to *V*, its true relation to *V* is shown in the same way. We designate these planes as follows:

Q is the plane perpendicular to *H* and oblique to *V*.
R is the plane perpendicular to *V* and oblique to *H*.

If a second removal is required:

X is the plane perpendicular to Q and oblique to *H*.
Y is the plane perpendicular to R and oblique to *V*.

The accompanying figure shows the projection of a line on Q; note that the Q-projections of the two points, A and B, are the same in elevation as their *V*-projections. This fact is the chief instrument.

FUNDAMENTAL PROBLEM I

24. Problem 1.—To project an oblique line on an auxiliary plane parallel to the given line, and perpendicular to H or V.

Let AB (Fig. 24) be the given line.

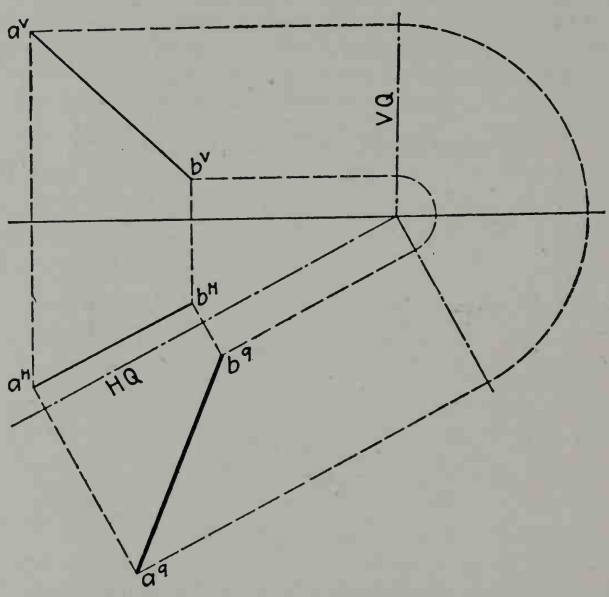


FIG. 24.

Construction.—1. Draw HQ parallel to a^Hb^H. (VQ is drawn here, but is not necessary in this problem.)

2. Run projectors from a^H and b^H perpendicular to HQ.

3. Locate a^Q on its projector the same distance from HQ that a^V is from *GL*. Locate b^Q similarly.

Conclusion.—The line a^Qb^Q is the projection of AB on Q. (Let the student give the full explanation.)

Corollary I.—The Q-projection of AB is the **true length** of the line. Why?

Corollary II.—The angle between $a^Q b^Q$ and HQ is the **true angle** between AB and H. Why?

Note.—When the other trace of Q or R is not needed, it will be omitted from figures hereafter.

25. Supplementary Problem.—To lay off a given distance on a given line from a given point.

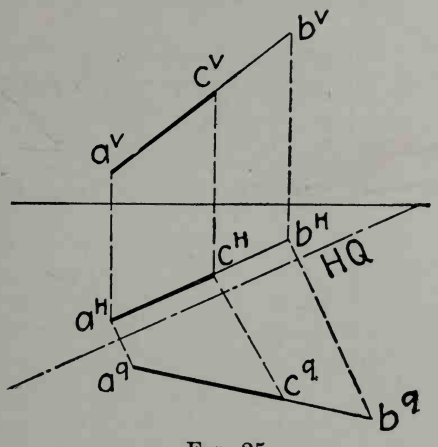


FIG. 25.

Required.—To lay off 1 in. on AB (Fig. 25) from A toward B.

Constructions.—1. Obtain the Q-projection of AB.

2. Lay off $a^Q c^Q$ equal to 1 in.

3. Project c^Q back to c^H, thence to c^V.

Conclusion.—AC is 1 in. long.

EXERCISES

26. Note.—Exercises throughout this book are presented in two ways, with dimensioned coördinates, and graphically. The various sets are not exact duplicates. The graphic "layouts" are not always given dimensions, that being left to the student or instructor.

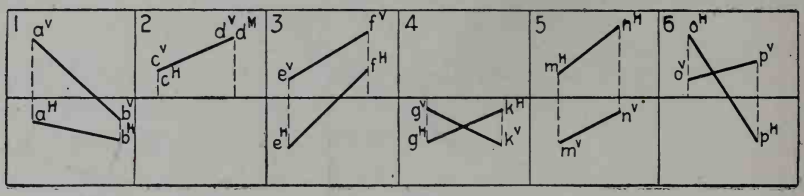
Project on Q the following lines and measure their true length and their angle with the horizontal.

57. A(2 + 2 − 1) B(4 − ½ − 2).
58. C(1 + 1 + 1) D(2 + 3 + 3).
59. E(5 + 1 + 2) F(3 − 3 + ½).
60. G(2 − 1 − 2) K(4 − 2 − 1).
61. L(1 − 3 + 1) M(3 − 1½ + 3)
62. O(4 + 2 − 3) P(2 + 1 + 2).

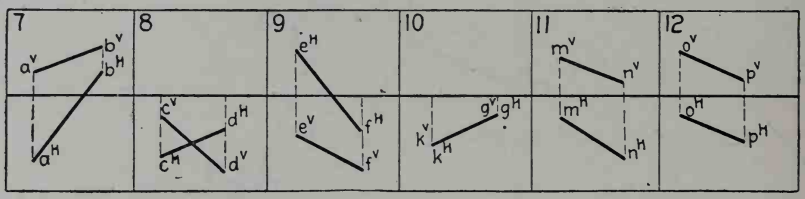
Project on R the following lines, and measure their true lengths and inclination from the vertical.

63. A(4 + 2 + 1) B(2 + 1 − 2).
64. C(1 − 1 − 2) D(3 − 2½ − 1).
65. E(2 − 1 + 2) F(3 − 2½ + 1).
66. G(3 − 3 − 3) K(5 − 1 − 1).
67. L(2 + 2 − ½) M(3½ + 1 − 3).
68. N(2 + 1 − 1) O(4 − 1 − 3).

Lay off on the above lines a distance of —— in. (amount specified by instructor) from the first point toward the second.



Graphic Layout No. 1.—Make Q-projections of these lines. Dimensions to suit.



Graphic Layout No. 2.—(*a*) Make R-projections of these lines. (*b*) Lay off on the above lines a distance of —— in., from the first point toward the second.

Note.—The requirements for these layouts may be transposed, if desired.

FUNDAMENTAL PROBLEM II

27. Problem 2.—To find the Q-trace or R-trace of a plane oblique to H and V.

Required, to find the Q-trace of the plane T in Fig. 26.

Construction.—1. Draw HQ perpendicular to HT, and VQ perpendicular to *GL*.

2. Construct a right triangle on HQ as a base, making the altitude equal to that part of VQ included between *GL* and VT.

Note.—This is equivalent to revolving into *H* the right triangle cut by *Q* from *H*, *V* and T.

Conclusion.—The hypothenuse of this triangle is the Q-trace of T, hereafter written QT.

Note.—Thus T is a projecting plane on Q, and all points in T are projected on Q in QT. This makes it a simple instrument for all problems relating to points in T.

28. Supplementary Problems.—To find the dihedral angle between an oblique plane and a plane of projection.

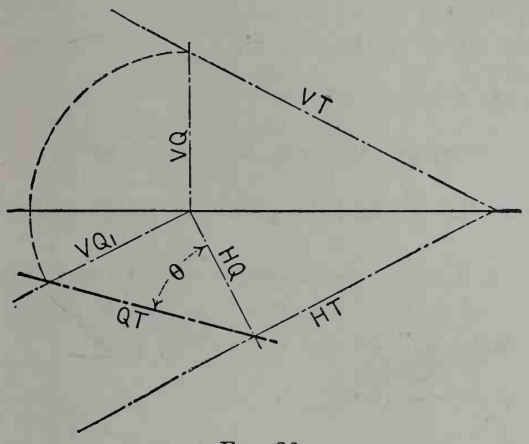
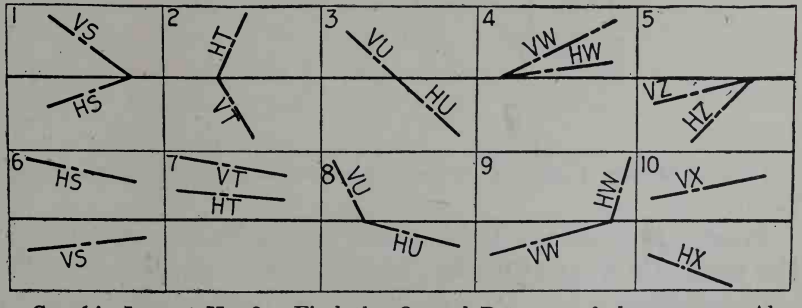


FIG. 26.

Conclusion.—The angle θ in Fig. 26 is the true measure of the angle between T and H. If the R-trace of T had been found, it would have yielded the angle between T and V.

Proof.—The plane Q is perpendicular to the intersection of T and H, therefore the angle between HQ and QT is the measure of the dihedral angle. R does the same for T and V.

29. EXERCISES



Graphic Layout No. 3.—Find the Q- and R-traces of plane ——. Also measure the dihedral angles between the planes and H and V, in degrees.

Find the Q-trace and R-trace of the various planes given here. Also measure the dihedral angle in degrees that each of these planes makes with H and V.

69. S(1 + 3) 4(1 − 2).

70. T(5 − 1) 2(5 + 2).

71. U(2 + 2) 3½(5 − 2).
72. W(3 − 1) 5(3 − 3).
73. Z(4 + 3) 1½(4 + 1½).
74. S(1 + 3) (5 + 2) (1 − 2¼) (5 − 1½).
75. T(5 − 3) (1 − 2) (5 + 1½) (1 + 1).
76. U(1 + 3) 2½(5 − 1).
77. W(1 − 1) 3(5 + 2½).

FUNDAMENTAL PROBLEM III

30. Problem 3.—To find the points in which a line pierces H and V.

Analysis.—1. Extend the *V*-projection of the line until it intersects *GL*, and project this point to the *H*-projection. The point thus found will be the *H*-piercing point.

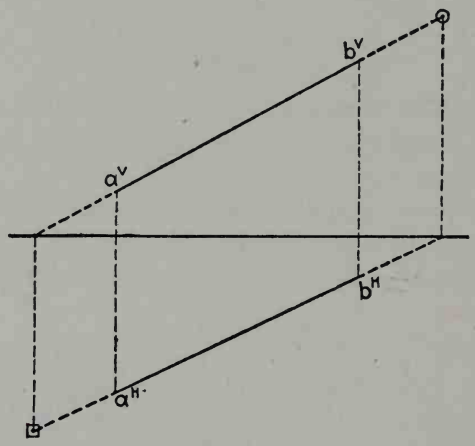


FIG. 27.

Interchanging *V* and *H* in the analysis will yield the *V*-piercing point.

Proof.—All points in *H* have their *V*-projections in *GL*, hence this point is in *H*, and therefore where the line pierces *H*.

Construction.—Let AB (Fig. 27) be any oblique line.

1. Extend a^Vb^V to *GL*.
2. Project the intersection to ⊡ on a^Hb^H, or its extension. (⊡ is the symbol hereafter used for the *H*-piercing point.)
3. Extend a^Hb^H to *GL*.
4. Project the intersection to ⊙ on a^Vb^V, or its extension. (⊙ is the symbol for *V*-piercing point.)

Axioms.—A line parallel to V has no ⊙, hence the H-projection of such a line is parallel to GL.

A line parallel to H has no ⊡, hence the V-projection of such a line is parallel to GL.

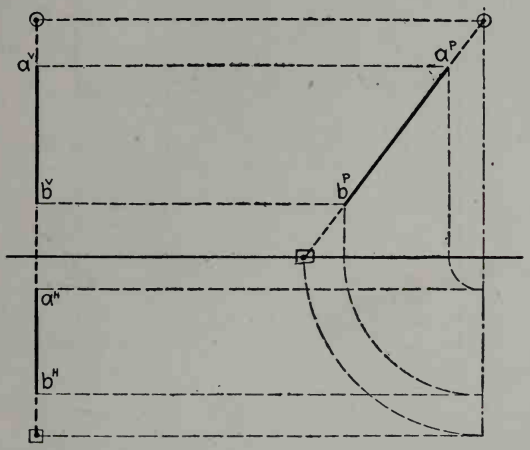


FIG. 28.—H- and V-piercing points of lines located in P-planes.

Construction.—An examination of Figs. 28 and 28*a* will be sufficient to show the operation in this case.

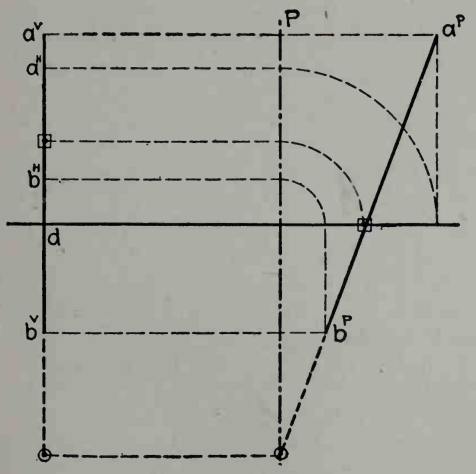


FIG. 28*a*.

31. Supplementary Problem.—To find the point where a line pierces a profile plane, and show it in P-projection.

Construction.—Let AB (Fig. 28*b*) be the given line. Required its *P*-piercing point. Locate *P* at any suitable point.

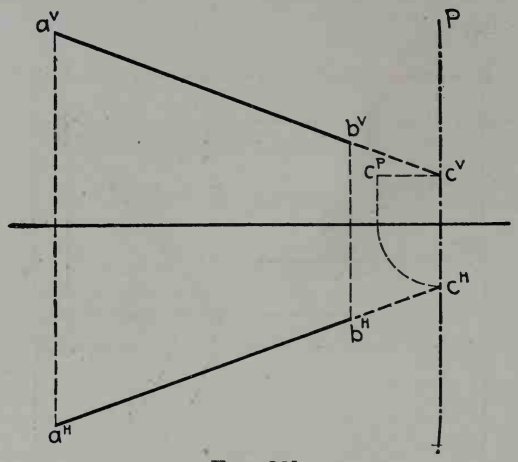


FIG. 28*b*.

1. Extend a^Vb^V to the *P*-trace at c^V.
2. Extend a^Hb^H to the *P*-trace at c^H. C is the piercing point.
3. Revolve *P* into *V*, locating c^P.

EXERCISES

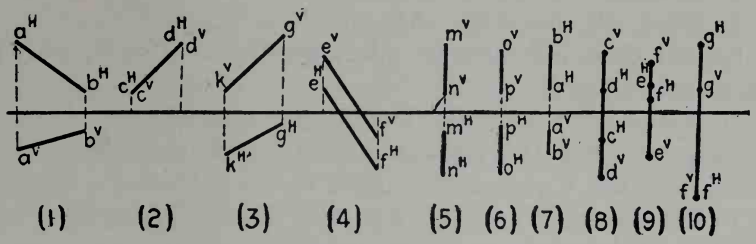
32. Locate the *H*- and *V*-piercing points of the following lines, and tell what angles they traverse. If *P* is required, place it at 4½ in. from the margin.

78. A(1 + 2 − ½) B(2½ + ½ − 2½), C(3½ + ½ + 2) D(5 + 1½ + ¼).
79. E(1 − ½ − 1¾) F(3 − 2 − ¼), G(3½ + 2 + 1) K(5 − 1 − 2).
80. M(1 + 2 + 2) N(2½ − 1 − 1), O(3 − 1 + 2) P(5 − 2 + ½).
81. A(1 + 2 + 1) B(3 + ½ − 2), C(3 + 2 + 1) D(4 + 1 + ½).
82. E(2 − ½ − 1¼) F(3 − 1½ − 2), G(3 − 1 + 2) K(5 − 3 − 1).
83. M(1 + 2 − 1½) N(1 + 1 − ½), O(2 − 2 − 2½) P(2 − 1 − ½).
84. A(1 + 2 + ½) B(1 + ¾ + 1½), C(2 − 2 + ½) D(2 − 1 − ½).
85. E(1 − 1 − 2) F(1 + 2½ + ½), G(2 − 2½ − ½) K(2 + ½ + 1).
86. M(1 + 2 − 2) N(1 − 1 − ½), O(2 + 2 + 1¼) P(2 − 1½ + ½).
87. A(1 + 2 − ½) B(1 − ½ − 1), C(2 + ½ − 2) D(2 − 2 + ½).
88. E(1 + 2 − 1) F(1 − 1 + 1½), G(2 − 2 − 1) K(2 + 1 + 1).

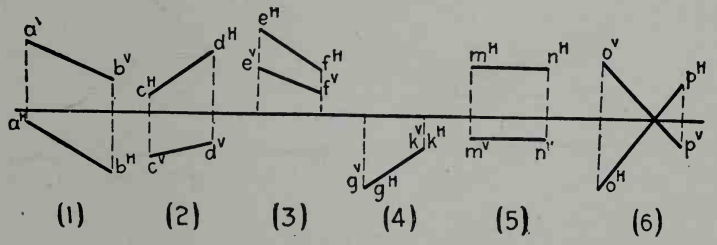
Find the *P*-piercing points of the following lines, locating *P* 4½ in. to the right.

89. A(2 + ½ + 2) B(3½ − 1 + ½).
90. C(2 + 1¼ − 1) D(3½ + 2½ − ¼).
91. E(1½ + 1 + 1½) F(3½ + 1 + 1½).
92. G(2 + ½ − 1) K(3½ + 1½ − 1).

93. M($1\frac{1}{2}$ − 2 + $\frac{1}{2}$) N(4 − 1 + 2).
94. O(2 − 3 + 2) P(4 − 1 + 1).
95. A(2 + 3 − 2) B(4 + 1 − 1).



Graphic Layout No. 4.—Find the ⊡ and ⊙ of the lines.



Graphic Layout No. 5.—Find the *P*-piercing points of the lines. Place *P* 4½ in. to right of margin.

LOCATING POINTS AND LINES IN PLANES

33. Problem 4.—To assume a line in a plane.

Analysis.—1. Assume a point in the *V*-trace.

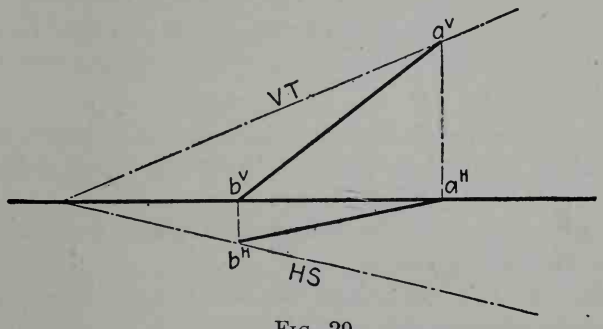


FIG. 29.

2. Assume a point in the *H*-trace.

3. Connect the respective projections of the two points.

Conclusion.—The line thus found will lie entirely in the plane, because two of its points lie in the plane.

Construction.—Let T (Fig. 29) be the plane.

1. Assume a^V in VT; a^H will be in *GL*.
2. Assume b^H in HT; b^V will be in *GL*.
3. Draw a^Vb^V and a^Hb^H. AB will be entirely in T.

Special Case.—To assume a line parallel to H or V, lying in an oblique plane.

If parallel to *H*, draw the *V*-projection parallel to *GL* from an assumed point in the *V*-trace. From the *H*-projection of this point draw the *H*-projection parallel to the *H*-trace.

In Fig. 29*a*, the line CD is a parallel line to *H* lying in S, and the line EF is parallel to *V* lying in R.

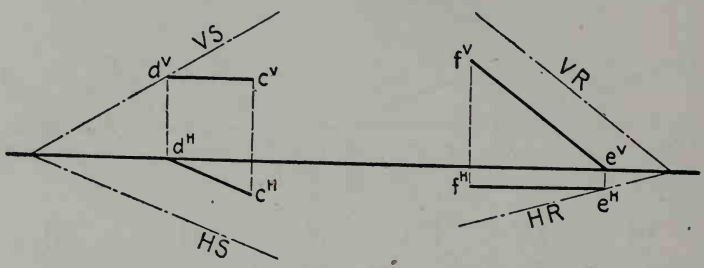


FIG. 29*a*.

34. Problem 5.—Given one projection of a point in a plane, to locate the other projection.

Construction.—Through the given projection draw the projection of a line and assume the line to be in the plane, thus locating the other projection. The required projection of the point will be on the second projection of the assumed line.

Hint.—The convenient line for most cases of the foregoing is a line parallel to *H* or *V*.

Note.—If the given projection of the point lies in the projecting trace of a projecting plane, the point is indeterminate, and can be one of an infinite number of points lying in the plane.

Problem 6.—To find the traces of a plane determined by three given points.

Analysis.—1. Join the points, two and two, by lines.

2. Find the piercing points of the lines.

3. Draw the *H*-trace through the *H*-piercing points, and the *V*-trace through the *V*-piercing points.

Proof.—The lines have their piercing points in the respective traces of the plane, therefore the lines (and all their points) lie in the plane thus determined.

Construction.—Let A, B, and C be the given points (Fig. 30).

1. Join the projections of AB and AC.

2. Find the four piercing points of AB and AC.

3. Through the *H*-piercing points draw HT, and through the *V*-piercing points draw VT.

Check.—1. The line BC should have its piercing points in the respective traces of T, and

2. The traces should meet in a point on *GL*, or be parallel to *GL*.

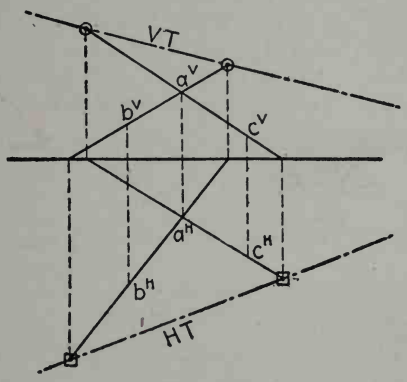


FIG. 30.

Hints.—1. Sometimes the points may be so placed that the piercing points are not all to be found within the limits of the problem. In such a case, an auxiliary line may be drawn from any of the three points to any point on the line joining the other two. For instance, if such a proceeding were necessary, a line could be drawn from B to any point, M, on the line AC. BM would lie in the required plane, and its piercing points would help to determine the traces.

2. Ordinarily, only three piercing points are necessary to be found, because when the *H*-trace is found, its intersection with *GL* may be joined to one of the *V*-piercing points.

If two intersecting lines are given.—This makes no change in the problem.

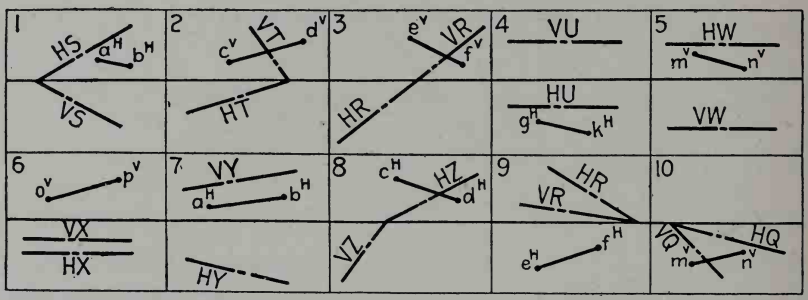
If two parallel lines are given.—Find their piercing points and draw the traces, as in the previous case.

If a point and a line are given.—The point may be connected with any point in the line, thus making two intersecting lines, or through the point a line may be drawn parallel to the given line, thus making two parallel lines.

EXERCISES

35. Locate the following lines in the planes given. The points to be determined are lettered as unknowns, x, y, z, etc., without + or − signs.

96. AB, any oblique line in T(1 + 3) 1(4 − 2).
97. CD, any oblique line in S(5 + 2) 2(2 − 3).
98. EF, any oblique line in any plane oblique to *GL*.
99. Draw a line in the third angle that lies in T(1 + 2) 3(1 − 3).
100. Locate C(2 + 1, x) D(3, + 2, y) in S(1 + 2) 4(1 − 2½). State values of x and y.
101. Locate E(4 +, x) F(5 + 2, y) in W(1 + 3) 3(5 − 1).
102. Draw a line in S(1 + 2) 4(1 − 2½) that is ‖ to *H*, 1 in. above *H*.
103. Draw a line in W(1 + 3) 3(5 − 1) that is ‖ to *V*, ½ in. behind *V*.
104. Locate M(5 + 1, x) N(3, 0, y) in W(1 + 3) 3(5 − 1).
105. Locate O(3 + 2, x) P(4 + ½, y) in T(1 + 3) 3(5 − 3).



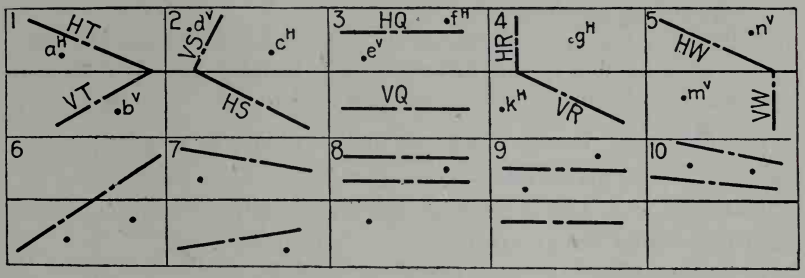
Graphic Layout No. 6.—Locate the other projections of the line in the given plane——.

106. Find the locus of all points in S(5 + 2) 2(5 − 3):
   (*a*) 1 in. above *H*.
   (*b*) 1 in. below *H*.
   (*c*) 2 in. behind *V*.
   (*d*) 2 in. before *V*.
107. Locate in R(+ 2) ∞ (− 1½) the following lines:
   A(1 + 1, x) B(2 + 1½, y).
   C(2, x, − 1) D(4, y − 2½).
   EF, the locus of all points 1 in. before *V*.
108. Locate the following lines in X(+ 1½) ∞ (− 1):
   F(1 − 1, x) G(2 − 2, y).
   K(2½ + 2½, x) L(4 +, y).
   MN, the locus of all points ¾ in. above *H*.
109. Locate O(2 + 2, x) P(4 + 1, y) in T(1 + 3) (5 + 1½) (1 − 2) (5 − 1).

Locate the following points in the planes given:

110. A(1, x − 1), B(3 + 3, y) in S(1 + 3) 5(1 − 1½).
111. C(1½ − 2, x), D(4, y + 2) in U(5 − 1) 1(3 + 3).
112. E(2 + 1, x), F(3, y − 3) in W(+ 2) ∞ (− 1).
113. G(2 + 1½, x), K(3 + 3, y) in Q(+ 2) ∞ (+ 1).

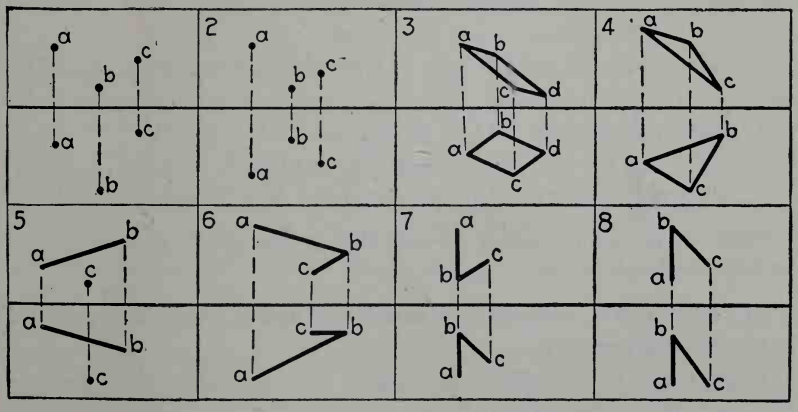
114. L(1 + 1, x), M(4½, y + 1) in R(1 + 2) 3(5 − 2).
115. N(1, x + ½), O(3 + 2, y) in S(1 + 3) 5(1 + 1).
116. P(1 + 1½, x), A(4, y − ½), B(4½ + 2, z) in T(1 + 3)(5 + 1½) (1 − 2)(5 − 1).



Graphic Layout No. 7.—Given one projection of points in planes, to locate the other.

Draw the traces of the planes containing the following points, lines, and plane figures:

117. A(2 + 2 − 1), B(3 + ½ − 3), and C(3½ + 1¼ − 1¼).
118. D(2 + 1 − ½), E(2½ − ½ + ½) and F(4 + 1½ + ¾).
119. G(2 − 2 + 1), K(3 − ½ + 3), and L(3½ − 1¼ + 1¼).
120. M(2½ + ½ − ½) N(3½ + 1¼ − ⅜), and O(4½ + ¼ − 1½).
121. A(1 − 1 + 2½) B(4 − ¾ + 1), and C(3 − 2 + ½).
122. The triangle D(1½ + 1 − 1½) E(3 + 1 + ½) F(4 − 1 + 2).
123. The parallelogram M(1½ − 1 + ⅝) N(2 − 1⅝ + ⅜) O(2⅝ − ⅜ + 1⅝) P(3⅛ − 1 + 1⅜).
124. HT is a line through A(4, 0, 0) and B(2, 0 − 2¼). T contains D(1 + 1 − 1½). Draw VT.



Graphic Layout No. 8.—Locate these points in either *I* or *III*, and draw the traces of the planes containing them.

125. The points D(2½ + 1½ − 1), E(3¼ + ¾ − 1), F(3¼ + 2¼ − 1), and G(4 + 1½ − 1) are the corners of a vertical base of a cube. Draw the projections of the cube and the traces of its six bounding planes.

126. L(3, 0 − 1) M(4, 0 − 2½) is the base diagonal of a square pyramid of 2-in. altitude, standing on *H*. Draw the traces of the four oblique planes forming its faces.

127. O(3, 0 − 1½) is the center of the base of a 2-in. hexagonal oblique pyramid, whose apex is P(2 + 2 − 1). Hexagon base is in *H*; draw the traces of the six oblique bounding planes.

36. Problem 7.—To pass a plane through a given point, parallel to two given lines.

Analysis.—1. Draw through the given point two lines, parallel respectively to the given lines.

2. Pass a plane through the lines thus drawn.

Conclusion.—This plane contains the given point and is parallel to both lines, from the geometrical theorem that a line,

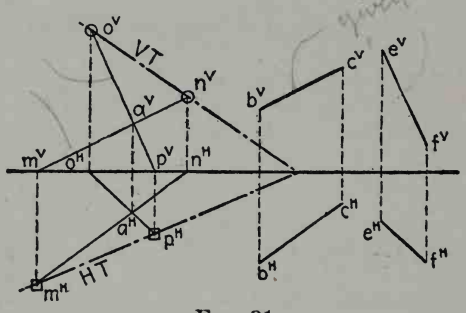


Fig. 31.

external to a plane, that is parallel to any line in that plane, is parallel to that plane.

Construction.—Let A (Fig. 31) be the given point, and BC and EF the given lines.

1. Through A draw MN ‖ to BC, and OP ‖ to EF.
2. Find the *H*- and *V*-piercing points of MN and OP.
3. Through these piercing points draw the traces of the plane.

Note.—If the given lines are parallel, the required plane is indefinite, and an infinite number of planes may be passed.

37. Problem 8.—To pass a plane through a given line parallel to another given line.

Analysis.—1. Through any point of the given line, draw a line parallel to the second line.

2. Pass the required plane through the two intersecting lines.

Let the student make the construction.

Note.—If a plane be passed through the second line parallel to the first, the two planes will be parallel.

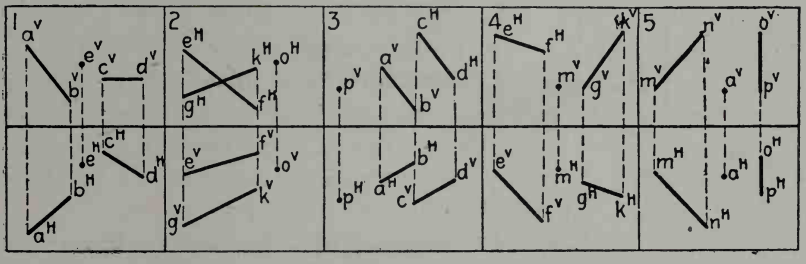
EXERCISES

38. Pass planes through the following points parallel to the lines given:

128. Through O(3 − 1 + 1) ‖ to A(1 − 1½ + 1) B(2 − ½ + 2) and C(4 − 1 + 1) D(5 − 1½ + 2).

129. Through A(4 + 1 − 1) ‖ to E(1 + 2 − ½) F(1 + 1 − 1) and G(2 + ½ − 1) K(3 + 2 − 2).

130. Through M(4 + 1 − 1½) ‖ to L(1½ + 1 − 1) N(2½, 0 − ½) and O(3 + 1 + ½) P(4 + 2 + 1½).

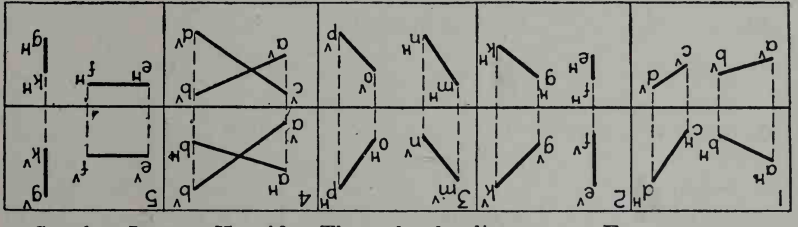


Graphic Layout No. 9.—Through the given point, pass a plane parallel to the two given lines.

131. Through A(1 + 3 − 2) B(2 + 1 + 1) pass a plane ‖ to C(3 + 1 −1) D(4 + 1½ − 2).

132. Through E(4 − 1 + 2) F(3 − 2 + 1) pass a plane ‖ to G(1 + 1½ − 2) K(2 + 1 − 1).

133. Through M(3 − 2 + 1) N(4 − ½ − 2) pass a plane ‖ to O(1 + 2 + 1) P(2 + 3 − 1).



Graphic Layout No. 10.—Through the line ——, Ex. ——, pass a plane parallel to line ——.

Revolution of Points

39. Definition.—The revolution of a point about a line (called its **axis**) consists in moving the point in a plane perpendicular to the axis, and keeping it at a constant distance from the axis.

The **Path of Revolution** is a circle.

FOURTH FUNDAMENTAL PROBLEM

40. Problem 9.—To revolve a point through any given angle about an axis perpendicular to H or V.

Construction.—Let O (Fig. 32) be the point, AB the axis perpendicular to H, and θ the given angle.

1. With $a^H b^H$ as a center draw a circle through o^H.

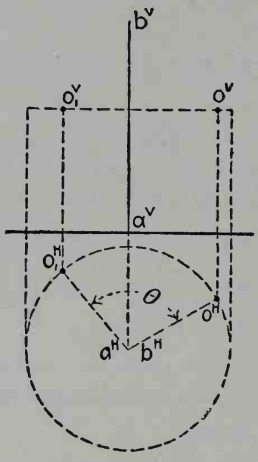


FIG. 32.

2. Through o^V draw a line parallel to *GL*. (These will be the *H*- and *V*-projections of the path of revolution.)

3. Move o^H through the given angle to o_1^H.

4. Project to o_1^V.

Conclusion.—This gives the new position of the point. If the axis were perpendicular to V, the circle would be projected on V in its true size and on H as a line parallel to *GL*.

41. Problem 10.—To revolve a line about a vertical or horizontal axis through any given angle.

Analysis.—1. As the line is the sum of all points in it, take any two points of it and revolve them about the axis, as in Problem 9.

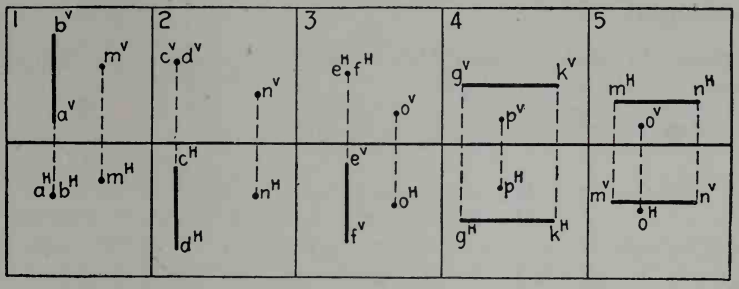
2. Join these revolved points, and the resulting projections will be projections of the line.

42. EXERCISES

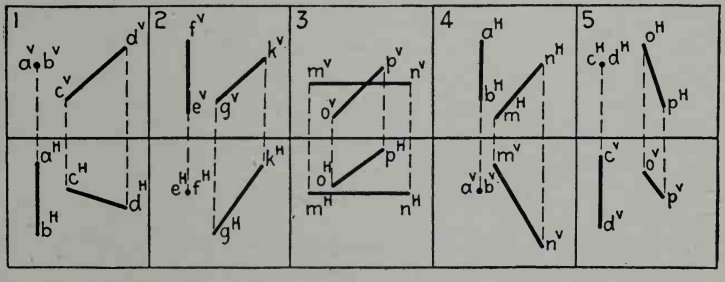
134. Revolve A(4 + 1 − 1) about B(3, 0 − 2) C(3 + 3 − 2) counter-clockwise 120°.
135. Revolve D(4 + 2 − 1) about E(3 + ½, 0) F(3 + ½ − 2) clockwise 240°.
136. Revolve G(1 + 1 − 1) about K(2 − 1 + 1) L(2 − 1 + 2) clockwise 135°, and M(5 − 1 + 1) about N(4, 0 + 2½) P(4 − 2 + 2½) until it reaches H.
137. Revolve A(3 − 1 − ½) clockwise 90° about B(4, 0 − 1) C(4 − 2 − 1). Continue the revolution until it returns to the fourth angle.
138. Revolve D(5 − ½ − 1) counter-clockwise 90° about E(4 + ½, 0) F(4 + ½ − 2). Continue the revolution until it reaches H again.
139. Revolve G(4 + ½ − 1) K(3 + 2 − 2) about P(3 + 1, 0) O(3 + 1 − 3) clockwise 135°.
140. Revolve L(2 + 2 − 1) M(3½ + 1 − 2½) about K(1½, 0 − 2) O(1½ + 3 − 2) into a position parallel to V.

141. Revolve A(1 − 1 + 2) B(3 − 2 + ½) clockwise 90° about C(2, 0 + 1) D(2 − 2 + 1).

142. Revolve E(1 + 1 + 1) F(2½ − 1 + 2) clockwise 75° about G(2 − 2 + 2) K(2 + 2 + 2). Also revolve it ‖ to V.



Graphic Layout No. 11.—Revolve the given point ——°—— clockwise about the given axis.



Graphic Layout No. 12.—Revolve given line ——, ——°—— clockwise about axis ——.

43. Problem 11.—To find the true length of a line in space, by revolution.

Discussion.—To find the true length of a line by projecting it on an auxiliary plane is solved in Problem I, Corollary I. Another method in quite as much use is that of revolving the line parallel to one of the planes of projection, using the principle of Problems 9 and 10.

Analysis.—1. Assume an axis through the given line perpendicular to H or V.

2. Revolve the line as in Problem 10 until it becomes parallel to the plane to which the axis is parallel.

Conclusion.—The projection on this plane is the true length of the line.

Proof.—The line remains unchanged in revolving about its axis, because every point in the line moves through the same angle

in a series of parallel circles. This means that every point remains in the same relation to the axis as every other point throughout the revolution, and therefore all points in the line remain in the same relation to each other, and the line is unchanged.

Note.—In revolving parallel to V, the H-projection is not changed in length, but the V-projection is.

Construction.—Fully shown in Fig. 33.

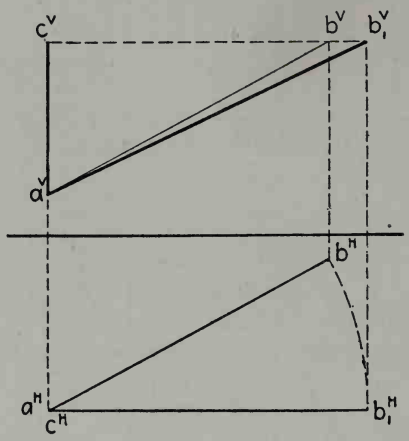


FIG. 33.

44. The true length of a line in a profile plane. The P-projection shows its true length.

45. EXERCISES

143. Measure A(1 + 1 − 1) B(2 + 3 − 2) by revolving ∥ to H, (2) ∥ to V. Check result by Problem 1, Corollary 1.
144. Measure C(5 − 1 + 3) D(3 − 3 + 2), (1) by revolving ∥ to V, (2) ∥ to H. Check.
145. Measure E(2 + 1 + 1) F(4 + 2½ + 2½), (1) by revolving ∥ to V, (2) ∥ to H. Check.
146. Measure G(3 − 3 − 3) K(1 + 1 − 1), (1) by revolving ∥ to H, (2) ∥ to V. Check.
147. Measure L(2 + 1 − 2) M(3 − 1 + 1), (1) by revolving ∥ to V, (2) ∥ to V. Check.
148. Measure M(4 − 2 + 1) N(5 + 1 + 1½), (1) by revolving ∥ to H, (2) ∥ to H. Check.

Measure the following lines. Place P at 4½.

149. A(1 − 2 + ½) B(1 − ½ + 1) and C(2 − 1 − 2) D(2 − 2 − ½).
150. E(1 + 3 − 1) F(1 + 1 − 2) and G(2 + ½ + 1¼) K(2 + 3 + ½).
151. M(1 + 2 − 1) N(1 − ½ − 2) and O(2 − 1 + 1) P(2 + 2 + ½).

152. A(1 − 2 + 1) B(1 + 2 − 2) and C(2 + 1 + 2) D(2 − 2 − ½).
153. E(1 + 2 + 1) F(1 + 1 − 1½) and G(2 − ½ − 1) K(2 − 2 + ½).
154. Lay off —— in. (amount specified by instructor) on any of the lines in Problems 143–153 from the first point toward the second.
155. Through M(1 + 1 − 3) draw a line —— in. long, parallel to O(3 + 1 − 2) P(4 + 3 − 1).
156. Through A(5 − 2 + 3) draw a line —— in. long, parallel to B(1 − 1 + 1) C(2 − 1½ + 2).
157. D(2 − 3 + 1) E(3½ − 3 + 2¼) is the base diagonal of a square pyramid, 2½-in. altitude, in III. Draw its projections.
(*a*) Measure one of its slanting edges.
(*b*) Measure the line running from a point 1¾ in. up one edge to a point ½ in. up the adjacent edge.
158. Scale 100 ft. = 1 in. A rope conveyor is to be constructed from A(75 ft., 0, − 180 ft.) to a loading point on a hill 225 ft. high, 200 ft. north, 60° east, of the dumping point. How much cable must be purchased for a duplex belt, allowing 30 ft. for pulleys, slack, etc.?

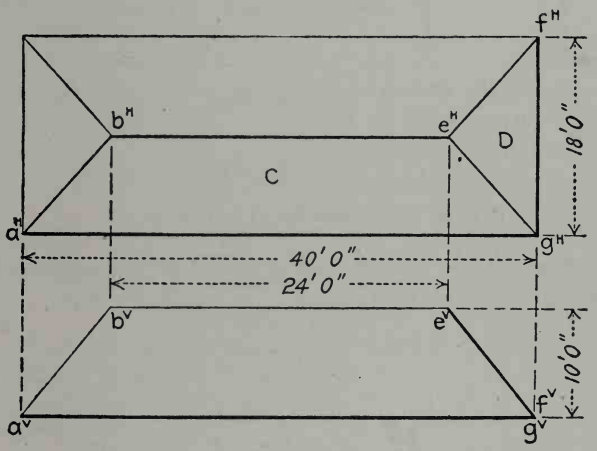


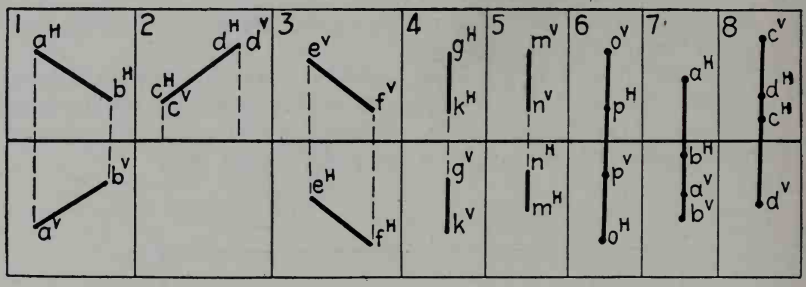
FIG. 34.

159. A pipe runs through a factory building from a point M(5 ft. + 10 ft. − 2 ft.) to N(30 ft. − 2 ft. 6 in. − 14 ft.) in the floor below.
(*a*) What is its length?
(*b*) At what point in the floor must the hole for it be made?
(*c*) At a point 6 ft. from the floor hole, measured along the pipe, is a support from the floor. How far from the wall (*V*) and the floor (*H*) is this point of support? Scale ¼ in. = 1 ft.
160. Fig. 34 shows the plan and elevation of a plain hip roof. Draw it to any convenient scale.
(*a*) What angle is it drawn in? Draw its *P*-projection.
(*b*) Locate the traces of the roof planes, C and D.
(*c*) What is the length of the hip rafter AB.

(*d*) Locate a point on the roof C, 3 ft. below the ridge and 10 ft. to the right of the center.

(*e*) Locate a point on the rafter, 3 ft. from F.

161. Scale 1 in. = 20 ft. A 60-ft. stack, 3 ft. in diameter, rises from A(60 ft., 0 — 35 ft.), on the top of a flat roof (*H*). Five guy-wires are attached to the stack at 40 ft. above the roof, and are anchored at points on the roof, respectively, B(30 ft., 0 — 10 ft.), C(85 ft., 0 — 7 ft. 6 in.), D(90 ft., 0 — 47 ft. 6 in.), E(70 ft., 0 — 67 ft. 6 in.), and F(25 ft., 0 — 52 ft. 6 in.). Draw the stack and guys, and measure the true length of the guys.



Graphic Layout No. 13.—(*a*) Measure the true length of given line ——. (*b*) Lay off from one point of given line —— a point —— in. distant on the line toward the second point.

FIFTH FUNDAMENTAL PROBLEM

46. Problem 12.—To revolve a point in space into H or V about an axis lying in that plane.

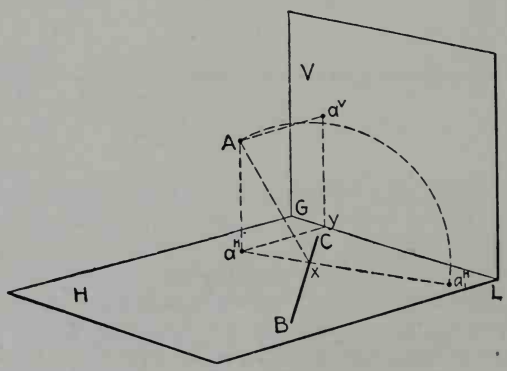


FIG. 35.

Process.—Fig. 35 shows a point A in space and a line BC in *H*. To revolve A about BC into *H*, it is necessary to draw the radius

of the circle through which A is to revolve. As will be seen in the picture, Ax is the radius, and it is the hypothenuse of the right triangle, $Aa^H - a^Hx$. By laying off this hypothenuse in either direction perpendicular to BC at x, the revolved position is obtained.

Analysis.—To revolve a point into *H* about an axis in *H*.

1. Draw the trace of the plane of revolution through the *H*-projection of the point, perpendicular to the axis.

2. Construct the right triangle as follows: (1) the base = the perpendicular distance from the *H*-projection to the line; (2) the altitude = the *V*-projection to *GL*.

3. Lay off the hypothenuse of this triangle from the axis on the trace of the plane of revolution.

Construction.—Let A (Fig. 36) be the given point and BC the given line in *H*.

1. Through a^H draw $xa_1{}^H$ perpendicular to b^Hc^H.

2. Construct the triangle $a^Hx - a^Hz(= a^Vy)$.

3. Lay off xz on $xa_1{}^H$, locating $a_1{}^H$.

Conclusion.—$a_1{}^H$ is the revolved position of A about BC.

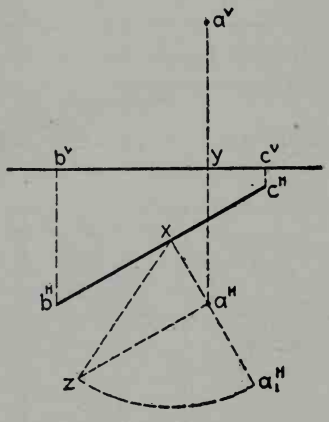


FIG. 36.

Notes.—To revolve a point into *V* about an axis in *V*, make an interchange of *H* and *V* in the foregoing.

2. If the point is directly over the axis, the base of the triangle becomes zero, and the hypothenuse will be equal to the altitude.

3. The "trace of the plane of revolution" and "the projection of the path of revolution" will be abbreviated hereafter to **"the path of revolution."**

47. EXERCISES

162. Revolve A(2 − 1 + 3) and B(4 + 2 − 2½) into *H* about C(1, 0 − 2) D(5, 0 − ½).

163. Revolve E(1 + 1 − 2) and F(4 − 1 + 2) into *V* about G(2 + 3, 0) K(5 + ½, 0).

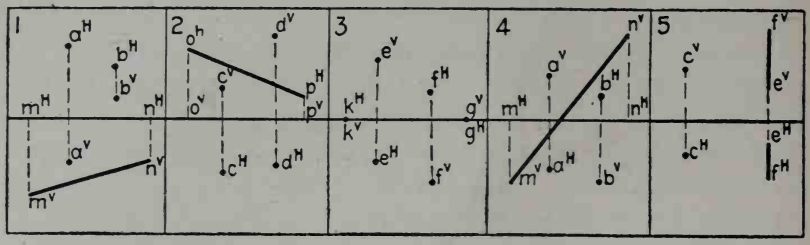
164. Revolve M(1 − 2 − 1), N(2 + 1 − 2), O(3 + 1 + 1), and P(4 − 1 + 1½) about *GL*, (1) into *V*; (2) into *H*.

165. Revolve K(2 + 1½ − 1) about L(1 + 3, 0) N(5, 0, 0) into *V*. Revolve M)3 − 2 + 1) about P(2, 0, 0) O(5, 0 − 1) into *H*.

166. Locate A(1 + 1, x) and B(2, y − 1) in T(1 + 3) 5(1 − 2), and revolve into *V* aboutVT, and into *H* about HT.

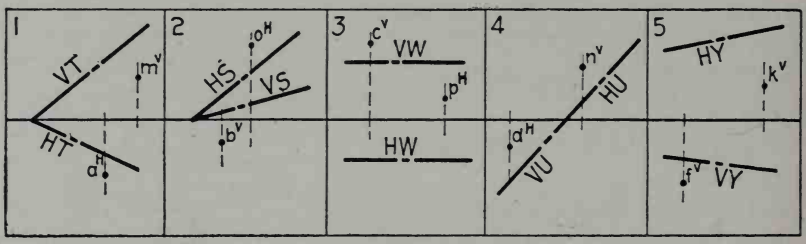
167. Locate C(4 + 2, x) and D(2, y − 1) in S(5 + 3) 1(5 − 1), and revolve into V about VS, and into H about HS.

168. Locate E(2, x − 2) and F(4 + 1, y) in U(+ 2) ∞ (− 1), and revolve them about VU and HU into V and H.



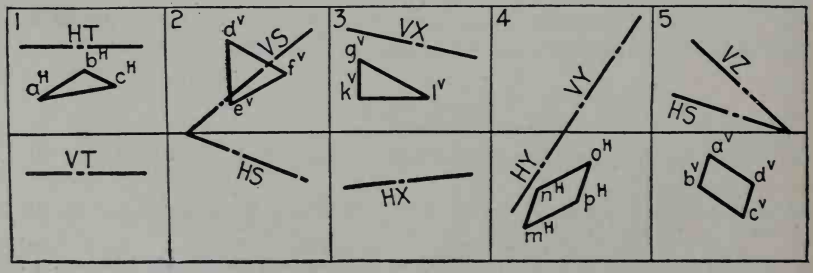
Graphic Layout No. 14.—Revolve the points in Ex. —— about the given line into the plane of the latter.

169. Locate K(2 − 2, x) and L(4, y + 1) in W (− 1) ∞ (+ 3), and revolve them about VW and HW into V and H.



Graphic Layout No. 15.—Locate the points in plane —— and revolve them into —— about the —— trace.

170. Locate O(+ 2) and P(+ 1) in the V-trace of T(1 + 3) 5(1 − 2) and revolve them into H about the H-trace.



Graphic Layout No. 16.—Locate the plane figure——in its plane, and revolve about one of the traces to show its true size.

171. Locate A(2 − 1, z) and B (4, 0 + 2) in S(− 1) ∞ (+ 2), and revolve A into H about HS, and B into V about VS.

172. Revolve the triangle M(1½ + 1½ − 1) N(2½ + 1 − 2¼) O(3½ + 1¼ − ½) into V about the V-trace of its plane. Will this give its true size, and the true angles at the corners?

173. Revolve the parallelogram A(3 − 1 + 1) B(3 ½ −¾ + 2) C(5 − 2¼ + 1½) D(4½ − 2½ + ½) into H about the H-trace of its plane. Will this give its true size?

Measurement of Angles

48. Problem 13.—To measure the true angle between any two intersecting lines.

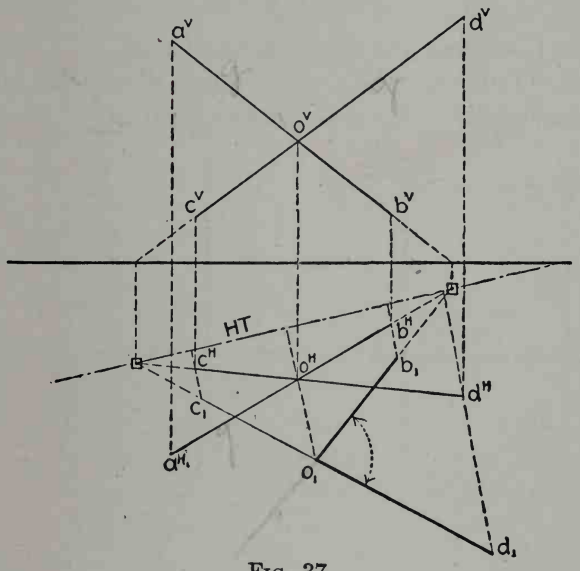


Fig. 37.

Two intersecting lines lie in one plane, and, if that plane be folded so as to coincide with H or V, the revolved projections will show the true relation of the lines.

Analysis.—1. Pass a plane through the two lines.

Note.—One trace only need be obtained.

2. Revolve the plane about its H-trace into H, or about its V-trace into V.

Note.—Revolving the various points in a plane is equivalent to revolving the plane.

Conclusion.—The angle between the lines in revolved position is the measure of the actual angle.

Construction.—Let AB and BC (Fig. 37) be the two intersecting lines whose angle is to be measured.

1. Pass the plane through the two lines; HT is the necessary trace.

2. Revolve AB and BC about the trace.

Conclusion.—The required angle is $b_1o_1d_1$.

Note 1.—It is usual to consider the acute angle as the measure, although the obtuse angle is not incorrect.

Note 2.—In case any of the necessary piercing points are inaccessible, it is proper to draw as many auxiliary lines from one line to the other as may be needed to obtain the trace.

SPECIAL CASES

49. Case I.—When one side is parallel to H.

Analysis.—1. Revolve the oblique line about the horizontal line as an axis until it becomes parallel to *H*.

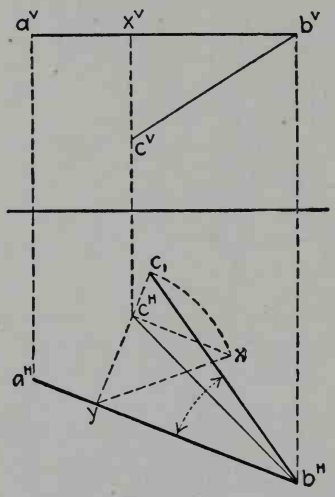


FIG. 37*a*.

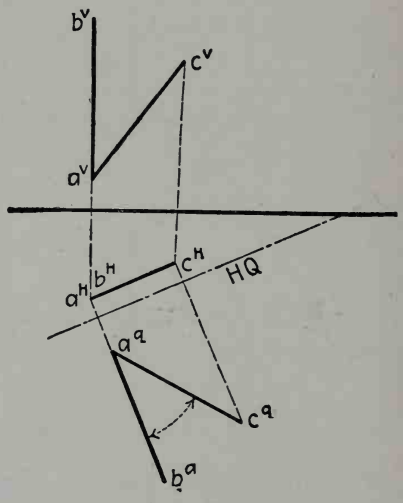


FIG. 37*b*.

Conclusion.—The projected angle is now equal to the true angle, since both are parallel to *H*, and their relations are unchanged.

Construction.—Required to measure the angle ABC (Fig. 37*a*), AB being parallel to *H*.

1. Revolve C about AB as an axis into the horizontal plane of AB.

Note 1.—This can be easiest done by transferring *GL* to a^vb^v, and applying Problem 12.

Note 2.—The same process can be performed when one of the lines is parallel to *V* or *P*.

Note 3.—If preferred, the regular processes can be used instead of this special one.

50. Case II.—When one side is perpendicular to H, V, or P.

Construction.—Project the lines on Q, parallel to the two lines. The Q-projection of the angle is the true size of the angle. See Fig. 37*b*.

51. Case III.—When both sides are parallel to P.

Conclusion.—The *P*-projection of the lines will show the angle in its true size.

52. Problem 14.—To find the projections of the bisector of any angle.

Construction.—Let ABC (Fig. 38) be the given angle. To find the bisector:

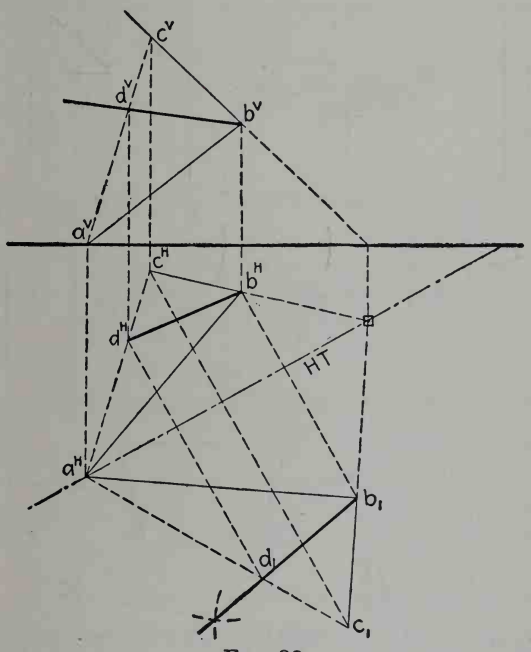


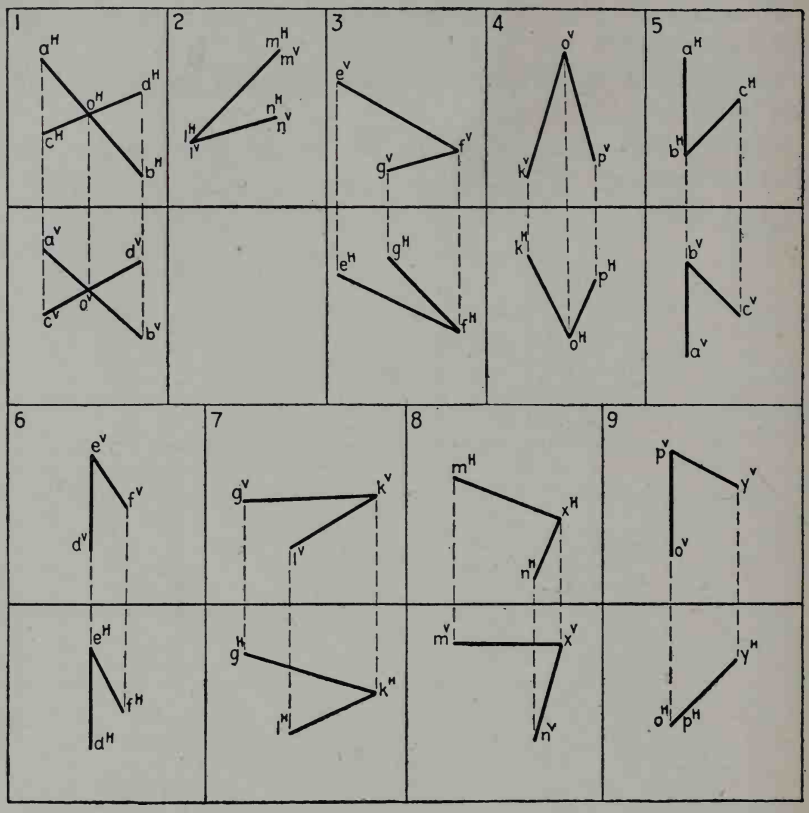
FIG. 38.

1. Obtain HT (or VT, if more convenient).
2. Revolve ABC into *H* about HT, making the true angle $a^Hb_1c_1$.
3. Bisect $a^Hb_1c_1$. The bisector is d_1b_1.
4. Draw an auxiliary line a^Hc_1, intersecting b_1d_1 in d_1.
5. Revolve a^Hc, back to its original position, a^Hc^H, a^Vc^V.
6. Revolve d, back to d^H on a^Hc^H, and project to d^V on a^Vc^V.
7. Connect the projections of B and D.

Conclusion.—BD is the bisector in original position.

Note.—If the angle can be measured by any of the special cases, the bisector may be found by the same process.

53. EXERCISES



Graphic Layout No. 17.—(*a*) Measure angles between given lines in Ex. ——. (*b*) Draw the projections of the bisector of the angle in Ex. ——.

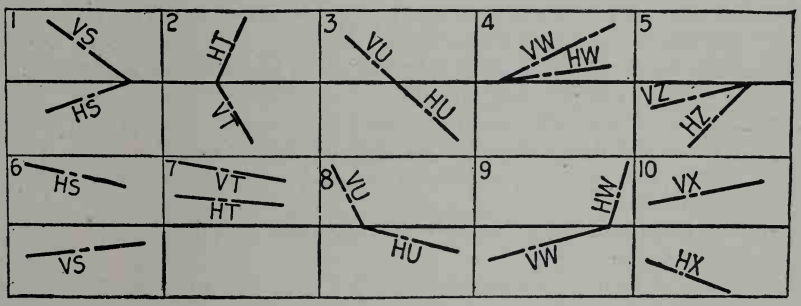
Measure the angles included between the following lines:

174. A(2 + 2 − 2) B(3½ + 1 − ½) and BC(4½ + 3 − 1½).
175. D(1 − 2 + 3) E(4 − 1½ + 1½) and EF(2 − 1 + 1).
176. G(1 + 2½ − 1) K(3½ + 1½ − 2) and KL(2 + 1 − 2½).
177. M(1½ + 3 − 2) O(2 + 1 − ½) and OP(5 + ½ − 1½).
178. A(2 + 3 − 2) B(2 + 1 − 1) and BC(3½ + 2 − 3).
179. D(4 + 1 − 1) E(4 + 2½ − 2) and EF(3 + 1 − 1).
180. G(3 + 1 − 3) K(3 + 3 − 1) and KL(2 + 3 − 1).
181. M(1 + 2 − 1) N(1 + 2 − 3) and NO(2½ + ½ − 1).
182. A(1 + 1½ − 2) B(4 + 1½ − 2) and BC(2 + 1 − 1).

183. D(3 + 3 − 1½) E(1 + 1 − ½) and EF(3 + 1 − 3) by revolving into the profile.
184. G(2 + 1½ − 3) K(4 + 1½ − 1) and KL(3 + 3 − 1½).
185. M(1 − 2½ + 1) N(4 − ½ + 1) and NO(2 − 1 + 2).
186. A(2 + 2 + 2) B(4 + ½ + ½) and BC(3 − 1 − 1).
187. D(4 − 2 − 2) E(2 + ½ + ½) and EF(3 + 2 − 1).
188. G(3 + 1 + 1) K(3 − 1½ − 1½) and KL(1 − 1 − 1)
189. M(4, 0, 0) N(2 − 1 + 1½) with *GL*.

Measure the angle between the traces of the following planes:

190. T(1 + 1) 4(1 − 2½).
191. S(1 + 3) 4(1 + 2).
192. R(4 − 1½) 1(4 + 2).
193. Q(1 + 2) 3(5 − 2).
194. U(4 + 2) 3(2 − 2).
195. W(5 + 3) 3½(1 − 1).
196. T(1 + 2) 1(3 − 3) and S(2 + 1½) 5(5 − 3)
197. R(1 + 3)(5 + 1½)(1 − 2)(5 − 1).
198. W(1 + 3)(5 + 1½)(1 + 2)(5 + 1).



Graphic Layout No. 18.—(*a*) Measure the angle between the *H*- and *V*-traces of plane ——. (*b*) Draw the bisector of the angle beween the traces of plane ——.

Measure the angles in the following triangles:

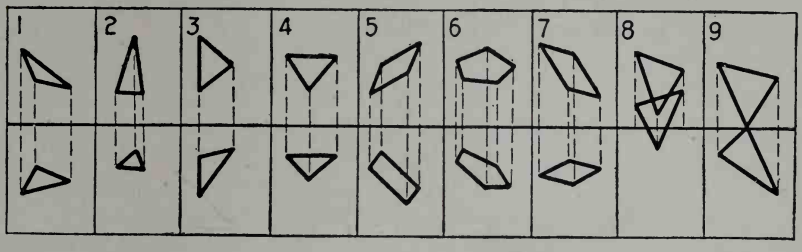
199. A(2 − 1 + 3) B(3 − 2 + 1½) C(5 − 1½ + 2).
200. D(2½ + 1 − 1) E(4 + 3 − ½) F(4 + 2 − 2).
201. G(2 + 2 − 2) K(2 + 1 − 1½) L(3 + 2 − 2).
202. M(2½ − 1½ + 2) O(3 + ¾ − 1) P(4 − ¾ + 1).
203. A(2 + 2 + 2) B(2½ + 1 + 1) C(4 + 1½ + 1½).
204. D(5 − 1 − 1) E(4 − 2 − 2) F(2½ + 1 − ½).
205. G(2 + 1½, 0) K(3½, 0, 0) L(3, 0 − 2).
206. Measure the angles and true size of the parallelogram M(3 + ½ − 1½) N(3 + 1 − ½) O(4 + 2 − 1) P(4 + 1½ − 2).
207. Measure the angles of the pentagon A(3½ + ½, x) B(4 + 1¼, y) C(4½ + ⅛, z) D(5½ + ¾, m) E(5 + 1¼, n) that lies in the plane T(6 + 3) 1(5 − 1½).

208. Prove graphically that A(2 + ¾ − 1) B(3 + 2 − 2) C(4½ + 2 − ½) is a right angle. Write an explanation.
209. Draw a line from D(2 + 2 − 1) intersecting E(2 + 1 − 2) F(5 + 2 − ½) at ——°(30, 45, 60, 75).
210. Draw a line from G(3 + 1 − 1) intersecting K(1 + 2 − 3) L(4 + 2 − 3) at ——°(30, 45, 60, 75).
211. Draw a line from M(2 + ½ − 2) intersecting N(1 + 2 − 1) O(3 + 1 − 1) at ——°(30, 45, 60, 75).
212. Draw a line from P(2 + 2 − 1) intersecting A(3 + 1 − 2) B(3 + 2 − ½) at ——°(30, 45, 60, 75).
213. Draw a perpendicular from C(2 + 2 − ½) to D(2 + ½ − ½) E(2 + 2 − 1½).
214. Measure the angle between two adjacent edges and the angle between an edge and a base line of the pyramid given in Ex. 157, Art. 45.
215. Measure the angle that E(2 + ½ − 2½) F(4 + 2 − 1) makes (1) with its own *V*-projection, (2) with its *H*-projection.
216. Draw the roof in Fig. 34 to a definite scale, and measure the following angles:
    (*a*) Hip rafter EF with hip rafter EG.
    (*b*) Hip rafter EF with eave FG.
    (*c*) Hip rafter AB with eave AG.
    (*d*) Hip rafter AB with ridge BE.
217. Draw the roof in Fig. 34 to a definite scale, and measure the true size of the roof planes C and D.
    Compute their areas from the scale.
    What would it cost to roof it at $1.25 per square yard?
218. The *V*-trace of T(1 + 3) 4(x, y) makes 60° with its *H*-trace. Draw the *H*-trace.
219. The *H*-trace of S(x, y) 3(5 − 1½) makes 120° with its *V*-trace. Draw the *V*-trace.
220. A ray of light passes through A(1 + 1 − 2) and is reflected from *H* to B(4 + 1½ − 1). At what point and angle does it strike *H*?
221. A ray of light passes through C(1 + 2 − 1) and, after reflecting from *V* and *H*, passes through D(5 + ½ − 2). At what points and angles does it strike *H* and *V*?
222. Draw a line from E(4, x − 1) in T(5 + 3) 2(5 − 2) making 60° with the *H*-trace.
223. Draw a line from K(2 + 1 − 1½) in the triangle KL(2 + 2 − 2) N(3 + 2 − 2) making 60° with the opposite side.
224. Find the center of the triangle A(2 + 1½ − 1) B(3 + 1 − ½) C(3½ + 3 − 2).
225. Find the center of the triangle D(1½ − 2 + 1) E(2½ − 1 + ½) F(3½ − 1½ + 2).

Draw the projections of the bisectors of the following angles (Ex. 226–247):

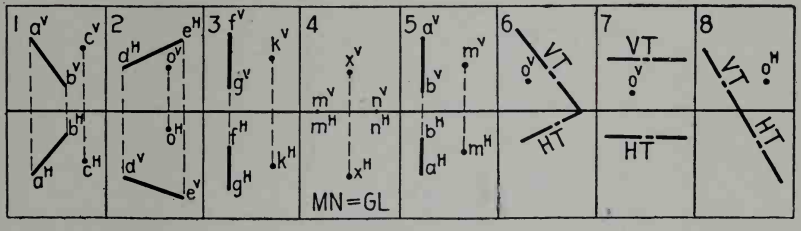
226. A(4 + 2½ − 1) B(2 + 1 − 1½) C(4 + 1¾ − ¼).
227. D(2 − 1 + 3) E(2¾ − 3 + 1½) F(4 − 1¼ + 1).
228. G(2 + 2 − 1) K(3½ + 3 − 3) L(2 + ½ − 1¾).

229. M(4 − ½ + 2) N(2 − 3 + 2½) O(4 −1½ + ½).
230. A(2 + 1½ − 3) B(2 + ½ − ¾) C(2 + 3 − 2).
231. D(2 − 1 + ½) E(2 − ½ + 1½) F(2 − 3 + ½)
232. G(1 + ½ − 3) K(2 + 1 − 1) L(4 + 2 − ½).
233. M(1 + 2½ − 2) N(1 + 1 − 1) O(3 + 2 − 2½).
234. A(2½ + 2½ − ½) B(2½ + ½ − 1¾) C(1 + 2 − 1).
235. D(2+ 1½ − 1) E(5, 0, 0) F(2, 0, 0).



Graphic Layout No. 19.—(*a*) Measure the angle of the polygon ——. (*b*) Locate the center of triangle —— (1, 2, 3, 4, 8, 9).

236. G(1 − 1 + ½) K(1 − 2½ + 2) L(1 − 1½ + 1½).
237. Between the *H*- and *V*-traces of T(1 + 3) 5(1 − 1½).
238. Between the *H*- and *V*-traces of S(1 + 3) 2(5 − 1).
239. Between the *H*- and *V*-traces of R(5 + 3) 1(5 + 2).
240. Between the *V*- and *P*-traces of R(5 + 3) 1(5 + 2)
241. Between the *V*- and *P*-traces of Q(+ 1) ∞(− 2).



Graphic Layout No. 20.—(*a*) Draw a perpendicular from point —— to line —— in Ex. ——. In Ex. 6–8 locate the point in the plane, and draw the perpendicular to the —— trace. (*b*) From the given point in Ex. —— draw a line making ——° (15, 30, 45, 60, 75) with the given line or trace.

242. Between the *H*- and *V*-traces of U(5 + 3)(1 − 1)(5 + 1½)(1 − ½).
243. Between the *H*- and *P*-traces of U(Ex. 242).
244. Between the *H*- and *V*-traces of W(2 + 3) 3(4 − 3).
245. Between the *H*- and *P*-traces of W(2 + 3) 3(4 − 3).
246. M(1 + 2 − 3) N(3 + 1 − ½) and its *V*-projection.
247. O(4 − 1 + 3) P(2 − 3+ 1½) and its *H*-projection.

54. Problem 15.—To find the shortest distance from a point to a line.

TWO METHODS

First Method.—Analysis.—1. Pass a plane through the given point and line.

2. Revolve the point and line about the trace of the auxiliary plane, as in the preceding problems.

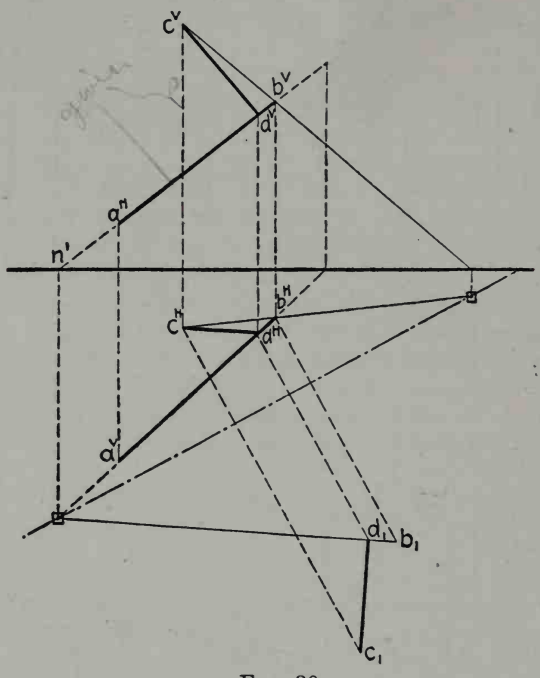


FIG. 39.

3. Erect the perpendicular from the point to the line.

Conclusion.—This perpendicular is the true shortest distance, as the point and line are in their true relation.

Construction.—Let C (Fig. 39) be the given point and AB the given line.

1. Pass the plane T through AB and C, obtaining HT only.
2. Revolve AB and C about HT to a, b, and c.
3. Erect the perpendicular c_1d_1 from c_1 to a_1b_1. This is the shortest distance.

Note.—If the **shortest line** is desired, c_1d_1 may be revolved back to the original projections of the line as shown in the figure.

55. Second Method by auxiliary planes.

Let AB (Fig. 40) be the given line and C the given point.

Construction.—1. Project AB and C on Q, which is taken parallel to AB (and perpendicular to H, **of course**).

2. Draw QX perpendicular to $a^q b^q$, and project AB and C on Q.

Conclusion.—The distance from c^x to $a^x b^x$ (a point in projection) is the true distance. Projecting back, CD is the shortest line.

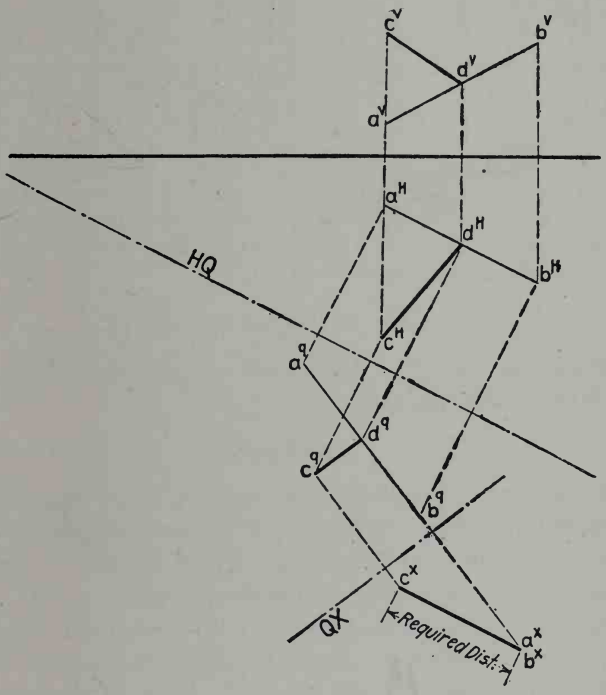


FIG. 40.

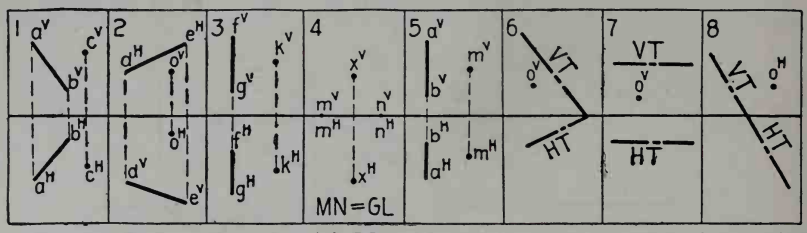
56. EXERCISES

Find the shortest distance from the following points to the following lines:

248. A(4 + 3 − 1) to B(2 + 2½ − ½) C(4 + 1 − 3).
249. E(1 − 3 + 2½) to D(1½ − ½ + 1) F(4½ − 3 + 2)
250. G(3 + 2 − 3 to K(1 − 2 + 3) L(4 − 1 + 1).
251. M(3 + 1 − 3) to N(2 + 2, 0) O(4 + 1, 0).
252. P(3 + 2 − 1½) to *GL*.
253. A(2 + 1 + 1) to B(1 − 3 − 3) C(3 − 1 − 1).
254. D(3 − 1 − 1) to E(2 + 2 + 2) F(4 + ½ + ½).

255. G(1 + 3, 0) on the *V*-trace of T(1 + 3) 4(1 + 2) to HT. Also K(2½, 0 − 1) on HT to VT.

256. A 2-in. steam pipe runs through a basement from a point in the floor, M(10 ft., 0, − 18 ft. 6 in.) to a point on one wall, N(50 ft. + 13 ft., 0). At a point O(22 ft. + 18 ft. − 2 ft.) in the ceiling it is desired to make the shortest possible connection, using 1½-in. pipe. Where will the joint be made, and how long will the branch pipe be, allowing 3 in. for connection?



Graphic Layout No. 21.—(*a*) Measure the distance from the given point in Ex. —— to the given line. (In Ex. 6–8 locate the point in the plane given, and measure the distance to the —— (*H*- or *V*-trace). (*b*) Draw the projections of the shortest possible line.

Figures in Oblique Planes

57. Problem 16.—To draw the projections of any plane figure in any oblique plane.

METHODS

There are two important processes of achieving this result. The student, by mastering both, will have complete command over any situation that will arise. The difference in the method is mainly in the manner of counter-revolving the points.

Analysis.—Assuming that a given point in a given oblique plane is to be the center of some regular polygon lying in the plane.

1. Revolve the given point about the trace of the given plane, as in Problem 12.

2. Construct the required polygon in its true size about the revolved position of the center.

3. Counter-revolve the points (corners) of the polygon into the original position of the plane.

Note.—The above is the analysis for both methods.

58. METHOD BY ORDINARY REVOLUTION

Construction.—Let O (Fig. 41) be a point in T. Let it be required to draw the projections of a 1-in. square in T, about O as a center, having two of its sides parallel to *H*.

1. Revolve O about HT to o_1.

2. Construct the square $a_1b_1c_1d_1$ in its true size about o, as a center with two sides parallel to HT.

3. Draw through o_1 a line through any point or points of the square ($b_1o_1c_1x$), extending it to HT.

4. Counter-revolve this line to the position xo^H. (This is possible, for X, being in the axis will remain stationary, and o_1 will counter-revolve to o^Ho^V.)

5. Counter-revolve c_1 and b_1 to this line on paths perpendicular to HT, also a_1 and d_1 by the same method.

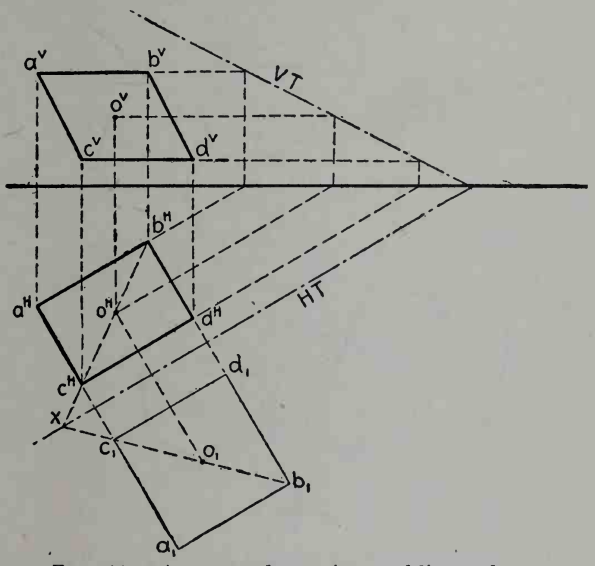


FIG. 41.—A square drawn in an oblique plane.

6. Locate the *V*-projections of A, B, C, and D by any of the known methods.

The square will then be drawn according to its specifications.

59. AUXILIARY PLANE METHOD

Construction.—Let O (Fig. 42) in T be the given center of a 1-in. regular hexagon.

(**Note.**—When the diameter of a hexagon is given, it means the diameter of the circumscribed circle, long diameter, unless short diameter or diameter "across the flats" is specified.)

1. Assume Q perpendicular to HT at any convenient point.

2. Obtain the Q-trace of T and the Q-projection of O.
3. Revolve O about HT to o_1 in the manner shown in Fig. 42.
4. Construct the required hexagon about o_1 as a center.
5. Counter-revolve the corners of the hexagon: (1) to QT; (2) to the *H*-projection; (3) to the *V*-projection.

Note 1.—The arrows, in Fig. 42, show the direction of this operation in the case of point A.

Note 2.—The *V*-projections of the various points are the same distance from *GL* as the Q-projections are from HQ.

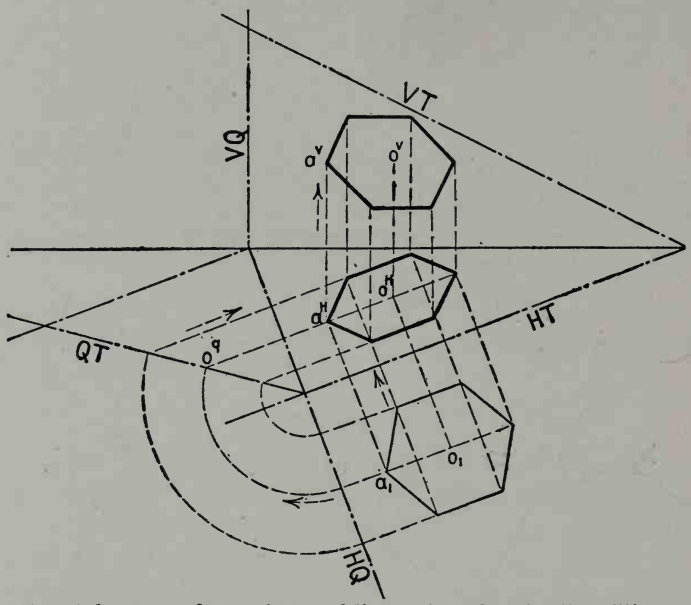


FIG. 42.—A hexagon drawn in an oblique plane by the "auxilliary plane" method.

60. Supplementary Problem.—To draw the projections of a circle in an oblique plane.

Construction.—Let O (Fig. 43) be the center of a 1½-in. circle lying in T.

1. Revolve O about HT to o_1.
2. Draw the circle full size about o_1 as a center.
3. Draw the diameters parallel and perpendicular to HT, a_1b_1 and c_1d_1 respectively.
4. Counter-revolve the points a_1, b_1, c_1, and d_1. They will be the four vertices of the ellipse, which is the *H*-projection of the circle. The major axis will be a^Hb^H and the minor axis c^Hd^H.

The ellipse may be drawn on these axes in any desired manner, preferably the "trammel method." (See Art. 147.)

For the *V*-projection, revolve O about VT and counter-revolve the points e_1, f_1, g_1, and k_1 in the same way; e^vf^v will then be the major axis, and g^vk^v will be the minor axis.

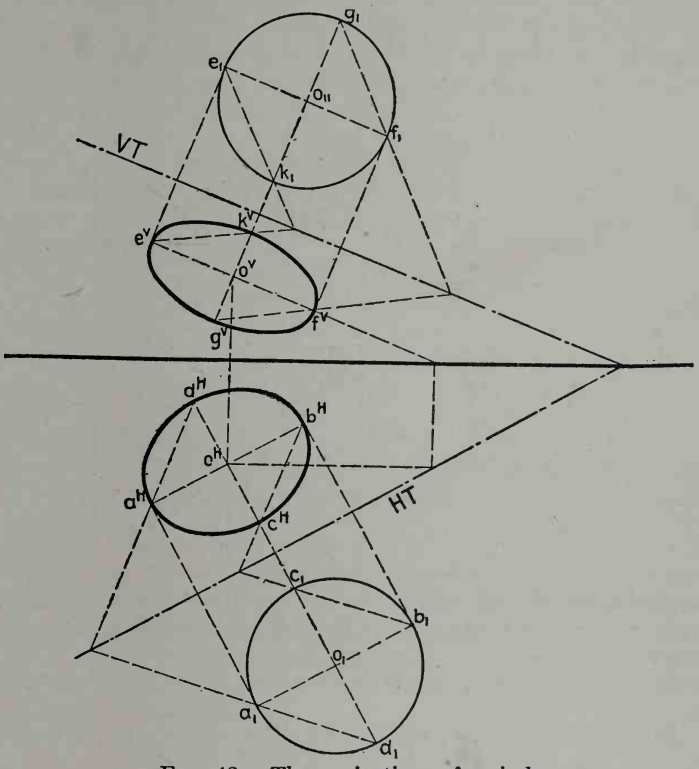


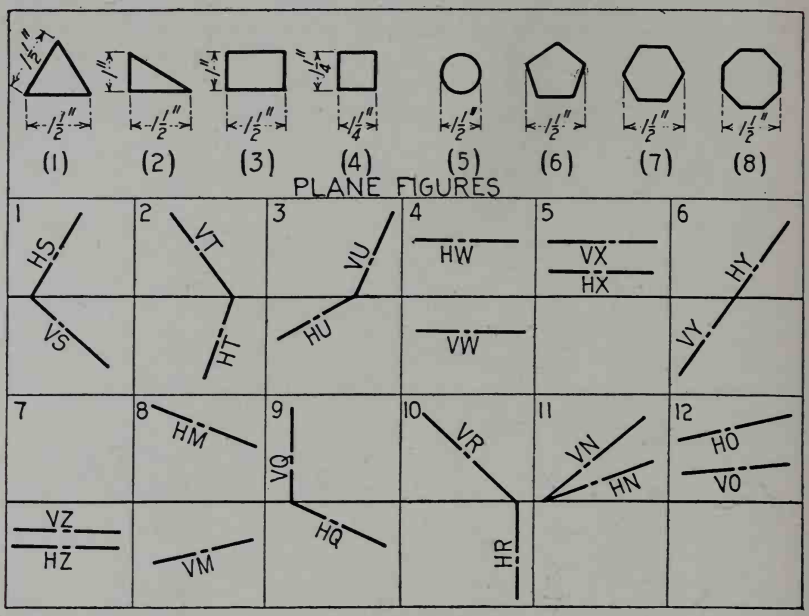
FIG. 43.—The projections of a circle.

61. EXERCISES

Draw the projections of the following plane figures in the planes given according to the stated conditions:

257. Equilateral triangle, 1½-in. sides. T(2 + 3) 2(5 − 1½). Center at A(3½ + 1, x) in T.
258. Triangle Ex. 257. U(2 + 2) 4(4 − 3). One side ∥ to H, 1 in. from HU.
259. Triangle Ex. 257. S(5 + 2) 2(5 − 1½). Center at O(4 + 1, x) in S.
260. Triangle Ex. 257. T(2 − 3) 4(2 + 2). One side ∥ to *V*, 1 in. from VT.
261. Triangle Ex. 257. U(+ 2) ∞ (− 1). Center at M(3, y − ½).
262. Triangle Ex. 257. W(− 3) ∞ (+ 2). One side ∥ to H, 1 in. from VW.

263. Triangle Ex. 257. X(+ ½) ∞ (− 2). Center at N(3 + 1½, y).
264. Triangle Ex. 257. Q(1 + 3) 3(5 − 1). Center at K(4 + 1, x).
265. Triangle Ex. 257. R(1 + 1) (5 + 2) (1 − ½) (5 − 1). Center at F(3 + 1, x).
266. Triangle Ex. 257. S(5 + 2) 1(5 + 1). Center at D(3, x − 1).



Graphic Layout No. 22.—Draw the projections of plane figure —— in given plane ——, under one of the following conditions, as selected by the instructor:

(*a*) Assume a point in the given plane, as the center of the given plane figure.

(*b*) Assume a line in the given plane as one of the sides of the plane figure.

(*c*) Assume a line in the given plane as the diagonal or diameter of the given plane figure.

(*d*) Draw one corner of the given figure in *H*, and one side inclined ——° (10, 15, 30, 40, 45) to the *H*-trace.

Note.—*V* may be substituted for *H* in condition (*d*).

(*e*) Circumscribe a circle about the plane figure —— in plane ——.

(*f*) Inscribe a circle in the plane figure —— in plane ——.

267. Triangle Ex. 257. Y(1 + 2½) 3(5 − 2½). Center at A(2½, x − 1).
268. Triangle Ex. 257. P(4 + 3) 4(4 − 3). One side ∥ to *H*, 1 in. below *H* in *III*.
269. Substitute a right triangle 1 in. × 1½ in. in any of the planes in Ex. 257–268, under the same conditions.
270. Inscribe a circle in any of the triangles in Ex. 257–269.

271. Substitute one of the following polygons for the triangle in Ex. 257–268:
(*a*) 1¼ in. square.
(*b*) 1-in. × 1½-in. rectangle.
(*c*) 1½-in. regular pentagon.
(*d*) 1½-in. regular hexagon.
(*e*) 1½-in. circle.
(*f*) 1½-in. octagon.

272. Draw the projections of polygon ——— in Ex. 271 in any of the planes given in Ex. 257–268 with one corner in H and another corner ½ in. from the H-trace.

273. Triangle in Ex. 269 in any of the planes in Ex. 257–268, with its hypothenuse ‖ to V, 1 in. from the V-trace.

274. Square, Ex. 271 (*a*) in any of the planes in Ex. 257–268, with its diagonal ‖ to H, ¾ in. above or below H.

275. Hexagon, 1¼ in. short diameter, with a ¾-in. circle in the center, drawn in any of the planes in Ex. 257–268, with its center at any chosen point in the given plane.

276. Circumscribe a circle about any of the foregoing polygons.

277. Square, Ex. 271 (*a*) in any of the planes in Ex. 257–268, with one corner in V, and a side inclined 30° to the V-trace.

278. The line M(3 + 1, x) N(4 + ½, y) in R(1 + 1) (5 + 2) (1 − ½) (5 − 1) is one of the diameters of a circle in R. Draw the projections of the circle.

279. An ellipse, major axis A(2, 0 − 1) B(3½, 0 − 1), minor axis C(2¾, 0 − ½) D(2¾, 0 − 1½) is the projection of a circle lying in a plane, whose H-trace is E(1, 0 − 2) F(4, 0 − 2). Draw the V-trace of the plane and the V-projection of the circle.

280. A 1¼-in. circle whose center in M(3, 0 − 1) is the H-projection of an ellipse lying in any of the planes in Ex. 258, 259, 261, 263, 264, 265, 266, 267. Draw the V-projection and the true size of the ellipse.

281. The roof of a tower is a hexagonal pyramid, base 14 ft., and altitude 18 ft. In each of the three front faces are rectangular windows, 2 ft. 9 in. × 6 ft. 3 in., each 3 ft. 3 in. from the tower base. Draw the projections. Scale ⅛ in. = 1 ft.

SIXTH FUNDAMENTAL PROBLEM

62. Problem 17.—To find the line of intersection of two planes.

Axioms.—1. The intersection of two planes is a straight line.

2. Two points determine a straight line.

Analysis.—1. Find both projections of the point where the V-traces of the given planes intersect.

2. Find both projections of the point where the H-traces intersect.

3. Join the respective projections of these two points, which are common to both planes.

Conclusion.—The line thus drawn is common to both planes, and therefore their intersection.

Note.—The H-projection of any V-trace is in GL, and therefore the H-projection of any point in the V-trace is in GL.

Proof.—The V-traces, being each a line in one of the given planes, intersect in a point common to both planes, and therefore in the line of intersection. Similarly for the intersection of the H-traces. Having found these two points, the line of intersection is determined.

Construction.—Left to the student. Refer to Fig. 44.

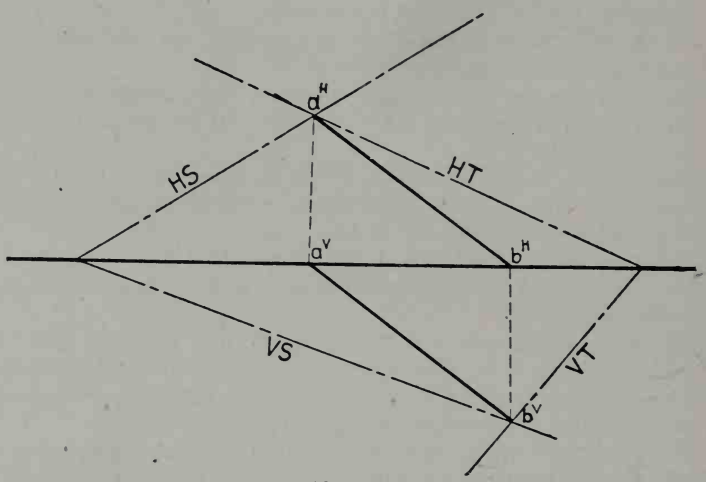


FIG. 44.

63. Special Cases.—When the traces do not intersect within the limits of the problem, or not at all.

Analysis.—1. **If one pair of traces intersect,** locate their point of intersection.

2. Draw an auxiliary plane that will intersect both given planes. The lines of intersection of the auxiliary and given planes will intersect in a point on the required line of intersection.

3. **If neither pair of traces intersect,** draw a second auxiliary plane which will determine another point on the required line, thus determining the line of intersection.

Construction.—For most oblique planes, such as the two shown in Fig. 45, it is quickest and most convenient to use planes parallel to H or V as auxiliaries. Such planes have one trace only, and that is parallel to GL. Such planes intersect oblique planes in lines parallel to their traces; that is, a plane parallel

to V will intersect an oblique plane in a line parallel to its V-trace, etc. Let the student give the reason from Solid Geometry.

1. Draw VQ (Q ∥ to H). Q will cut CM and DM from T and S respectively.

2. Locate m^V and m^H, the intersection of CM and DM.

3. Draw HR (R ∥ to V). R will cut AO and BO from T and S respectively.

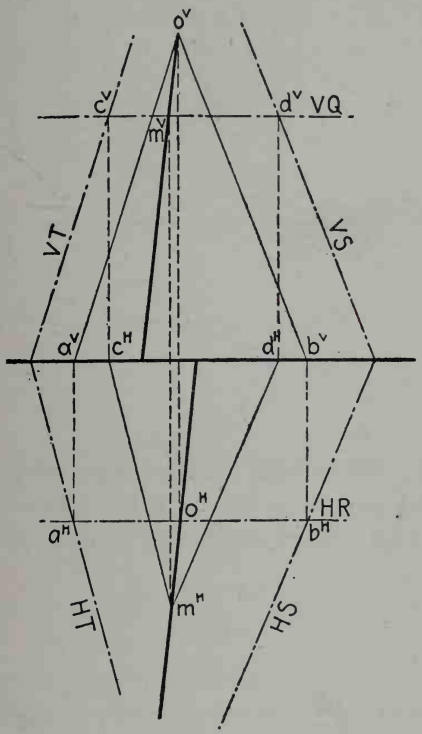


Fig. 45.

4. Locate o^V and o^H, the intersection of AO and BO.

5. Draw o^Vm^V and o^Hm^H, the projections of the required intersection.

64. Case II.—When it is convenient to use an auxiliary plane parallel to one of the given planes.

Fig. 46 shows a plane Q, parallel to S, being used to determine the direction of the line of intersection of T and S.

Construction.—1. Pass a plane Q parallel to S, so that its intersection XY with T will come inside the problem limits.

2. Draw XY, the intersection of T and Q.

3. Find M (one point on the intersection of T and S).
4. Draw MN parallel to XY.

Conclusion.—MN is the required intersection of T and S.

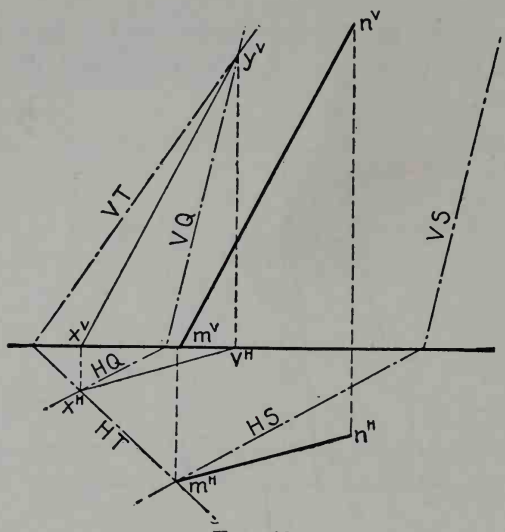


Fig. 46.

65. Case III.—When both planes are parallel to GL.

Although the planes intersect, their traces are all parallel. Inasmuch as both planes are parallel to *GL*, their intersection

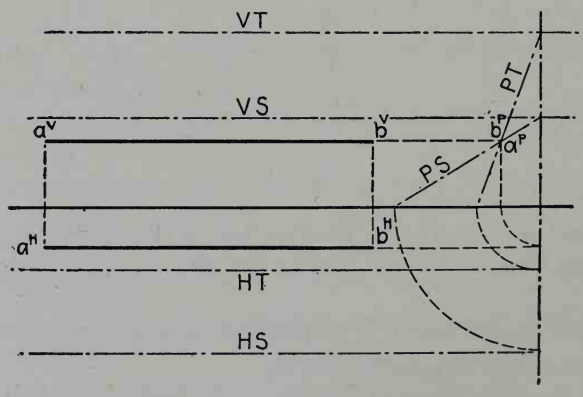


Fig. 47.

will also be parallel to *GL*. (See Solid Geometry problems). Hence only one point on the intersection is required.

Figs. 47 and 48 show two different auxiliary planes that may

be used. In Fig. 47 a profile plane is used, and the profile traces yield the intersecting point.

In Fig. 48 an oblique plane, Q, is used, which intersects T in the line CD, and S in the line EF. The lines CD and EF intersect in the point G, through which the line of intersection, AB, may be drawn.

Note.—The foregoing solutions give an idea of the multitude of resources open to the student in the solving of varieties of problems. Sometimes one method or auxiliary plane will be useful, which would be awkward under other conditions. No stereotyped method of performing these solutions can be given, for each problem will bring up conditions, where there is a preference of methods. The analyses are exactly alike, and they must be learned, but not memorized.

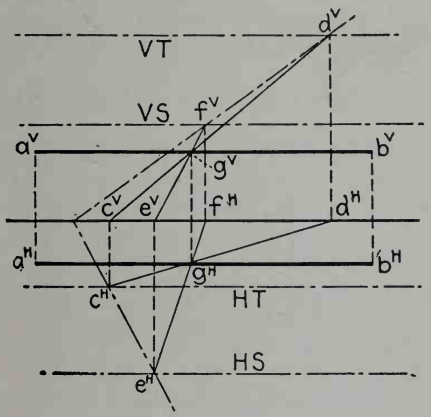


FIG. 48.

EXERCISES

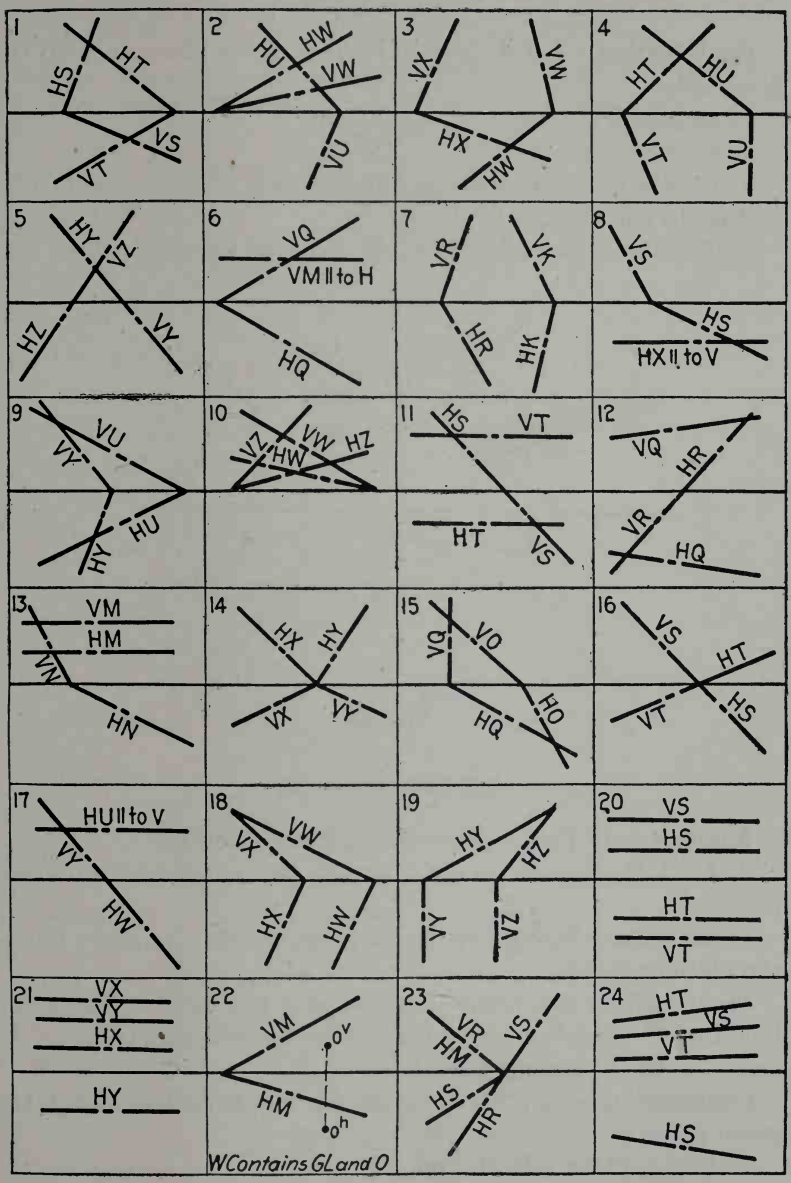
66. Draw the intersections of the following planes.

282. Q(1 + 3) 4(1 − 1) and R(5 + 3) 1(3 − 3).
283. S(1 + 3) 1(4 − 3) and T(1 + 2) 3(3 − 3).
284. U(5 + 3) 1(5 − 1) and W(3 + 3) 3(5 − 2).
285. Y(1 + 2) 4(1 − 1½) and Z(4 + 3) 1½(1½ − 3).
286. X(+ 1) ∞ (− 2) and Q(2 + 3) 5(1 − 2½).
287. R(− 1) ∞ (+ 3) and S(1 + 2) 3(1 − 1).
288. T(+ 1) ∞ (+ 1) and U(1 + 1½) 2(5 − 3).
289. W(+ 1) ∞ (− 3) and Y(+ 2) ∞ (− 2).
290. Z(+ 1) ∞ (− 1½) and Q(+ ½) ∞ (− 2).
291. R(− 2) ∞ (+ 1) and T(− 1) ∞ (− 2).
292. X(− 3) ∞ (+ 1) and S(+ 2) ∞ (− 3).
293. W(+ 2) ∞ (− 1) and Z(− 1) ∞ (+ 2).
294. X(1 + 3) 4(5 − 1) and Y(+ 2) ∞ (− ¾).
295. R(5 + 3) 3(1 − 3) and T(1 + 3) 4(5 − 1).

296. S(+ 1½, parallel to *H*) and X(5 + 2) 3(1 − 2).
297. U(+ 1, parallel to *V*) and W(5 + 2) 3½(1 − 3).
298. X(1 + 2) 3(1 − 1) and Y(5 + 1½) 3(5 − 3).
299. R(1 + 3) 3(1 − 2) and S(5 + 2) 3(5 + 1).
300. T(1 + 2) 3(5 − 2) and W(2 − 3) 3(4 + 3).
301. U(4 + 2) 3(1 − 1) and X(5 + 2) 4(5 − 2).
302. Y(1 + 3) 3(5 − 2) and Z(2 + 3) 4(1 − 1).
303. R(1 + 3) 5(3 − 3) and Q(5 + 2) 3½(1½ − 3).
304. S(1 + 3) 2(1 − 3) and U(5 + 2) 3(5 − 3).
305. T(1 + 3) 2(5 − 1) and X(5 + 2) 4(1 − 2).
306. W(1 − 3) 2½(5 + 2) and Y(4½ − 3) 4(1 + 1).
307. Q(3 + 3) 1(2 − 3) and Z (4 − 3) 5(3 − 3).
308. R(1 + ¾) (5 + 2¼) (1 − ½) (5 − 1½) and S(1 + 3) 3(5 − 3).
309. T(2 + 3) 2(5 − 2) and U(5 + 2) 3(1 − 3).
310. W(5 − 3) 1(1 − 2) and X(1 − 2½) 3(5 + 1¼).
311. Y(3 + 3) 1½(4 − 2) and Z(5 + 2) 2(4½ − 2).
312. Q(1 + 2) (5 + 3) (1 − 1) (5 − 1½) and R(1 + 1) (5 + 1½) (1 − 2) (5 − 3).
313. S(1 + 1) (5 + 3) (1 − 1½) (3 − 2) and T(1 − 1½) (3 − 2) (1 − 1) (5 − 3).
314. U(1 + 2) 5(1 − 3) and a 30° plane which contains *GL*.
314. W(5 − 1) 1(4 + 3) and a plane containing *GL* and A(3 + 1 − 2).
315. X(1 + 1) (5 + 3) (1 − ½) (5 + 1½) and a 45° plane through *GL*.
316. Draw a line through M(4 + 1 − 1) parallel to T(1 + 3) 4(1 − 1) and S(5 + 3) 1(3 − 3).
317. Draw a line 2 in. long through O, (4 + 1 − 2) parallel to W(1 + 3) 2(5 − 1) and X(5 + 3) 3(1 − 1½).
318. Draw a line 2½ in. long through A(2 + 3 − 3) parallel to X(1 + 3) 3(5 − 3) and Y(3 + 3) 2(1 − 3).
319. Find the point of intersection of the three planes, R(1 + 3) 5(1 − 2), S(− 2) ∞ (− ½), and T(5 + 2) 1(3 − 3).
320. Draw the projections of the tetrahedron bounded by *H* and the three planes in Ex. 319.
321. Develop the surface of the tetrahedron in Ex. 320.
322. Draw the projections of the frustum of the tetrahedron in Ex. 320 made by W(+ ½) ∞ (− 2).
323. Develop the surface of the frustum of the tetrahedron in Ex. 322.
324. Draw the projections of the tetrahedron formed by the intersection of the planes S(3 + 2) ½(5 + 1½), T(3 − 3) 2(4 + 2), R(3 − 3) 4½(2 + 2¾) and the plane Q, 2 in. below and parallel to *H*.
325. Develop the surface of tetrahedron in Ex. 324.
326. Draw the projections of the frustum of the tetrahedron in Ex. 324, made by the plane Y(− 1¾) ∞ (+ 3).
327. Develop the surface of the frustum in Ex. 326.

The Piercing Point of a Line and Plane

67. Problem 18.—To find the point where an oblique plane is pierced by a line.



Graphic Layout No. 23.—Draw the projections of the line of intersection of the planes given in Ex. ——.

FIRST METHOD

Construction.—Let T (Fig. 49) be the given plane and AB the given line.

1. Obtain QT.
2. Obtain $a^q b^q$.
3. Locate O^q (the piercing point) where $a^q b^q$ intersects QT, and project it back to $O^H O^V$.

Conclusion.—The point O is the required piercing point.

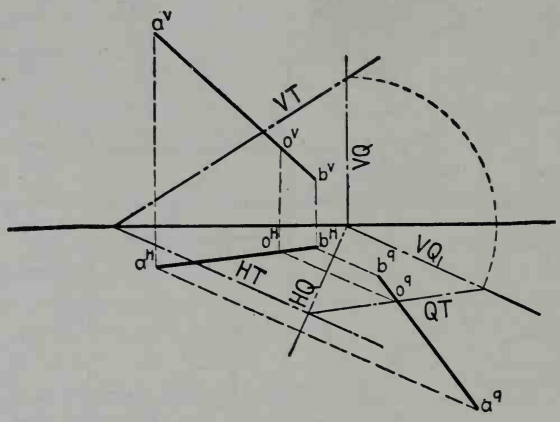


FIG. 49.

SECOND METHOD

Analysis.—1. Pass a plane through the given line.

2. Find the line of intersection of the auxiliary and given planes.

Conclusion.—Where this intersection crosses the given line is the piercing point.

Proof.—Since the point thus found is not only on the given line, but in the given plane, it must be the common point of the two, *i.e.*, the "piercing point."

Construction.—Let AB (Fig. 50) be the given line and T the given plane.

1. Pass a plane (Q) through AB.

Note.—While any of the infinite number of planes containing AB may be used, it is simpler and preferable to use the *H*- or *V*-projecting plane. In this construction the *H*-projecting plane (Q) is used.

2. Obtain MN, the intersection of Q and T.

3. Locate C, where MN crosses AB.

Conclusion.—C is the point in which AB pierces T.

Note.—This problem figures in most of the succeeding problems, and should be thoroughly mastered.

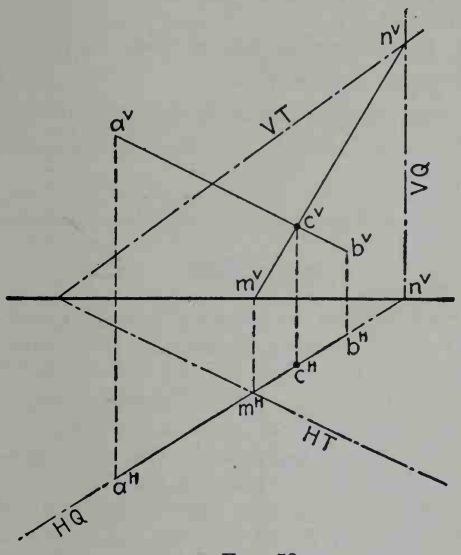


FIG. 50.

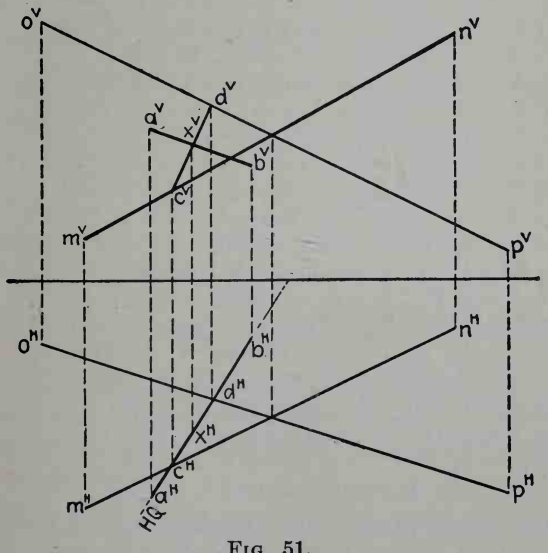


FIG. 51.

68. Problem 19.—To solve Problem 18 without the traces of the given plane.

Construction.—Let the two intersecting lines MN and OP determine the given plane, and AB be the given line (Fig. 51).

1. Pass the projecting plane Q through AB.

(Note.—HQ is sufficient.)

2. Project the Q-piercing points of MN and OP, c^H and d^H, to their V-projections, c^V and d^V.

3. Draw c^Vd^V, the intersection of Q and the plane MNOP.

4. Where c^Vd^V crosses a^Vb^V, at x^V, the piercing point is located. Project to x^H on a^Hb^H.

Note.—This problem is of great value in surface intersections.

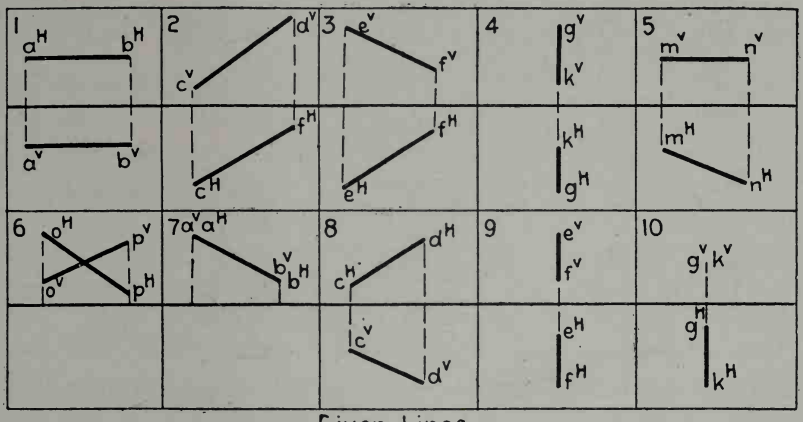
EXERCISES

69. Find the piercing points of the following lines and planes. Familiarize yourself with both methods by doing the exercises both ways.

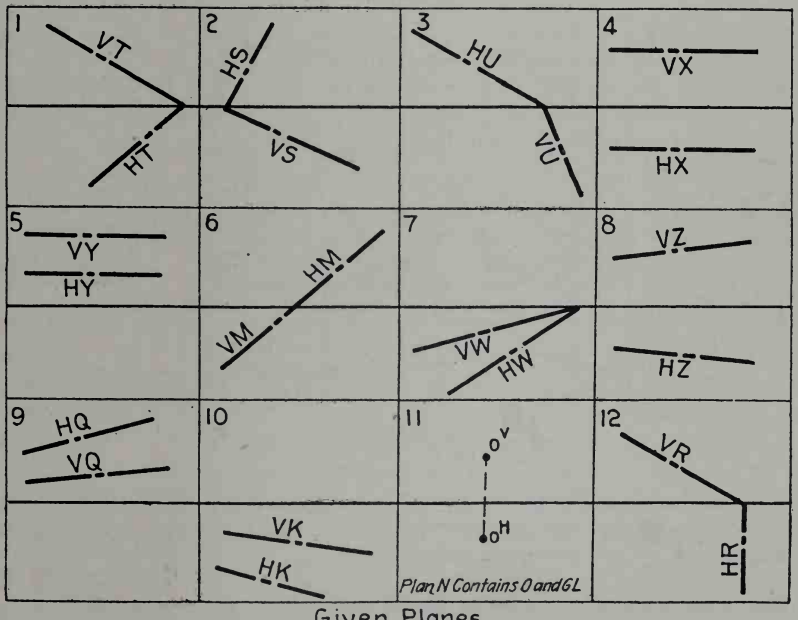
328. A(2 + 3 − 2) B(4 + 1 − 1) and S(5 + 3) 1(5 − 1½).
329. C(1 + 1½ − ½) D(4 + 1½ − ½) and S in Ex. 328.
330. D(3 + ½ − 1¼) E(5 + 2 − 2) and S in Ex. 328.
331. F(3 + 2, 0) G(3 + 2 − 3) and S in Ex. 328.
332. K(3 + ½ + ½) L(4 + 3 + 3) and S in Ex. 328.
333. M(3 − 1 + 1) N(4 − 2 + 3) and T(2 − 2) 5(3 + 3).
334. O(1 − 1 + 2) P(5 − 1 + 2) and T in Ex. 333.
335. A(1 + ½ + 1½) B(5 + ½ + 1½) and T in Ex. 333.
336. C(3½ − 2 + 3) D(3½ − ½ + ½) and T in Ex. 333.
337. E(3½ + 1 + 3¼) F(5 + 2 + 1) and T in Ex. 333.
338. G(2 + 2 − 1) K(4 + 1 − 3) and X(5 + 3) 3(1 − 3).
339. M(1 + 2 − 1) N(4 + 2 − 1) and X, Ex. 338.
340. O(4 + 3 − 1) P(2 − 3 + 1) and X, Ex. 338.
341. A(3 − 3 + 2) B(1 − 1 + 1) and Y(− 1) ∞ (+ 2).
342. C(1 − 2 + 3) D(3 − ½ + ½) and Y, Ex. 341.
343. E(3 + 2 − 2) F(3 + ½ − 1½) and U(+ 1½) ∞ (− 1).
344. G(1 + 2 − 2) K(3½ + 1 − ¾) and U(Ex. 343).
345. M(2 + 2 − 1) N(3 + 1 − 1½) and R(5 + 2) 1(5 + 1).
346. O(3 + 3 − ½) P(3 + 1 − 2) and R(5 + 2) 1(5 + 1).
347. A(1 + 1 − 1) B(5 + 1 − 1) and R (Ex. 346).
348. C(3 − 1 + 2) D(5, 0 + 3) and R(Ex. 346).
349. E(2 + 3 − 3) F(2 + ½ − 1½) and T(5 + 2) (1 + 1) (5 − 3) (1 − 1½).
350. G(2 + 2 − ½) K(3 + ½ − 1½) and T, Ex. 349.
351. M(1 + 2 − ½) N(5 + 2 − ½) and T, Ex. 349.
352. A(4 + 2 − 1) B(3 + 1 − 3) and S(1 + 3) 2(5 − 2).
353. C(1 + 1 − 2) D(4 + 1 − 2) and S, Ex. 352.
354 E(2 + 2 − ½) F(2 + 1 − 2) and S, Ex. 352.
355. G(3 + 2 − 1½) K(3¾, 0 − 2) and S, Ex. 352.

Locate the piercing points of the following lines with the planes of the various given lines, without drawing the traces of the planes.

356. M(2 + 2 − 1) N(3½ + 1 − 2½) and the plane of O(5 + 3 − 1) P(1 + 1 − 1½) K(4 + ½ − 2).



Given Lines

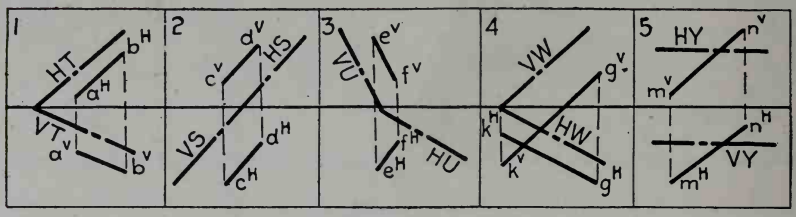


Given Planes

Graphic Layout No. 24.—Locate the point in which the line —— pierces the plane ——. **Note.**—Draw the line so that it is likely to pierce the plane. The H and V projections and traces may be transposed, if desired.

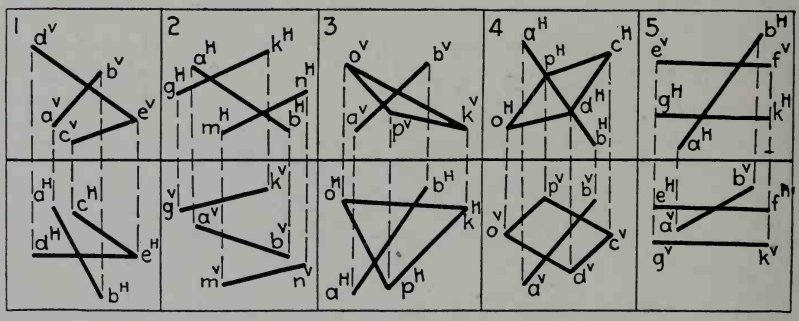
357. A(5 − ½ + 3) B(2 − 2½ + 1) and the plane of C(5 − 2 + 2) D(1 − 1 + 1½) E(5, 0 + 1).

358. P(3 + 2 − 3) O(3 + 1 − ½) and the plane of G(3 + 2 − 2) K(1 + 1 − 1½) L(5 + 1, 0).
359. M(2 − 3 + 2) N(2 − ½ + 1) and the plane of A(1 − 2 + 2½) B(4 − 1 + 1) C(1 − ½ + 1½).
360. D(1 + 3 − 1) E(3½ + 1 − 2) and the plane of F(2 + 1½ − 3) G(4 + 2½ − 1) and K(2½ + ½ − 2) L(4½ + 1½, 0).
361. M(3 + 2 − ½) N(3 + ½ − 1½) and the plane of FG − KL, Ex. 360.



Graphic Layout No. 25.—Find where the given line pierces the given plane in Ex. ——.

362. O(4 − 2 + ½) P(2½ − ½ + 2½) and the plane of A(2 − 1½, 0) B(4 − ½ + 2) and C(2 − 2½ + 1) D(4 − 1½ + 3).
363. From the middle point A of the line B(1 + 2 − 1) C(1 + 1 − 3) draw a line which shall intersect M(2 + ½ − 1) N(4 + 3 − 3) and O(3½ + 1½ − 1) P(4 + ½ − 1½).



Graphic Layout No. 26.—Find where the given line AB pierces the plane of the intersecting or parallel lines in Ex. ——, without drawing the plane-traces.

364. Draw a right hexagonal prism, its center line E(1 + ¾ − 1) F(3½ + ¾ − 2), with one of its faces in *H*. Find the points in which it is pierced by G(1½ + 2 − ½) K(3, 0 − 2¾).
365. Draw the projections of the shortest line on the surface of the hexagonal prism (Ex. 364) between the piercing points of GK.
366. Hexagonal prism in Ex. 364. Find the piercing points of the three parallel lines A(1, 0 − 2) B(3 + 2 − 2), C(1¼, 0 − 1) D(3¼ + 2 − 1), and E(2, 0 − 1½) F(4 + 2 − 1½).

367. Hexagonal prism in Ex. 364. Find the piercing points of the three parallel lines G(1½, 0 − 1) K(1½ + 2 − 1), L(2, 0 − 2) M(2 + 2 − 2), and N(2½, 0 − 1) P(2½ + 2 − 1).

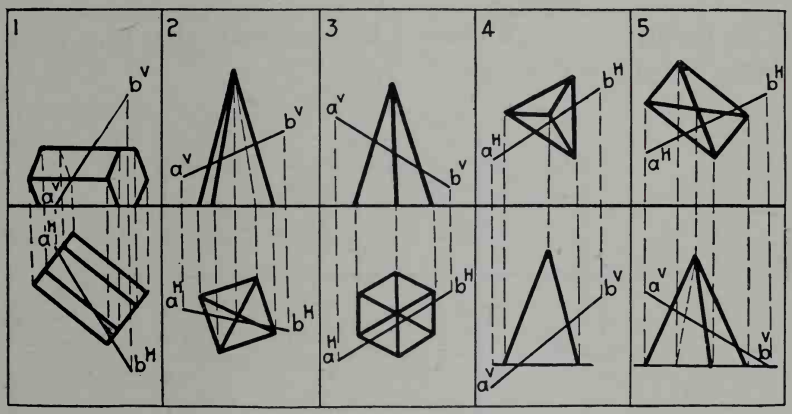
368 Draw a right square pyramid whose base diagonal is A(2, 0 − 1) B(4, 0 − 2) and apex C(3 + 2½ − 1½). Find the points in which it is pierced by the line E(1½, 0 − 2) F(4 + 2 − ½).

369. Pyramid in Ex. 368. Find the points in which it is pierced by four lines parallel to *GL*, through the points M(4 + 1 − 1¾), N(4 + ½ − 1¼), O(4 + 1 − ¾), and P(4 + 1½ − 1¼).

370. Pyramid in Ex. 368. Find the points in which it is pierced by four lines ∥ to *V* and 45° to *H*, through the points M, N, O and P in Ex. 369.

371. Pyramid in Ex. 368. Find its intersection with the solid whose four edges are the lines in Ex. 370.

372. Pyramid in Ex. 368. Find its intersection with the prism whose four edges are the lines in Ex. 369.



Graphic Layout No. 27.—(*a*) Find where the given line pierces the given solid in Ex. ——.

(*b*) Find the intersection of solid in Ex. —— by the —— (*H*, *V*) projecting plane of AB. Find its true size.

(*c*) Pass a plane through solid in Ex. ——, its traces ∥ to the respective projections of AB, Ex. ——. Find the intersection and true size.

(*d*) Pass a profile plane ½ in. to the right of the center of solid ——. Find the intersection and true size.

(*e*) Develop the truncated portion of the solid made by the plane in one of the foregoing exercises.

373. Pyramid in Ex. 368. Find its intersection with the plane T(5 + 2) ½(3 − 3). Crosshatch the section and show it in its true size.

374. Develop the surface of the pyramid in Ex. 373, between *H* and T. Make a paper model of it.

375. Pyramid in Ex. 368. Find its intersection with the plane S(+ 1½) ∞(− 3). Crosshatch the section and show it in its true size.

376. Develop the surface of the pyramid in Ex. 375 between H and S. Make a paper model of it.
377. Prism in Ex. 364. Find its intersection with the plane U(2 + 3) 2-(3 − 3). Crosshatch the section and show it in its true size.
378. Develop the surface of the prism in Ex. 377 between one of its bases and U. Make a paper model of it.
379. Prism in Ex. 364. Find its intersection, and the true size of the intersection, with a profile plane through A(2½, 0, 0).
380. Develop the surface of the prism in Ex. 379 between one base and the profile plane. Make a paper model.
381. Prism in Ex. 364. Draw its intersection by the plane S(2 + 3) 4½-(1 − 3). Crosshatch the intersection and show its true size.
382. Develop the prism in Ex. 381 between one of its bases and S. Make a paper model.

Perpendiculars to Planes

70. Problem 20.—To draw a line perpendicular to a given plane, through a given point.

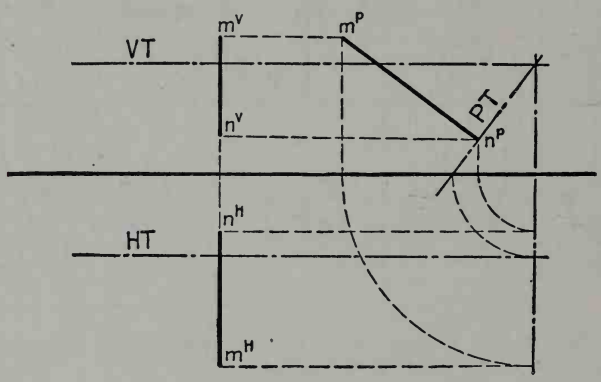


Fig. 52.

Analysis.—1. Through the V-projection of the given point draw the V-projection of the required line perpendicular to the V-trace of the given plane.

2. Through the H-projection of the given point draw the H-projection of the required line perpendicular to the H-trace of the given plane.

Construction.—Let the student make the construction for the ordinary case.

Special Case.—When the given plane is parallel to GL.

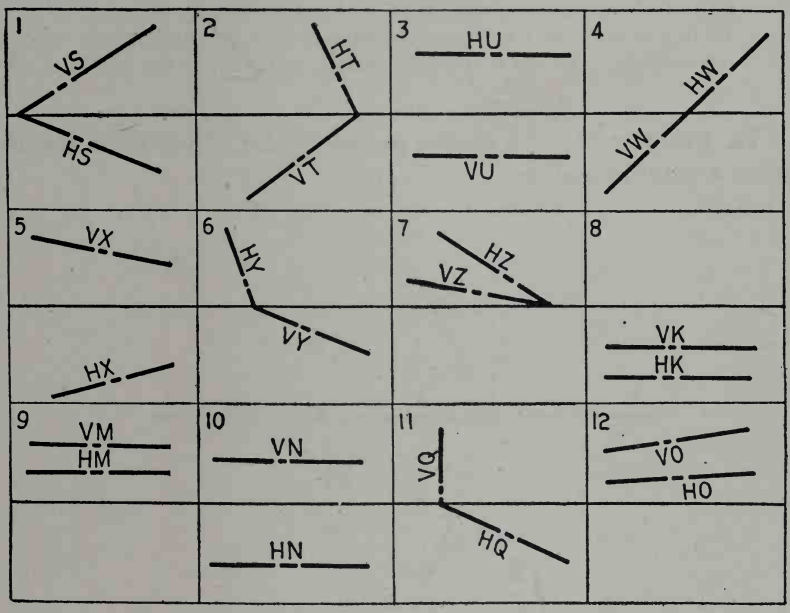
Construction.—Let M (Fig. 52) and T be the given point and plane.

1. Obtain the *P*-projection of M.
2. Obtain the *P*-trace of T.
3. Draw the perpendicular m^Pn^P to PT.
4. Project n^P to n^V and n^H on a line through M perpendicular to the traces of T.

Conclusion.—MN is perpendicular to T.

Note.—N need not be a point in the plane, but any point in the line.

71. EXERCISES



Graphic Layout No. 28.—From A(3 + 2 − 2) (or assume the point, if preferred) draw a perpendicular to plane ——. Find where the perpendicular pierces the given plane. Find the true length of the perpendicular from the point to the plane.

383. From A(2 + 2 − 2½) draw a perpendicular to T(5 + 2) 1(4 − 3), and find where it pierces it.
384. From B(1 − 2 + 1) draw a perpendicular to S(5 − 3) 5(2 + 3), and find where it pierces it.
385. From C(4 − 1 − 2) draw a perpendicular to R(3 − 3) 1(4 − 3), and find where it pierces it.
386. From D(2 + 1 − ½) draw a perpendicular to X(+ 2) ∞ (− 2), and find where it pierces it.

387. From E(2 − 1 + ½) draw a perpendicular to U(5 − 2) (1 − 1) (5+3) (1 − 1½), and find where it pierces it.
388. From F(3 + 2 − 2) draw a perpendicular to W(1 + 2) 3(5 − 2), and find its W-piercing point.
389. From G(3, 0, 0) draw a perpendicular to Y(1 + 2) 4(1 − 3), and find where it pierces it.
390. From K(3, 0, 0) draw a perpendicular to T(− 2) ∞ (− 1).
391. Scale ⅛ in. = 1 ft. The roof of a building contains two points: A (16 ft., 0, − 8 ft.), and B(22 ft., + 14 ft. 6 in., − 2 ft.). The ridge and lower edge run 24 ft. horizontally to the southeast. At C(28 ft., 0 − 6 ft.) it is desired to erect a rafter brace perpendicular to the roof. At what point will it support the roof, and how long will it be?
392. Scale ⅛ in. = 1 ft. A tower, a hexagonal pyramid, 18-ft. base, 16-ft. altitude, base center at O(24 ft., 0 − 8 ft.). From B(42 ft. + 20 ft. − 18 ft.) it is desired to run a telephone wire perpendicularly to the nearest face. Where is the hole drilled, and what is the length of the wire?

72. Problem 21.—To erect a perpendicular of any given length from a point in a plane.

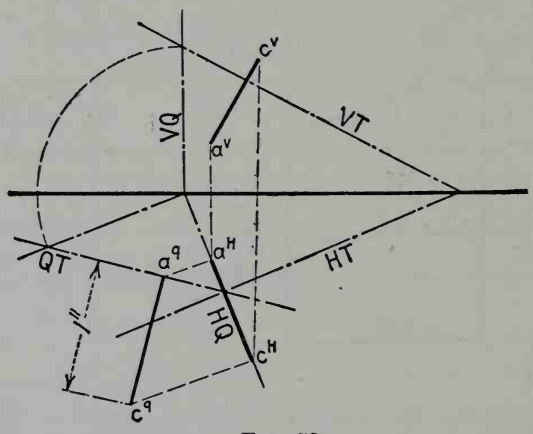


FIG. 53.

Construction.—Let A (Fig. 53) be the given point, and T the given plane. Required to erect a perpendicular 1 in. long.

1. Draw a perpendicular to T of indefinite length through A.
2. Obtain QT.
3. Obtain a^q.
4. Erect the perpendicular a^qc^q, 1 in. long.
5. Project c^q back to c^H, thence to c^v.

Conclusion.—AC is perpendicular to T, and 1 in. long.

SECOND METHOD

Analysis.—1. Erect a perpendicular of indefinite length from the given point, according to Problem 20.

2. Assume a point on the perpendicular.

3. Revolve the definite line, thus drawn, so as to show its true length.

4. Lay off the required distance on the true length of the line from the given point.

5. Return the limiting point to its proper position in space.

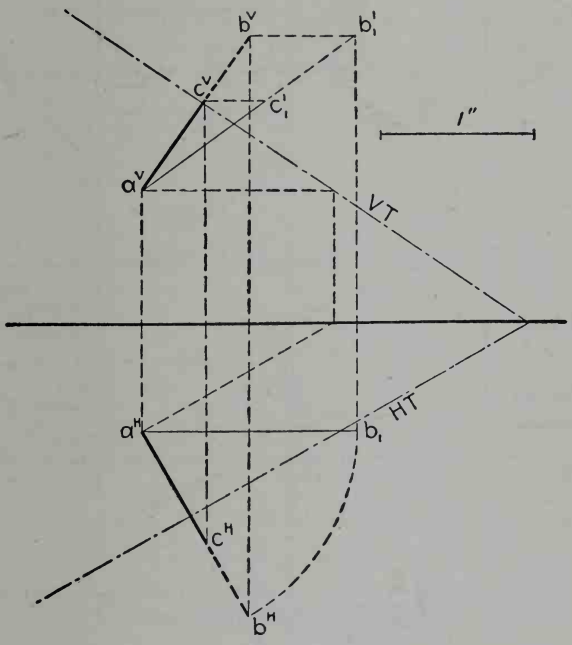


FIG. 54.

Construction.—To erect a perpendicular 1 in. long at A (Fig. 54) in the plane T.

1. Draw a^Hb^H and a^vb^v, respectively perpendicular to HT and VT.

2. Having assumed B on the line, revolve it parallel to *V*, to show its true length.

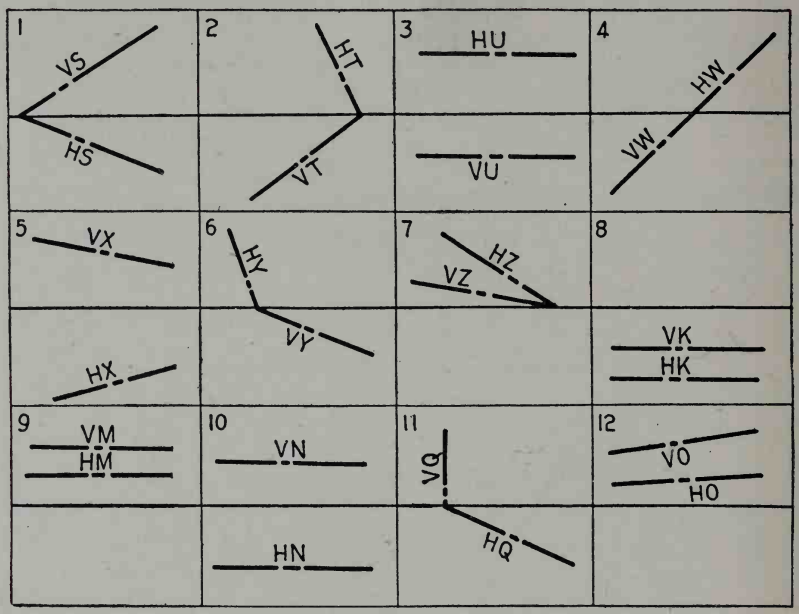
3. Lay off 1 in. on this revolved line $a^vc_1{}^v$.

4. Return $c_1{}^v$ to c^vc^H on AB.

Conclusion.—AC is perpendicular to T, and 1 in. long.

74. EXERCISES

393. From A(2 + 1, x) in T(1 + 3) 4(1 − 2) erect a perpendicular 2 in. long.
394. From B(1, y − 1) in S(+ 2) ∞ (− 1) erect a perpendicular 1½ in. long.
395. From C(2, z − 1) in R(1 + 2½) 5(1 + 1) erect a perpendicular 2 in. long.
396. From D(3 − 1, y) in Q(1 − 3) (5 − 1) (1 + 1½) (5 + ½) erect a perpendicular 1½ in. long.



Graphic Layout No. 29.—Assume a point in plane ——, and erect a 1½-in. perpendicular to the plane from the assumed point.

397. From E(2 − 1½, x) in U(2 − 3) 3½(5 + 3) erect a perpendicular 1 in. long.
398. Draw the projections of a right prism, 1¾-in. altitude, having M(1 + 2 − 1¼) O(1¾ + 1 − 2) P(2½ + 1½ − 1) as its base.
399. Assume the projections of any plane figure in space and erect a right prism of 2-in. altitude on it as a base.

The Projections of Oblique Solids

75. Problem 22.—To draw the projections of a solid having its base in an oblique plane.

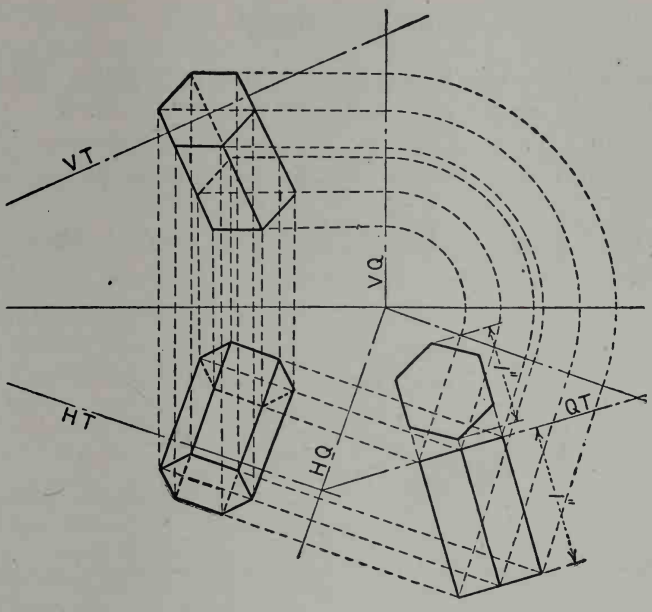


FIG. 55.

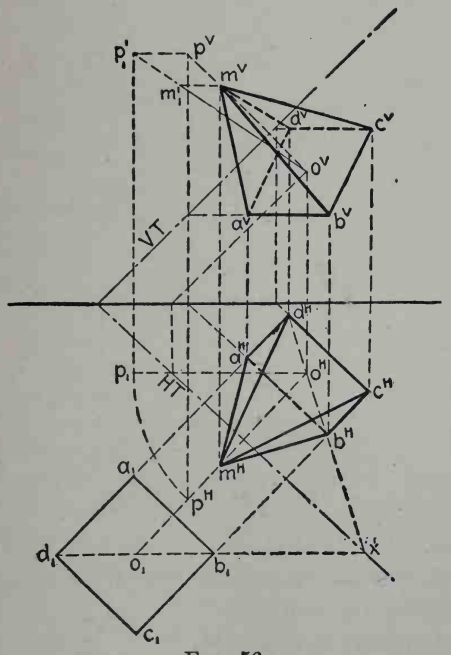


FIG. 56.

FIRST METHOD

Construction.—Let T (Fig. 55) be the given plane, and let it be required to draw a 1-in. × 1½-in. hexagonal prism.

1. Obtain QT.

2. With QT as a ground line draw the solid in its natural position.

3. Project it back on *H* and *V*, following the dotted projectors, as shown in the figure.

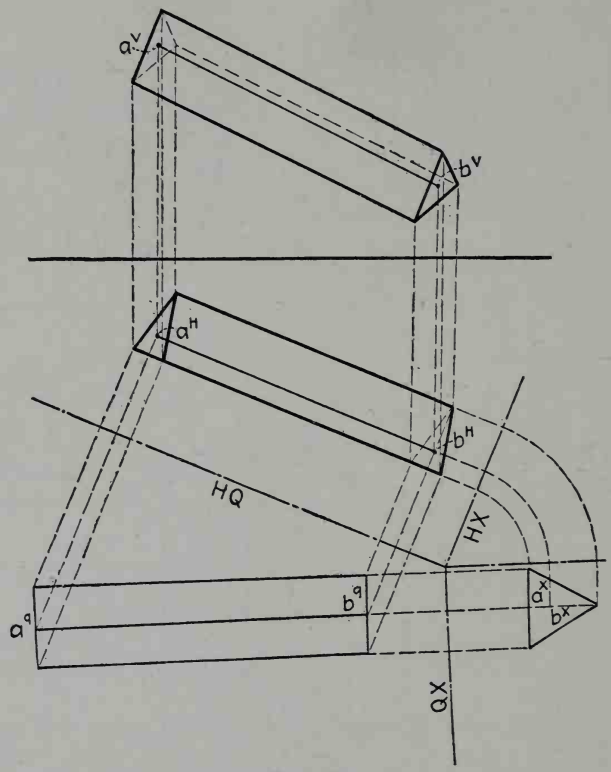


FIG. 57.

76. SECOND METHOD

Construction.—Let O (Fig. 56) be the center of a 1-in. square in the plane T. Let it be required to draw a pyramid of 2-in. altitude on this square as a base.

1. Revolve O about HT into *H*.

2. Construct a 1-in. square about O, as a center.
3. Counter-revolve the square into T, as in Problem 16.
4. At O erect OM, a 2-in. perpendicular to T.
5. Connect M with the four corners of the square.

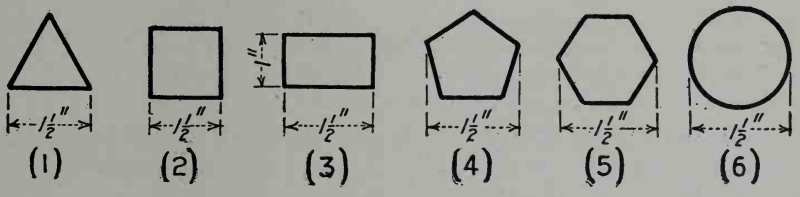
77. Supplementary Problem.—To draw the projections of a solid, having its center line given.

Construction.—Let AB (Fig. 57) be the center line of an equilateral triangular prism, faces 1 in. × 3 in. altitude.

1. Locate the Q-projection of AB.
2. Draw QX perpendicular to AB, locating $a^x b^x$, a point.
3. Draw the Q- and X-projections of the prism.
4. Project these views on *H*, and thence on *V*.

78. EXERCISES

In the following exercises, twelve typical solids are given, to be erected with their bases in the given planes with their base centers at given points in the planes. Where the base diameter is given it signifies the diameter of the circumscribing circle. **Consider the solids opaque, and dot the invisible lines.** (See Figs. 55, 56 and 57.)



The solids to be erected:

(*a*) Equilateral triangular prism, 2-in. altitude.
(*b*) Equilateral triangular pyramid, 2-in. altitude.
(*c*) Square prism, 2-in. altitude.
(*d*) Square pyramid, 2-in. altitude.
(*e*) Rectangular prism, 2-in. altitude.
(*f*) Rectangular pyramid, 1¾-in. altitude.
(*g*) Pentagonal prism, 2-in. altitude.
(*k*) Pentagonal pyramid, 2-in. altitude.
(*l*) Hexagonal prism, 2-in. altitude.
(*m*) Hexagonal pyramid, 2-in. altitude.
(*n*) Circular cylinder, 2-in. altitude.
(*o*) Circular cone, 2-in. altitude.

Place one of the foregoing solids (the instructor's assignment) on the following planes, the center at the points given, and the base diagonal parallel to *H* or *V*, according to assignment.

400. Solid ——, Q(1 + 3) 1(4 − 3), Center O(3 + 1, x).

401. Solid ——, R(1 − 3) 1(5 + 2), A(4 − 1, y).
402. Solid ——, S(4 + 2) 2(4 − 1), B(4, x − ½).
403. Solid ——, T(1 − 3) 3(1 + 2), C(1½ − 1, x).
404. Solid ——, U(1 + 2) 2½(4 − 2), D(2½, y − 1).
405. Solid ——, W(5½ − 2) 4(2½ + 2), E(3½ − ¾, x).
406. Solid ——, X(4 + 3) 1(5 + 1), F(3 + y − ½).
407. Solid ——, Y(1 + 1½) (5 + ¾) (1 − 1) (5 − ½), G(2 + 1, x).
408. Solid ——, Z(5 − 3) (1 − 2) (5 + 2¼) (1 + 1½), K(3 − 1¾, y).
409. Solid ——, S(1 + 3) 2(5 − 1½), L(4 + 2, x).
410. Solid ——, T(5 − 2½) 4(1 + 1), M(2, y + 1½).
411. Solid ——, U(+ 2) ∞ (− 1), N(3 + 1, x).
412. Solid ——, W(− 3) ∞ (+ 1), O(3 − 1, y).
413. Solid ——, X(+ 1) ∞ (− ½), P(2, x − 2).

Erect one of the foregoing solids on one of the following center lines. Draw one side of the base ‖ to *H* or *V*, according to assignment.

414. Solid ——, Center line A(1 + 2 − 1) B(3 + 2 − 1).
415. Solid ——, Center line C(2½ − 1 + 1½) D(1 − 1 + 1½).
416. Solid ——, Center line E(1 − 1 + 1½) F(3 − 2 + 1½).
417. Solid ——, Center line G(5 − 1 + 2) K(3 − 1 + 1).
418. Solid ——, Center line M(1 + 2 − ½) N(2 + ½ − 2½).
419. Solid ——, Center line O(4 − 3 + 1) P(5 − 1 + 2½).
420. Solid ——, Center line A(1 + 3 − 3) B(2 + 1 + ½).
421. Solid ——, Center line C(4 − 1 + 1) D(5 − 2½ + 3).
422. Solid ——, Center line E(2 + 2 − ½) F(2 + 1 − 2).
423. Solid ——, Center line G(2 + 2½ − 2½) K(2 + 1 − 1).
424. Solid ——, Center line M(2 − 2 + ¾) N(2 − ¾ + 2).
425. Solid ——, Center line O(2 − 1 + ¾) P(2 − ¾ + 2½).

Erect solid b, d, f, k, m, or o, according to assignment, with its apex in the point given, and its base in the plane given in the following exercises. Disregard the altitudes given, as they may not fit.

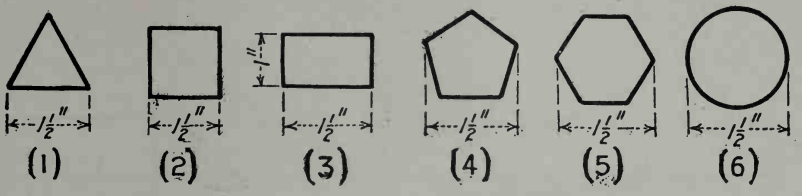
426. Solid ——, Apex A(5 + 1½ − ½), Base T(1 + 3) 1(4 − 2).
427. Solid ——, Apex B(3 + 2 − 2), Base S(1 + 2) 2½(2½ − 3).
428. Solid ——, Apex C(1 + 3 − 2), Base U(2 + 2) 5(2 − 1).
429. Solid ——, Apex D(3 − 2½ + 3), Base W(5 − 2) 3(5 + 3).
430. Solid ——, Apex E(1 + 2 − ½), Base X (5 + 2) 4(1 − 1).
431. Solid ——, Apex F(3 − 1 + 2), Base Y(1 − 3) 2(5 + 1).
432. Solid ——, Apex G(3 + 1 − 2½), Base Z(5 + 3) 1(5 + 1).
433. Solid ——, Apex K(3, 0, 0), Base Q(1 + 3) (5 + 2) (1 − 2¼) (5 − 1½).
434. Solid ——, Apex M(4 − 3 + 2½), Base R(1 − 1½)(5 − ¾) (1 − 2) (5 − 1).
435. Solid ——, Apex N(5 + 2 − ½), Base S(1 + 3) 3(5 − 3).
436. Solid ——, Apex O(1 − 3 + 1), Base T(5 − 2) 3½(2 + 2).
437. Solid ——, Apex P(3 + 3 − 3), Base U(+ 2) ∞ (− 1).
438. Solid ——, Apex A(2 − 2 + 2), Base W(− 2½) ∞ (+ 1).
439. Solid ——, Apex B(2 − 2 + 1), Base X(+ ½) ∞ (+ 1¼).
440. Erect a 1-in. cube on T(1 + 2) 4(1 − 1), with its base center at O(2 + 1, x), diagonal ‖ to *H*.

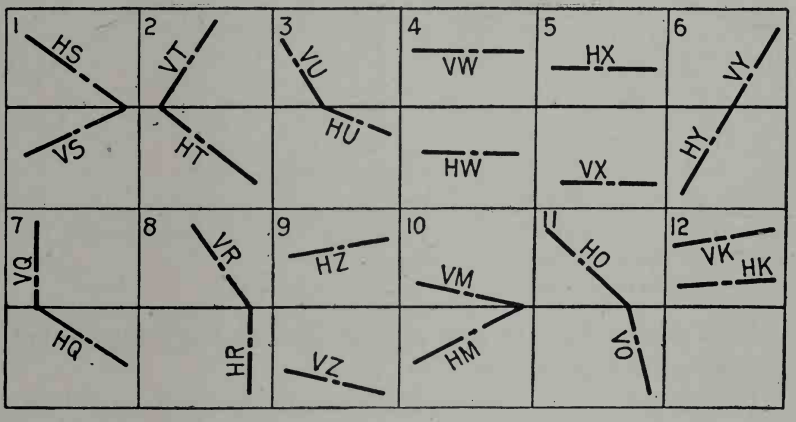
441. Draw a 1-in. cube on S(+ 2) ∞ (− 1) with one corner at A(3 + ¼, x) and another touching H.

442. Draw a cube, one of whose edges is the line A(4 + 1 − 2) B(3 + 1 − 1) and another touches H.

443. The center line of a cube is C(2 + 2 − 2) D(3 + 1 − 1½). Draw the cube, (*a*) with an edge ‖ to H; (*b*) with its base diagonal ‖ to H.

444. The line E(2 + 1½, x) F(3 + ¾, y) in S(+ 2) ∞ (− 1½) is one edge of a cube, base in S. Draw the projections of the cube.





Graphic Layout No. 30.—Let the polygons be bases of 2-in. prisms or pyramids (or cones or cylinders for No. 6).

(*a*) Erect prism (or cylinder), base —— on plane ——, assuming a center in the plane.

(*b*) Erect pyramid (or cone), base —— on plane ——, assuming a center in the plane.

(*c*) With the base —— on plane —— under any of the conditions given in Graphic Layout No. 22, erect a 2-in. pyramid or prism.

(*d*) Assume a convenient point in space (*e.g.*, A(3 + 2 − 2) or B(3 − 2 + 2)) as the apex of pyramid ——, or cone ——, to be drawn with its base in plane ——.

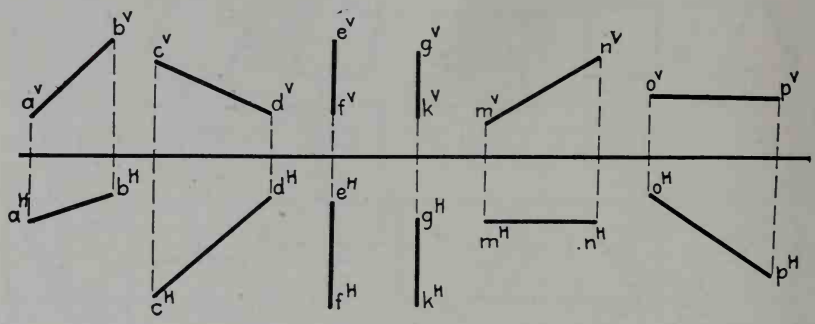
(*e*) Draw prism, base ——, with its center line parallel to —— (H, V, or P), one of its faces touching plane ——.

(*f*) Draw hollow prism or cylinder, with ¼-in. walls, under the conditions (*a*) or (*e*).

445. The lines G(2 + 1 − 1) K(1 + ½ − 1¼) and KL(2½ + 1¼ − 1½) form two sides of a parallelogram, which is the base of a prism of 1½-in altitude. Draw the prism.

446. The line M(1 + 1 − 1) N(2 + ¼ − ¾) is one side of a rectangle which forms the base of a prism 1½-in. altitude. The opposite side passes through D(2 + 1 − 1). Draw the prism.

447. Draw a line from P(2½ + 2 − ½) that shall intersect M(1 + 2 − 1½) N(3 + ¼ − 1) and be at a distance of 2 in. from P(3½ + 1 − 2½).



Center lines for solids.

Graphic Layout No. 31.—(*a*) With line —— as center line, draw 2-in. prism (or cylinder) with base ——. (*b*) With line —— as center line, draw a 2-in. pyramid (or cone) with base ——.

Note.—The signs on these lines may be transposed.

79. Problem 23.—To pass a plane through a given point perpendicular to a given line.

Principle.—The traces of a plane perpendicular to any line are respectively perpendicular to the projections of the line; hence we know the **direction of the traces.**

Analysis.—Apply the principle given.

Construction.—Let O (Fig. 58) be the given point, and AB the given line.

1. Draw $o^H p^H$ perpendicular to $a^H b^H$.
2. Draw $o^V p^V$ parallel to *GL*.
3. Obtain ⊙ (*P*) of OP.
4. Through ⊙ draw VT perpendicular to $a^V b^V$.
5. Where VT intersects *GL* draw HT, perpendicular to $a^H b^H$.

Conclusion.—T contains O, and is perpendicular to AB.

Notes.—1. If the first trace does not intersect *GL* within the problem limits, draw a second auxiliary line parallel to the second required trace.

2. If the given line lies in a profile plane, the simplest solution will be accomplished through the use of the *P*-projection.

80. EXERCISES

448. Through A(3 + ½ − 1) pass a plane perpendicular to B(1 + 1 − C(2 + 3 − 2).
449. Through D(4 − 1 + 1) pass a plane ⊥ to E(2 − 3 + 1) F(5 − 1 + 2).
450. Through G(2 − 1 + 1) pass a plane ⊥ to K(4 + 1 − 1) L(5 + 3 − 2).
451. Through M(3 + 1 − 1) pass a plane ⊥ to N(4½ + ½ − 1) P(5 + 3 − 2½).
452. Through O(2 − 1 + 2) pass a plane ⊥ to A(3 + 2 − ½) B(3 + 1 − 3).

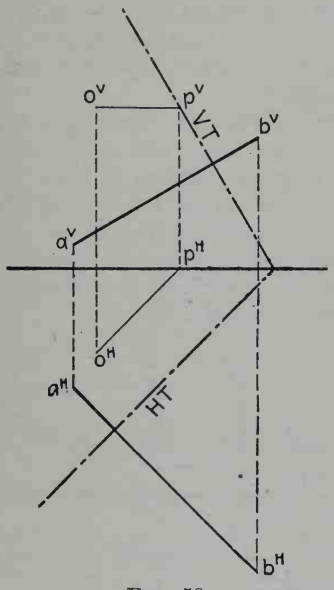


Fig. 58.

453. Through P(2 + 1½ − 2) pass a plane ⊥ to C(3 + 2 + 1) D(3 − 1 + ½).
454. Through K(5 + 1 − 1) pass a plane ⊥ to E(3 + 3 − 1) F(2 + 1 − 3).
455. Pass a plane ⊥ to G(2 + 1 − 1) K(3 + 2 − 3) through the middle point of the line.
456. Pass a plane ⊥ to M(1 + 1 − ½) N(4 + 3 − 1½) through the point M.
457. One edge of a cube is the line A(3 + 1½ − 1½) B(4 + ½ − 1) and one of its corners touches *H*. Draw its projections.
458. M(3½ + 2 − 2½) N(1 + 1½ − 1½) is the base of an isosceles triangle whose vertex is in *GL*. Draw its projections and revolve to show its true size.

459. The center line of a stick 1½ in. square is the line O(2 + 3 − 2) P(3 + 1 − 1). Two ½-in. holes are drilled straight through the stick, ¾ in. from the ends. Draw the projections of the stick and holes, dotting invisible lines.

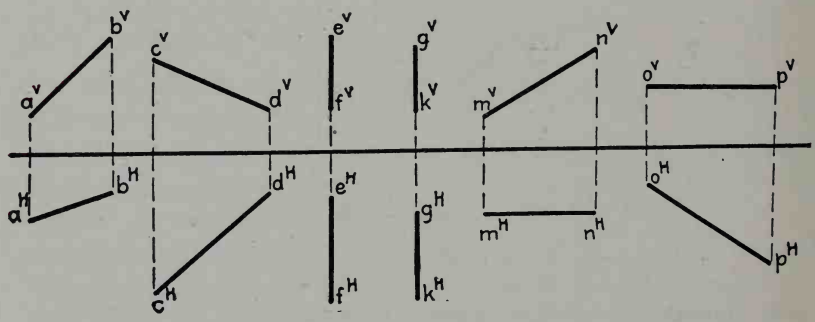
Note.—This exercise can be performed by the method of the Supplementary Problem, Problem 22. Conversely, Ex. 414–425 can also be constructed by the use of Problem 23.

460. The upper surface of a vein of coal is found to be perpendicular to a shaft bored in the direction of C(3 − 1 + ½) D(1 − 3 + 2). The ore is located at a point O(2, x, y) on CD. Draw the traces of the plane of the ore.

461. Scale 1 in. = 10 ft. A 4-ft. stack on a building has its center line A(15 ft. − 30 ft. + 15 ft.) B(15 ft., O + 15 ft.), a guy wire runs from a point 15 ft. up the stack to a point C(23 ft. − 22 ft. + 7 ft. 6 in.) in a roof perpendicular to the wire. Find the traces of the roof plane, and draw the projections of the roof, a rectangle, 40 ft. long by 20 ft. slant height, with C as its center.

462. Scale 1 in. = 6 ft. A 3-in. shaft carrying a 54-in. pulley (8-in. face) mounted 7 ft. from the upper end, runs from D(18 ft. O + 12 ft.) in the ceiling of a factory building to E(18 ft., 18 ft., O) in a wall. Draw the projections of the shaft and pulley (without spokes).

463. Scale 1 in. = 6 ft. A guide pulley, 54-in. × 12-in. face, is running on an oblique 2-in. shaft, whose center line is F(12 ft. − 6 ft. + 8 ft. 6 in.) G(24 ft. − 12 ft. + 19 ft. 6 in.) Draw the projections of the pulley at the center of FG.



Graphic Layout No. 32.—(*a*) Assume a point in space, and pass a plane through it perpendicular to line ——.

(*b*) Pass a plane ⊥ to line —— at its middle point.

(*c*) Let line —— be one edge of a cube, with one corner (or edge) touching —— (*H* or *V*). Draw the projections of the cube.

(*d*) Let line —— be the center line of a 1½-in. square stick, with a ½-in. hole drilled lengthwise through the center. Draw its projections.

(*e*) Let line —— be the center line of a 1½-in. square stick with a ½-in. hole drilled crosswise through the center. Draw its projections.

(*f*) Let line —— be the center line of a 2-in. shaft, carrying a 24-in. × 8-in. pulley. Draw the projections of the pulley to any convenient scale.

Distance of a Point to a Plane

81. Problem 24.—To find the distance from a point to a plane.

Construction.—Let O (Fig. 59) and T be the given point and plane.

1. Obtain QT and O^q.
2. Draw a perpendicular from O^q to QT.

Conclusion.—This perpendicular will be the distance required.

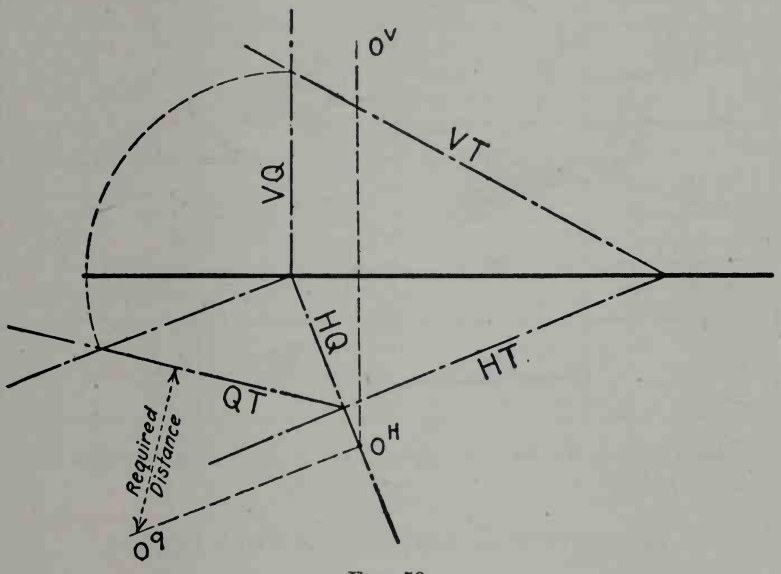
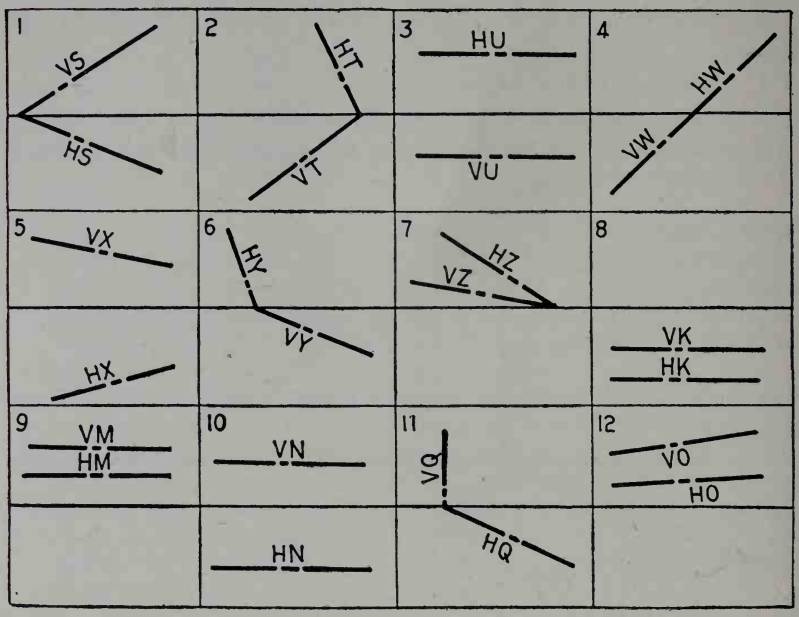


Fig. 59.

82. EXERCISES

464. Shortest distance A(1 + ½ − 2) to T(1 + 2) 4(4 − 3).
465. Shortest distance B(1 − 3 + 2) to S(4 − 2) 1(1 + 3).
466. Shortest distance C(2 + 1 − 3) to R(5 + 2) 2(5 − 2).
467. Shortest distance D(2 + ½ − ½) to Q(1 + 3) (5 + 2) (1 − 2¼) (5 − 1½).
468. Shortest distance E(1 + ½ − 3) to U(1 + 2) 3(5 − 2).
469. Shortest distance F(4 − 1 + 1) to W(5 + 3) 2(5 − 1).
470. Shortest distance M(4 − 2 + 2) to X(1 + 3) 2(5 − 1).
471. Shortest distance N(2 + 3 − 2) to Y(+ 1½) ∞(+ 1).
472. Shortest distance O(3 − 1 + 1) to Z (+ 1 ∞(− 2).
473. Shortest distance upper right-hand rear corner of a 3-in. cube with faces in *H* and *V*, to the plane of the three adjacent corners.
474. Scale 1½-in. = 1 ft. An 8-in. circular flue runs from a point A(1 ft., 0 − 8 in.) in a floor perpendicular to a plane whose traces intersect

at a point 4 ft. to the right, the *V*-trace being inclined 67½ to *GL*, and the *H*-trace 45°. Draw the projections of the roof and floor openings for the flue.



Graphic Layout No. 33.—Assume a point in space, and measure its distance from plane —·—.

PROJECTION OF LINES ON OBLIQUE PLANES

83. Problem 25.—To project a line on an oblique plane.

Construction.—Let AB (Fig. 60) be projected on T.

1. Let fall perpendiculars from A and B to T.
2. Obtain QT and a^qb^q.
3. Draw the projectors a^qm^q and b^qn^q, perpendicular to QT.
4. Project the points m^q and n^q back to the *H*- and *V*-projections of the perpendiculars drawn in operation (1).

Conclusion.—MN is the projection of AB on T.

Check.—If AB intersects MN at the point in which AB pierces T, the operation is probably correct. Why?

84. EXERCISES

475. Project A(1½ + 2 − 1) B(3 + 1 − 2) on Q(5 + 3) 5(3 − 2).
476. Project C(1 − 2 + 1) D(2 − ½ + 2) on R(3 − 3) 4(4 + 3)
477. Project E(2 + 1 − 1) F(4 + 1 − 1) on S(1 + 3) 4(1 − 2).

478. Project G(2 + 2 − ½) K(3 + 1 − 3) on T(+ 2) ∞ (− 1).
479. Project L(3 − 1 + 1) M(3 − 2 + 3) on U(+ 1) ∞ (− 2½).
480. Project N(2 − ½ + 1) P(4 − 1 + 2) or W(− 1) ∞ (+ 2).
481. Project G(3 + 2 − 1) K(4½ + 1 − 3) on X(1 + 2) 3(5 − 2).
482. Project A(1 + 1 − 1) B(3 + 3 − 2) on Y(1 + 3) (5 + 1½) (1 − 2) (5 − 1).
483. Project C(3 − 1 + 2) D(4 − 2 + ½) on Z(1 + 3) 5(1 + 1).
484. Project E(2 + 2 − 3) F(4 + 1 − 1) on R(1 + 3) 1(5 − 1).

Note.—Any of the above lines may be projected on any of the given planes.

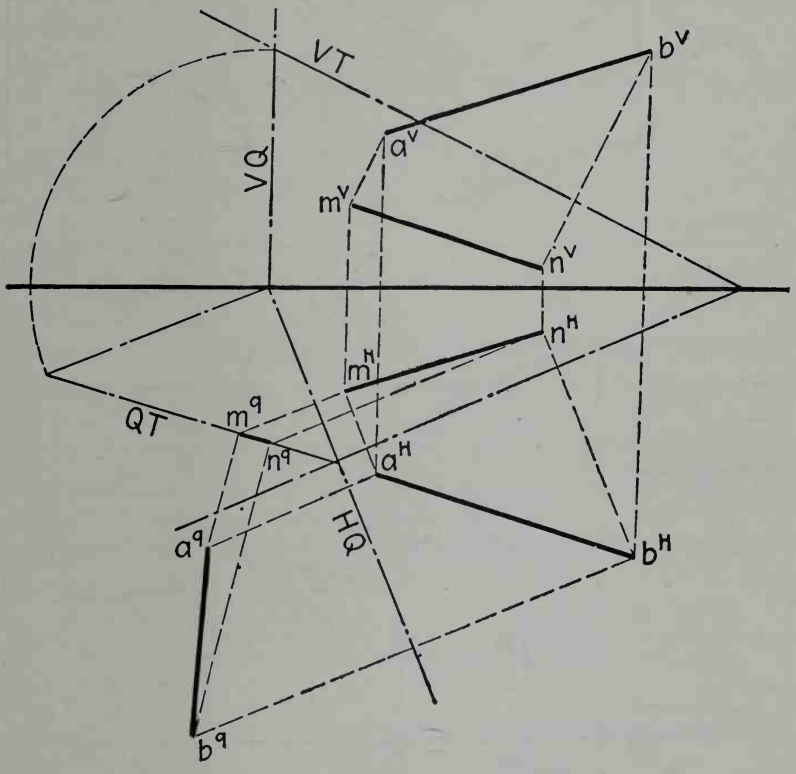
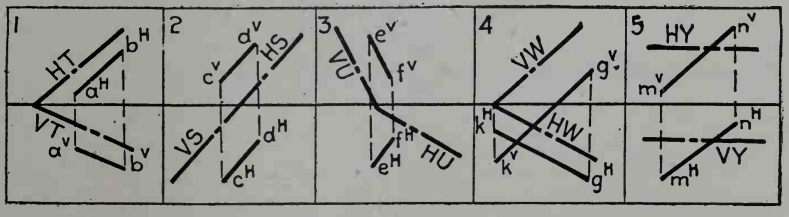
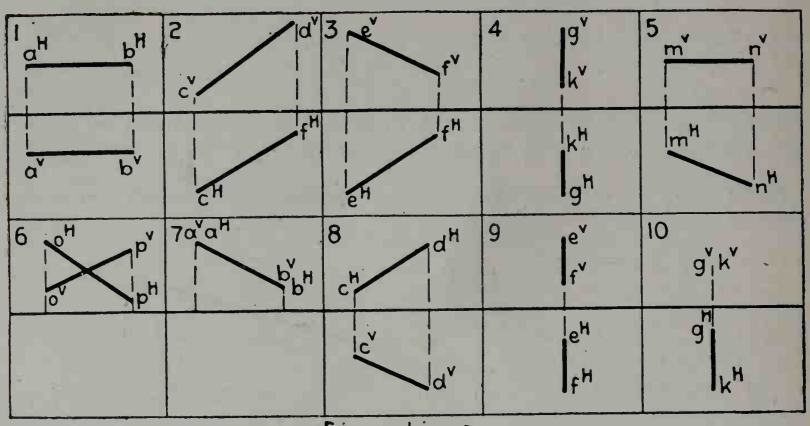


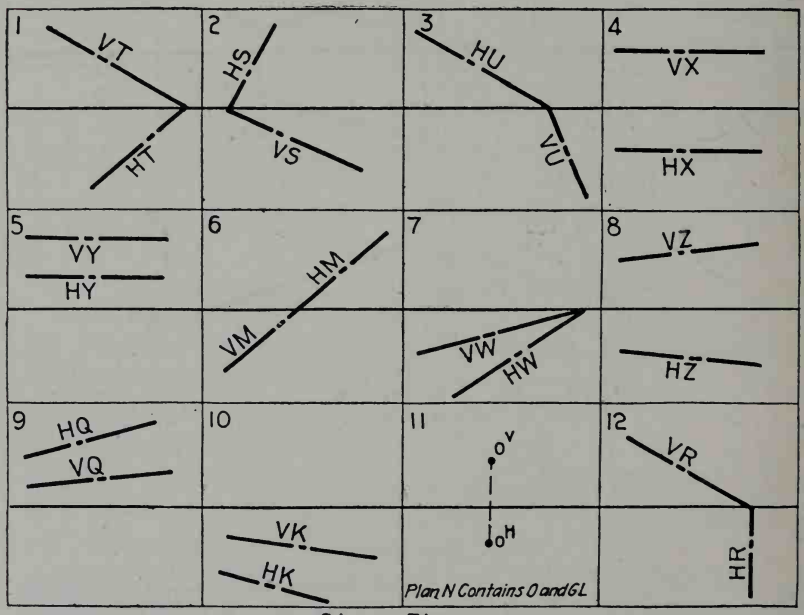
FIG. 60.



Graphic Layout No. 34.—Project given line on given plane in Ex. ——.



Given Lines



Given Planes

Graphic Layout No. 35.—Project given line —— on plane ——.

Angles Between Lines and Planes

85. Problem 26.—To measure the angle between any line and any plane.

First Method.—The measure of the angle between a line and a plane is the angle between the line and its projection on that plane. Therefore one analysis suggests itself; as follows:

Analysis.—1. Project the line on the plane by Problem 25.

2. Measure the angle between the given line and its projection thus found, by Problem 13.

Let the student make the construction.

85. Second Method.—This method, while not so obvious is much quicker and better.

Analysis.—1. From any point in the given line erect a perpendicular to the given plane.

2. Measure the angle between the given line and this perpendicular.

3. Subtract this angle from 90°, which gives the required angle between the line and plane.

Proof.—By letting fall a perpendicular to the plane, a right-angled triangle is formed between the given line, its projection, and the perpendicular. The angles between the line and its projection, and between the line and the perpendicular are complementary, and add up to 90°. Hence this method might be called the "complementary method."

Construction.—Let it be required to measure the angle between AB (Fig. 61) and the plane T.

1. From B draw the line $b^V p^V - b^H p^H$ perpendicular to T.

2. Through the *H*-piercing points of AB and BP (o^H and p^H) draw HQ.

Note.—If the *V*-piercing points are more convenient, they may be used in preference.

3. Revolve B into *H* about HQ to b_1.

4. Lay off a perpendicular $b_1 m_1$ to $p^H b_1$.

Conclusion.—The required angle will be θ, the complement of the angle between the line and the projector.

86. EXERCISES

Measure the angles that the following lines make with the planes given.

485. A(1 + 2 − 3) B(3 + 1 − 1) and T(2 + 3) 2(5 − 2).
486. C(2 + 3 − ½) D(4 + 1 − 1) and S(5 + 3) 5(1 − 2).
487. E(2 + 1 − 2)F(4 + 1 − 2) and R(1 + 3) 4(1 − 2).
488. G(2 + 1 − ½) K(2 + 2 − 3) and R(Ex. 487).
489. L(4 + 3 − ½) M(1 + 1 − 2) and R(Ex. 487).
490. N(2 − 1½ + 1) P(4 − 1½ + 1) and Q(5 − 2) 2(5 − 1½).
491. O(1 + 2 − 1) K(3 + 1 − 1) and Q(Ex. 490).
492. A(2 + 2 − 1) B(1 + ½ − 3) and U(+ 2) ∞ (− 1).
493. C(2 + 3 − 2) D(2 + 1, 0) and U(Ex. 492).

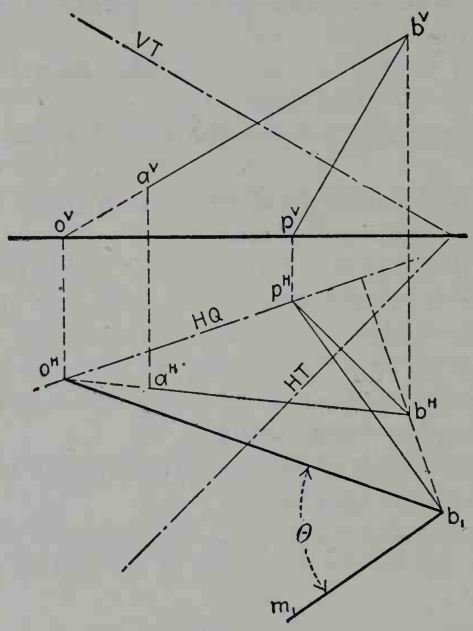
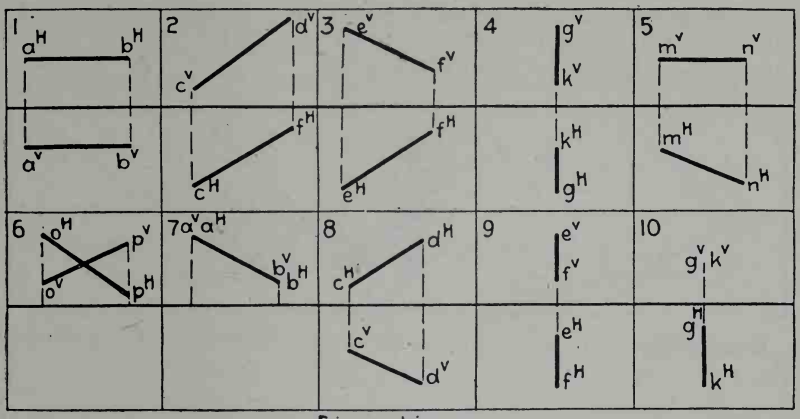
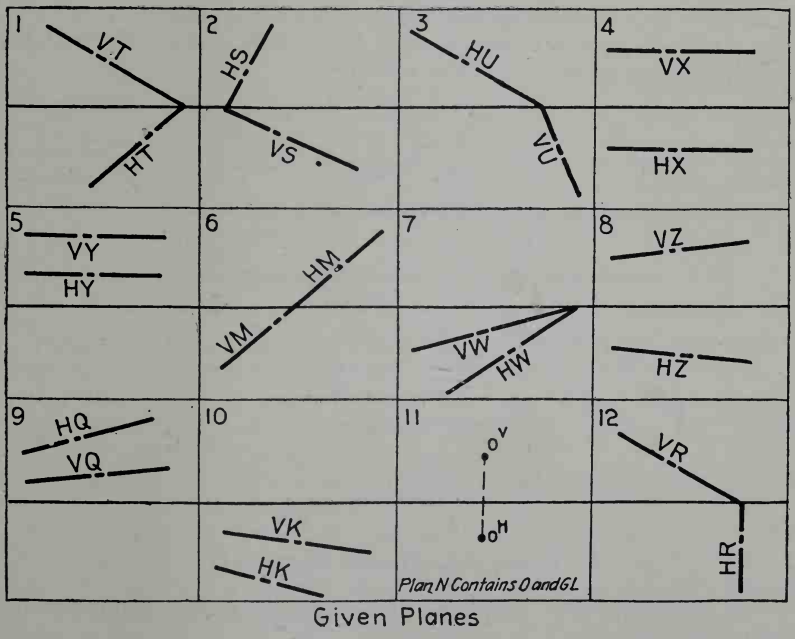


FIG. 61.

494. E(1 − 3 + ½) F(2 − 2 + 2) and W(+ 1) ∞ (− 2).
495. G(3 + 1 − 1) K(4 + ½ − 1½) and X(1 + 1) 3(5 − 1).
496. L(1 + 2 − ½) E(4 + 2 − ½) and X(Ex. 495).
497. M(2 − 1 − 2) N(3 − 2 − ½) and Y(1 + 3) 5(1 + 1).
498. O(2 + 2 − ½) P(2 + ½ − 3) and Y(Ex. 497).
499. A(1 + ½ − ½) B(2 + 3 − 3) and Z(1 + 3) (5 + 2) (1 − 2¼) (1 − 1½).
500. C(2 + 2 − 2) D(2 + 1 − ½) and Z(Ex. 499).
501. E(2 + 1 − 3) F(3 + 3 − 2) and Q(1 + 3) 2(5 − 1).
502. A(1 + 2 − 2½) B(2 + 1 − 1) with *H*, *V*, and *P*.
503. Any of the foregoing lines with *H*, *V*, and *P*.

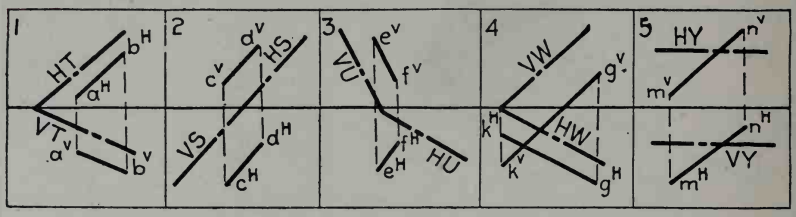
504. The angle of the coal shaft (Ex. 460) and H, V, and P.
505. The angle of the guy wire in Ex. 461, and H, V, and P.
506. The angle of the pulley shaft (Ex. 462) and H, V, and P.
507. The angle of the steam pipe (Ex. 256) and H, V, and P.
508. The angle of the hip-rafter AB (Fig. 34) with H, V, and P.





Graphic Layout No. 36.—Measure the angle between line —— and plane ——.

509. Draw a line from M(4 + 3 − 3) making an angle of 60° with T(1 + 2) 5(1 − 3) intersecting the line A(3 + 2, 0) B(1, 0 − 3). What angle does it make with AB?
510. Draw the projections of the locus of all lines from O(3 + 2 − 3) making 67½° with S(5 + 2) 2(5 − 1½).
511. Draw a line from N(2 + 2 − 2) making 45° with R(+ 2) ∞ (− 1) intersecting C(2 + 2, 0) D(4, 0 − 1).
512. Draw a line from P(2 + 3 − 2) at 60° from U(1 + 1½) 3(1 − 1½), intersecting E(1 + 1½, 0) F(1, 0 − 1½).



Graphic Layout No. 37.—Measure the angle between the line and plane in Ex. ——.

513. Draw the projections of the locus of all lines through M(3 + 2 − 1) making 60° with W(− 1) ∞ (+ 2).
514. Draw a line from P(3, 0, 0) making 45° with T(1 + 2) (5 + 1) (1 − 3) (5 − 1½), intersecting Q(2 + 1¾, 0) K(3, 0 − 2¼).
515. Draw the locus of all lines through N(3 + 2, 0) making 67½° with the plane through *GL* and F(2 + 1 − 2).

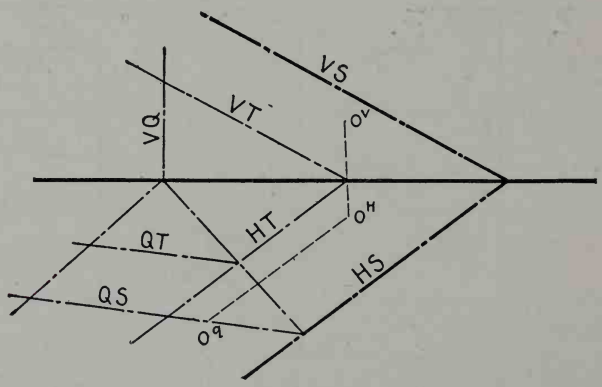


FIG. 62.

PARALLEL PLANE PROBLEMS

87. Problem 27.—To pass a plane through a given point parallel to a given plane.

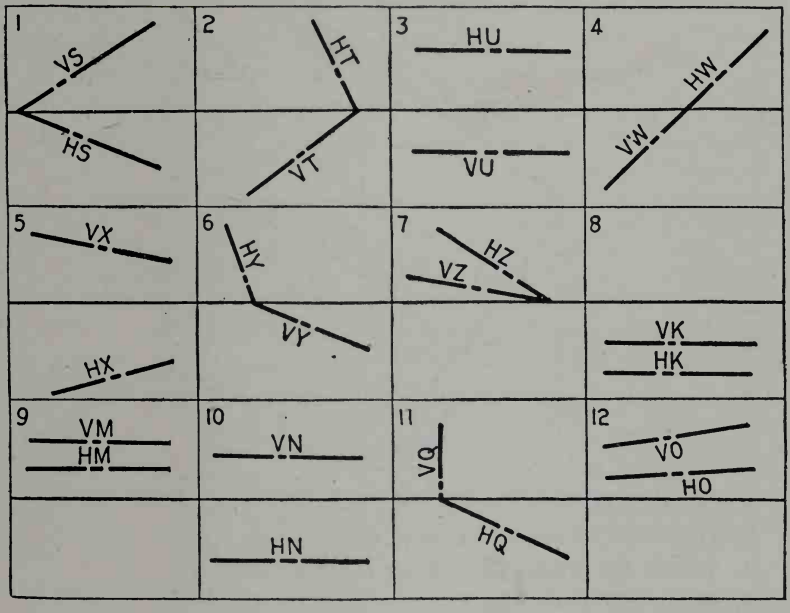
Principle.—Corresponding traces of parallel planes are parallel.

Construction.—Let O (Fig. 62) be the given point. Required to pass a plane through O parallel to T.

1. Locate QT.
2. Locate o^q.
3. Draw QS through o^q parallel to QT.
4. Where QS intersects HQ, draw HS parallel to HT. Then draw VS parallel to VT.

Conclusions.—The plane S contains O and is parallel to T.

88. EXERCISES



Graphic Layout No. 38.—Assume a point in space, and pass a plane through it ∥ to plane ——. What is the distance between the planes?

516. Through A(1 + 2 − ½) pass a plane ∥ to R(3 + 3) 3(5 − 3). What is the distance between planes?
517. Through B(3 + 2 − 2) pass a plane ∥ to Q(1 + 1) 3(1 − 3). What is the distance between planes?
518. Through C(1 − 1 + 1½) pass a plane ∥ to S(5 + 2) 1(5 − 1½). What is the distance between planes?
519. Through D(2 + 1 − ½) pass a plane ∥ to T(+ 3) ∞ (− 2). What is the distance between planes?
520. Through E(1 + 3 − 1) pass a plane ∥ to the plane containing *GL* and P(3 + 1 − 2). What is the distance between planes?

521. Through F(4 + 1 − ½) pass a plane ‖ to U(1 + 2) 3(5 − 2). What is the distance between planes?
522. Through G(2 + 1 − 2) pass a plane ‖ to W(1 + 2½) 4(1 + 1). What is the distance between planes?
523. Through K(3 + ½ − 1) pass a plane ‖ to X(1 + 3) (5 − 2) (1 − 3) (5 − 2). What is the distance between planes?

89. Problem 28.—To pass a plane parallel to a given plane at a given distance from it.

Construction.—Let T (Fig. 63) be the given plane. Required to pass a plane 1 in. from T and parallel to it.

It is obvious that two such planes can always be passed.

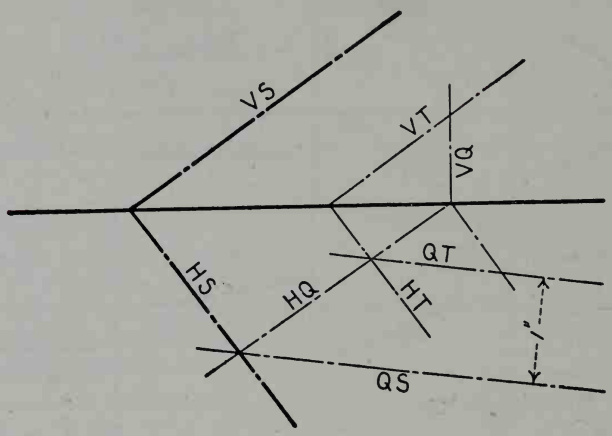


FIG. 63.

1. Obtain QT.
2. Draw QS ‖ to and 1 in. from QT.
3. Where QS crosses HQ draw HS ‖ to HT.
4. Draw VS ‖ to HT.

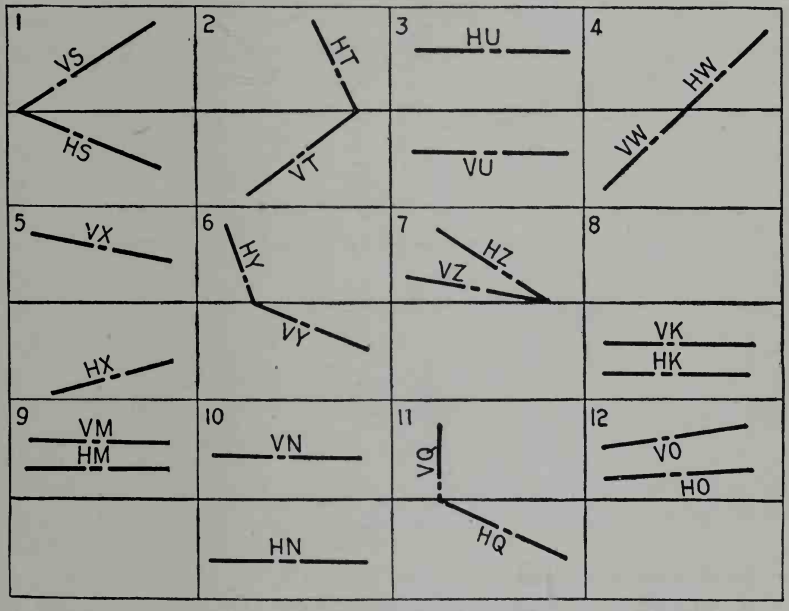
Conclusion.—The plane S is parallel to T and 1 in. from it.

EXERCISES

90. Draw one or two planes (according to directions from instructor) parallel to and 1 in. from the following planes.

524. T(1 + 3) 1(4 − 2).
525. S(2 + 3) 5(2 − 2).
526. Q(4 − 2) 1(5 + 3).
527. R(+ 2) ∞ (− 1).
528. U(+ 1) ∞ (+ 2).
529. W(− 1) ∞ (+ 1½).

530. X(− 2) ∞ (− 1).
531. Y(5 + 2) 3(1 − 2).
532. Z(5 + 3) 1(5 + 1).
533. Q(5 + 2) (1 + 1) (5 − 3) (1 − 1½).
534. R(5 + ½) (1 + 1½) (5 + 2) (1 + 3).
535. Scale ⅛-in. = 1 ft. A saw-tooth roof is being erected for a greenhouse. The planes are all parallel to T(42 ft. + 8 ft.) 34 ft. (42 ft. − 4 ft.) Draw as many of the roof planes as possible in your space at a distance of 9 ft. between each.
536. The alternate planes in the foregoing exercise are perpendicular to T. Let the ridge line be 7 ft. 6 in. above and parallel to *H*. Draw as much of the outline of the greenhouse as is possible in your space.



Graphic Layout No. 39.—Pass a plane ‖ to plane ——, making the intervening distance —— (1, 1¼, 1½, 1¾, 2) in.

Dihedral Angles

91. Problem 29.—To measure the dihedral angle between two planes.

The angle between two planes is considered to be equal to that angle included between the perpendiculars to the line of intersection, that lie in each plane. It is evident, then, that the plane of the measuring angle is perpendicular to the line

of intersection, and therefore to both of the given planes. Two methods of finding this angle are given herewith.

First Method.—Analysis.—1. Find the intersection of the given planes.

2. At any assumed point on the line of intersection pass a plane perpendicular to it.

Note.—Only one trace of this auxiliary plane is usually necessary.

3. Measure the angle between the intersections of the auxiliary plane with the two given planes.

Construction.—Let S and T (Fig. 64), be the two planes.

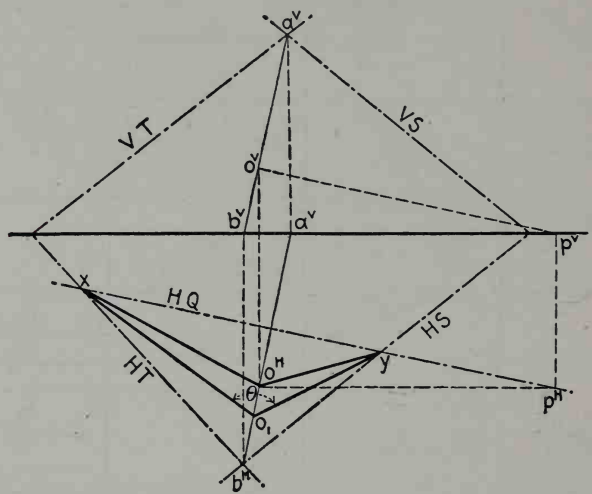


FIG. 64.

1. Obtain their line of intersection, AB.

2. Through O, assumed on AB, draw OP, perpendicular to AB and parallel to *V*.

3. Obtain P, the *H*-piercing point of OP.

4. Draw HQ through p^H, perpendicular to a^Hb^H.

5. Connect o^H with x and y (intersections of HQ with HT and HS).

6. Revolve O to o_1.

Conclusion.—$XO_1y(\theta)$ is the required angle.

Second Method.—Analysis.—1. From any point let fall two perpendiculars, one to each of the two given planes.

2. Measure the angle included between these perpendiculars.

This will be the correct measure of the required dihedral angle.

Proof.—The two perpendiculars from the point to the planes determine a plane perpendicular to both planes. Fig. 65 shows a view of two intersecting planes S and T. O is taken anywhere, and perpendiculars OM and ON are let fall to T and S respectively. The plane of MON cuts lines MP and NP from T and S, and the four lines, OM, ON, MP and NP form a quadrilateral, the sum of whose angles is 360°. Prove this. The angles at M and N are right angles by construction, and therefore add up to 180°. This leaves 180° as the sum of angles ϕ and θ, at O and P respectively. The angle ϕ is therefore the supplement of θ, which is the angle between T and S. But the measure of any angle, plane or dihedral, may be either the acute angle or its supplement, so the angle between the planes S and T may be either θ or ϕ,

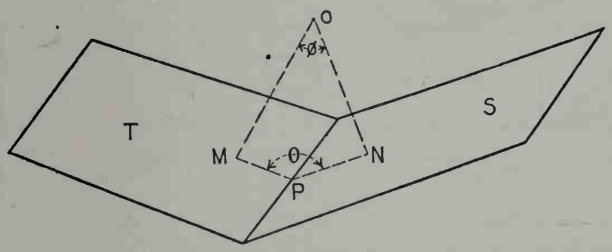


FIG. 65.

and as ϕ is the angle between the perpendiculars, it is also the measure of the given dihedral angle.

Note.—Although the angle between two lines, or two planes, may be either the acute or the obtuse angle, as stated in the foregoing, the acute angle is usually taken as the true measure.

92. EXERCISES

Dimensions of planes for following exercises:

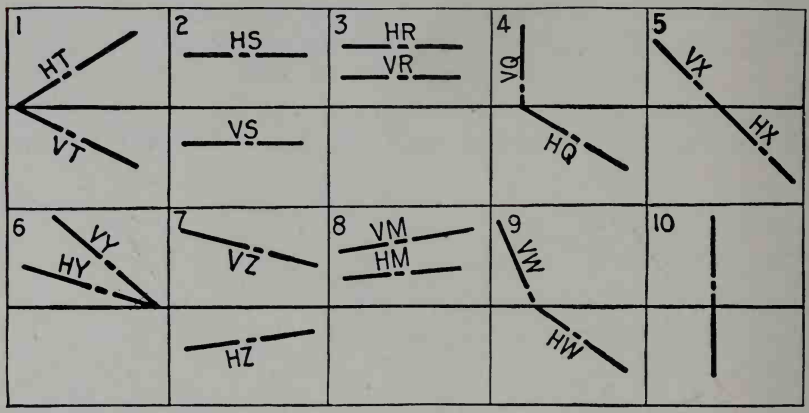
S(5 + 2) 1(5 − 3).
T(+ 2) ∞ (− 1).
R(− 1) ∞ (− 1½).
Q(5 + 3) 5(5 − 2).
X(1 + 2) 3(5 − 2).
W(5 − 2) 4(1 − 1).
Y(1 − 3) 4(1 − 1).
Z(5 − 3) (1 − 2) (5 + 1½) (1 + 1).
P(3 + 3) 3(3 − 3).
M(1 − 3) (5 − 1½) (1 − 2) (5 − 1).
U containing A(2 + 1 − 1½) and *GL*.

537. Measure the dihedral angle of the planes —— and ——.

538. Measure the angles that the plane —— makes with H, V, and P.

539. Measure the hip roof angle between the roofs C and D in Fig. 34, Art. 45.

540. A room is 18 ft. long, 10 ft. wide, and 10 ft. high. The center line of a chute runs from a point in the ceiling 2 ft. from the right end wall and 1½ ft. from the rear, to a point in the floor, 5 ft. from the left end wall and 2 ft. from the front. The chute is 2 ft. square, with the diagonal parallel to the floor.
(*a*) Find the floor opening for the chute.
(*b*) Find the angles of its faces with the floor. Scale 1 in. = 3 ft.



Graphic Layout No. 40.—Exercises 537 and 538 apply to either graphic or dimensioned layout.

541. A hexagonal roof tower, 12-ft. base diameter, 16-ft. altitude, has specially rolled angle irons for its frame. What is the angle at which they must be rolled?

93. Problem 30.—To find the traces of a plane that shall make a given angle with a given plane and contain a given line of that plane.

Analysis.—Assume a point in the given line, and pass a plane through the point perpendicular to the line.

2. Revolve the line cut by this auxiliary plane from the given plane about the trace of the auxiliary plane into H or V, as the case may require.

3. From the revolved position of the assumed point on the given line lay off a line, making the given angle with the revolved line.

4. Where this new line crosses the trace of the auxiliary plane will be a point on the trace of the required plane.

5. Since the required plane must contain the given line, its

traces will pass through the *H*- and *V*-piercing points of the given line.

6. Draw the traces of the required plane through the point thus found and the piercing points of the given line.

Construction.—Let T (Fig. 64) be the given plane, AB the given line, and θ the given angle.

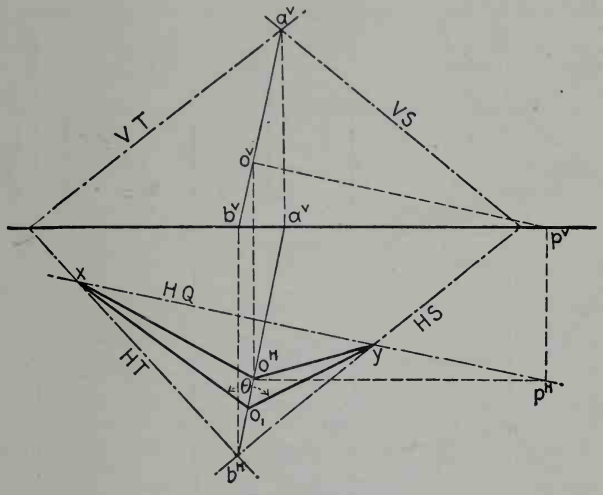


FIG. 64.

1. Assume O, any point in AB.
2. Through O pass a plane perpendicular to AB. HQ is the trace of this plane.
3. Revolve O into *H* about HQ to o_1.
4. Lay off o_1y at angle θ with o_1x.
5. Through b^H and y draw HS.
6. Draw VS through a^V.

Conclusion.—S is the plane containing AB making the angle θ with T.

Note.—There can be two planes passed through AB making θ with T, except when $\theta = 90°$.

Supplementary Problem.—Through a given line to pass a plane perpendicular to a given plane.

Analysis.—1. Through any point of the given line draw a line perpendicular to the given plane.

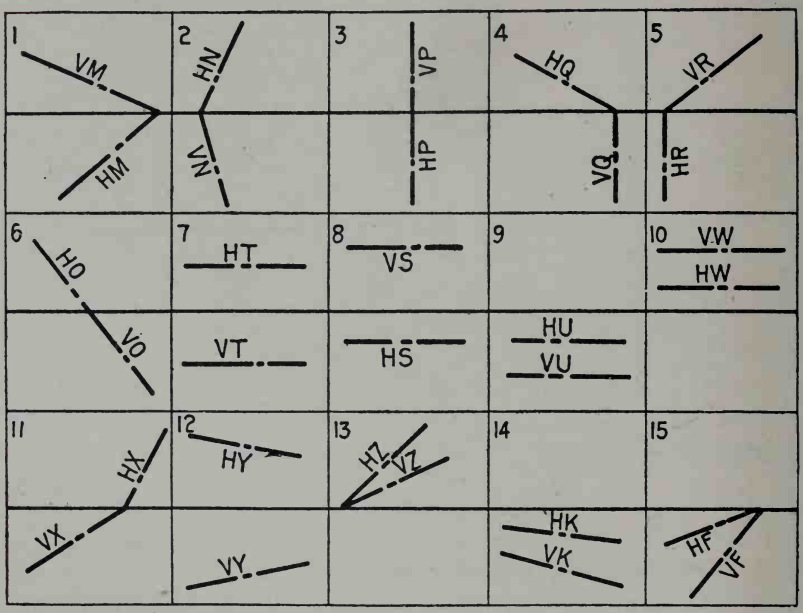
2. Pass a plane through the two lines.

Conclusion.—This plane will be perpendicular to the given plane. Why?

Let the student make the construction.

Note.—If the given line is perpendicular to the given plane, the number of solutions is infinite.

EXERCISES



Graphic Layout No. 41.—(*a*) Plane ——. Draw traces of a plane under the conditions of Ex. 542.

(*b*) Plane ——. Draw traces of a plane under the conditions of Ex. 543.

(*c*) Plane ——. Draw traces of a plane under the conditions of Ex. 544.

94. Use the following planes in Ex. 542–544, and 546.

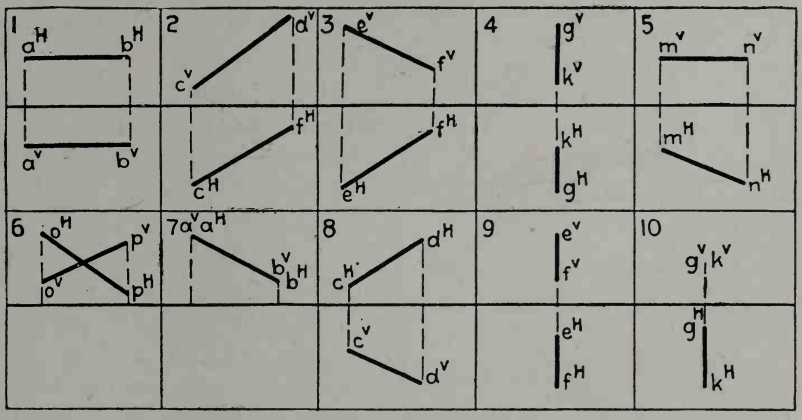
M(1 + 2) 5(1 − 3).
N(5 − 1) 2(5 + 2½).
O(1 + 3) (5 + 1½) (1 + 2) (5 + 1).
P(3 + 3) 3(3 − 3).
Q(2 − 3) 2(5 + 2).
R(1 + 1) 4½(4½ + 3).
S(5 + 2) 3(1 − 2).
T(+ 2) ∞ (− 1).
U(+ 1½) ∞ (+ 2).
W(+ 2) ∞ (+ 1).
X(+ 1½) ∞ (− 2½).
Y(1 + 2) (5 + 1) (1 − 2½) (5 − 1¼).

Z(5 + 1) 1(5 + 2½).
K(1 + 3) 2(5 − 1).

542. Assume a line 1 in. from and parallel to —— (*H* or *V*) in the plane ——. Draw the traces of plane that shall contain this line and make a dihedral angle of ——° with the given plane.

543. Plane ——. Assume a line running obliquely to *H*, *V*, and *P* in the plane, and draw the traces of a plane, which contains the line, and makes ——° with the given plane.

544. Plane ——. Assume a profile line in the plane, and draw the traces of a plane, which contains the line, and makes ——° with the given plane.



Graphic Layout No. 42.—Draw the traces of two planes containing line ——, and making a dihedral angle of ——° (15, 30, 45, 60, 75, 90).

545. The dihedral angle between two adjacent faces of a right pyramid, base a 2-in. equilateral triangle, is ——° (95, 100, 105, 110, 120, 135, 150). Draw the projections of the pyramid.

546. Assume a line in space, and pass a plane through the line, perpendicular to given plane ——. (Use either dimensioned or graphic layout).

PROBLEMS UTILIZING THE RIGHT CIRCULAR CONE

95. Properties of the Right Circular Cone.

Three properties of the cone are particularly useful in the solution of certain problems.

1. The elements of the cone all make the same angle with the plane of the base, and they are all of the same length, because one way of generating a cone is to revolve a right-angled triangle about one of its perpendicular sides.

2. All planes tangent to a cone make **the same angle** with the plane of the base, because a tangent plane is perpendicular to the generating triangle of the cone, and contains its hypothenuse.

3. Every element of the cone and every plane tangent to the cone pass through the apex.

With these properties understood, it is a simple matter to solve the six following problems:

96. Problem 31.—To draw a line through a point so as to make given angles with H and V.

Limits.—The sum of these angles cannot be more than 90°. If a line be parallel to *GL*, the sum is 0°, and if it lie in a profile plane, the sum is 90°. For any other position, the sum will be less than 90°.

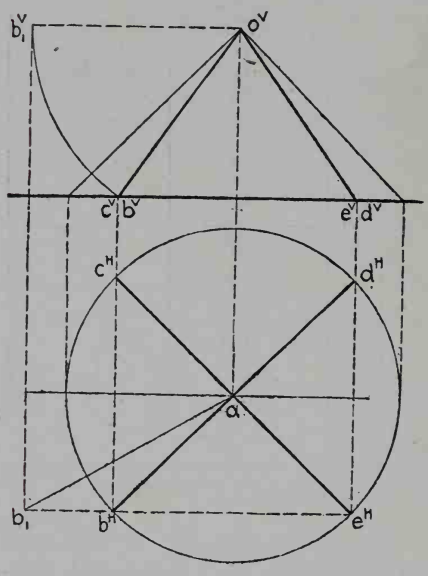


FIG. 66.

Construction.—Let it be required to draw a line through A, Fig. 66, that shall make 45° with *H*, and 30° with *V*.

1. Make A the apex of a right circular cone, whose base is in *H*, and whose elements make 45° with *H*.

All the elements make 45° with *H*, and all pass through A. Therefore, it remains to be found which are the four possible elements that make 30° with *V*.

2. From A draw a line a^Vb^V, a^Hb_1 parallel to *H*, making 30° with *V*, and equal in length to the elements of the cone.

3. Let this line be an element of a second cone with its apex at A, and base parallel to *V*. Draw the projections of the cone (whole or in part).

4. Where the bases of the cones intersect (B or E), the common element can be drawn.

Conclusion.—The line AB (as well as the lines AC, AD, and AE) is inclined 45° to *H*, and 30° to *V*.

Let the student give the reason for this and supply a proof and a means of checking its correctness.

97. EXERCISES

547. Through M(3 + 1½ − 2) draw four lines, each making 60° with *H* and 15° with *V*.

548. Through O(2 + 2 − 1) draw four lines, each inclined 30° to H and V.
549. Draw a line 4 in. long, terminating in H and V, inclined 20° to H and 40° to V.
550. Draw a line through K(4 + 1 − 1) making 35° with H and 55° with V.
551. Draw a line through P(3½, 0, 0) making 20° with H and 50° with V.

98. Problem 32.—To measure the angle an oblique plane makes with H or V.

Construction.—To measure the angle between T and H (Fig. 67).

1. With any assumed point (O) on VT as an apex, draw a cone with its base tangent to HT.

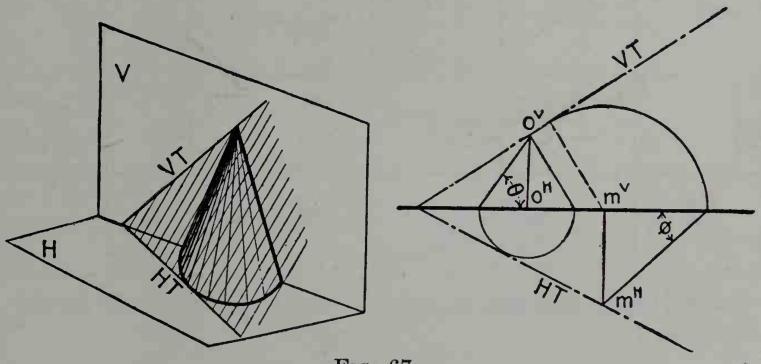


FIG. 67.

Conclusion.—The angle θ, the angle between the elements and the base plane, is the measure of the inclination of T to the horizontal.

Notes.—1. The angle ϕ shown in Fig. 67 is the measure of the angle between T and V.

2. This problem was also solved incidentally in Problem 2.

99. EXERCISES

Measure the angles that the following planes make with H and V.

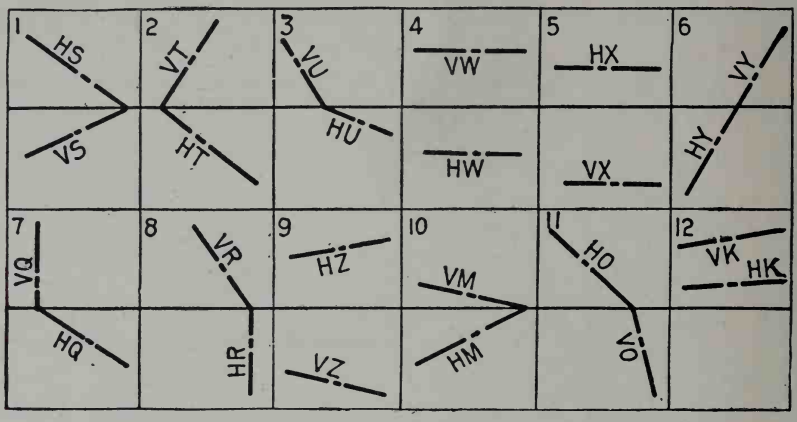
552. Q(1 + 2) 5(1 − 3).
553. R(5 − 1) 2(5 + 2½).
554. O(1 + 3) (5 + 1½) (1 − 2) (5 − 1).
555. S(5 + 2) 3(1 − 2).
556. T(+ 2) ∞ (− 1).
557. U(− 1½) ∞ (+ 2)
558. W(+ 2) ∞ (+ ½)
559. X(− 1½) ∞ (− 1).
560. Y(1 + 2) (5 + 1) (1 + 2½) (5 + 1¼).
561. Z(5 + 1) 2(5 + 3).

562. K(1 + 3) 2(5 − 1).

563. N through *GL* and E(2 + 2 − ½).

564. A tetrahedron is limited by the points A(2, 0 − 1), B(3½, 0 − 1½), C(4, 0 − 3), and D(3 + 1½ − 2½). Measure the dihedral angle between face —— and face ——.

565. Tetrahedron (Ex. 564). Measure the angles of the three oblique faces with *H* and *V*.



Graphic Layout No. 42.—Measure the angles between given plane —— and *H* and *V*.

100. Supplementary Problem 33(a).—Given one trace of a plane and the angle between the plane and the plane of projection, to find the other trace.

Analysis.—1. Construct a cone, whose elements make the given angle with its base, tangent to the given trace, with the center of its base in *GL*.

2. Draw the other trace through the apex of the cone and the intersection of the given trace with *GL*.

Notes.—1. If the given trace does not intersect *GL* within bounds, two cones can be constructed, and the trace drawn through the two vertices.

2. If the given trace is parallel to *GL*, the other trace will also be parallel, and the quickest solution is effected through the profile plane.

3. By inverting the cone, another plane could be found that would fulfil the conditions.

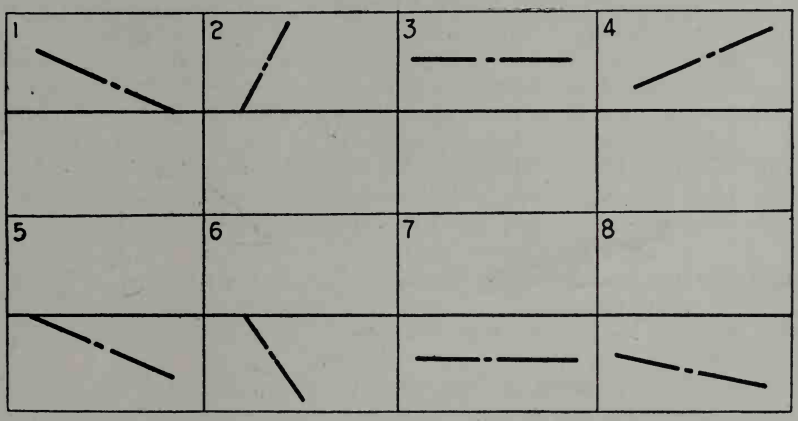
101. Supplementary Problem 33(b).—Given the H-trace of a plane and its angle with V, to find the V-trace.

Analysis.—1. From any point in the given trace as an apex, construct a cone of the required elemental angle, having its base in *V*.

2. From the intersection of the given trace with *GL* draw the *V*-trace tangent to the base of the auxiliary cone.

Note.—See notes following 33(*a*).

102. **EXERCISES**



Graphic Layout No. 43.—*H*- or *V*-traces of planes to be used with Ex. 566–579.

566. Given *H*-traces: S5(1 − 2), T4(2 − 3), U ∞(− 1), W(1 − 3) (5 − 2), X5(1 + 1½), Y ∞(+ 1½), Z(1 + 2) (5 + 1).
Draw the *V*-trace of plane ——, when the plane makes ——° with *H*.
567. Draw the *V*-trace of plane —— (Ex. 566), when the plane makes ——° with *V*.
568. The *H*-trace —— (Ex. 566) is the *H*-trace of two planes, each inclined to *H* at ——°. Draw their *V*-traces. Measure the angle between the two planes.
569. Draw the traces of the bisecting plane of the dihedral angle formed by the two required planes in Ex. 568.
570. Given trace —— (Ex. 566). This is the *H*-trace of two planes making ——° with *V*. Draw their *V*-traces.
571. Measure the dihedral angle between the two required planes in Ex. 570.
572. Draw the traces of the bisecting plane of the dihedral angle between the required planes in Ex. 570.
573. Given *V*-traces: X(3 + 3)2, Y(5 + 1)1, Z(+ 2) ∞, M(5 + 2)(1 + 1), S(1 − 1½)5, T(2 − 3)3, U(− 1) ∞, W(1 − 2½) (5 − 1). Draw the *H*-trace of ——, when the plane makes ——° with *V*.
574. Draw the *H*-trace of —— (Ex. 573), when the plane makes ——° with *H*. What is its angle with *V*?
575. The *V*-trace —— (Ex. 573) is the trace of the two planes making ——° with *V*. Draw their *H*-traces.
576. The *V*-trace —— (Ex. 573) is the trace of a plane making ——° with *H* and another plane making ——°. Draw their *H*-traces.

577. Measure the dihedral angle between the two planes required in Ex. 575.
578. Measure the dihedral angle between the two planes required in Ex. 576.
579. Draw the traces of the plane bisecting the dihedral angle between the two planes required in Ex. 575 or Ex. 576.
580. Through a point A(3, 0 − 1) in a vein of ore a horizontal line on the surface runs north, 37° west. The "dip" (inclination of the ore to the horizontal) is ——°. Find the V-trace of the plane of the ore.
581. The lower edge of a rectangular roof is 30 ft. long and inclined 40° to the V-plane. The roof is inclined 60° to the horizontal and the ridge is 8 ft. higher than the lower edge. Draw the H- and V-projections of the roof, and insert three windows, each 2 ft. 6 in. × 6 ft. 3 in., symmetrically placed. Draw to a scale ⅛ in. = 1 ft.
582. Given the line —— as the —— trace of a plane inclined ——° to ——. Draw the other trace. What angle does it make with ——?
583. The line —— is the —— trace of two planes making ——° with ——. Draw their traces.
584. The line —— is the —— trace of two planes, one making ——° with ——, and the other ——° with ——. Draw their traces.
585. Measure the dihedral angle between the required planes in Ex. 583 or 584.
586. Draw the traces of the bisector of the dihedral angle of the required planes in Ex. 583 or 584.

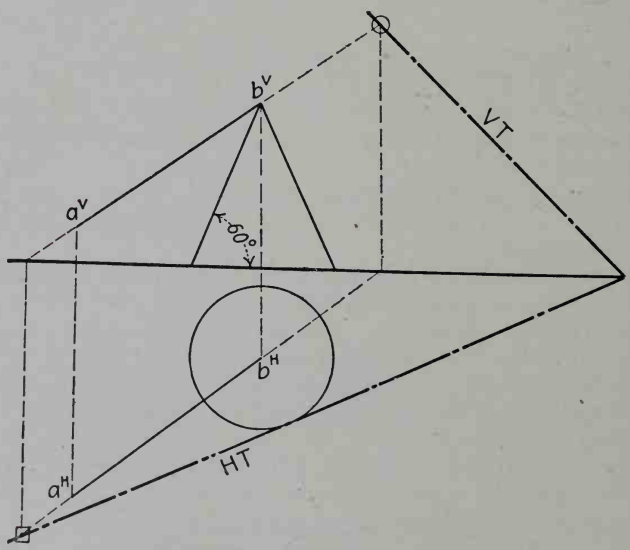


FIG. 68.

103. Problem 34.—To pass a plane through a given line, so as to make any required angle with a plane of projection.

Construction.—Let AB (Fig. 68) be the given line, and let it be required to pass a plane through it, which is inclined 60° to the horizontal.

1. Locate the *H*- and *V*-piercing points of AB.
2. Using any point (as B) as the apex of a 60° cone on *H*, draw the projections of the cone, as shown.
3. Draw HT through ⊡ tangent to the base of the cone.
4. Draw VT from T through ⊙.

Note.—There are two possible solutions, except when the given angle is 90°.

Limitation.—If the inclination of the given line exceeds the angle of the required plane with the given plane of projection, the problem is impossible. Why?

104. EXERCISES

587. Given lines: A(2 + 3 − 2) B(4 + 1 − ½).
C(2 + 1 − 3) D(4 + 1 − ½).
E(1 + 2½ − ½) F(4 + 1¼ − 1½).
G(3 + 1 − ½) K(3 + 2 − 3).
L(3 + 2 − ½) M(3 + ½ − 3).
N(2 + 1½ − 1) O(4 + 1½ − 1).
P(2 − 2 + ¾) X(4, 0 + ¾).
Y(3 − 3 + 1) Z(5, 0 + 2).
M(3 − 1 + 2) P(3 + 2 + ½).
A(2 + 2 + 2) F(4 + 2 + 2).
Pass a plane through line —— making ——° with ——. Measure its angle with the other planes of projection.

588. Draw the traces of two planes containing the line —— (Ex. 587), each making ——° with ——. Measure their angles with the other planes of projection.

589. Draw the traces of two planes containing line —— (Ex. 587), one inclined ——° to ——, and the other ——° to ——. Measure their angles with the other planes of projection.

590. Measure the dihedral angle between the planes required in Ex. 588 or 589.

591. The line E(2 + 3 − 1) F(4, 0 − 2) is the hip rafter of a roof each of whose sides is an isosceles triangle, and is inclined ——° to *H*. Draw the projections of the roofs. Measure the hip angle. Draw the projections of a window in the center of each.

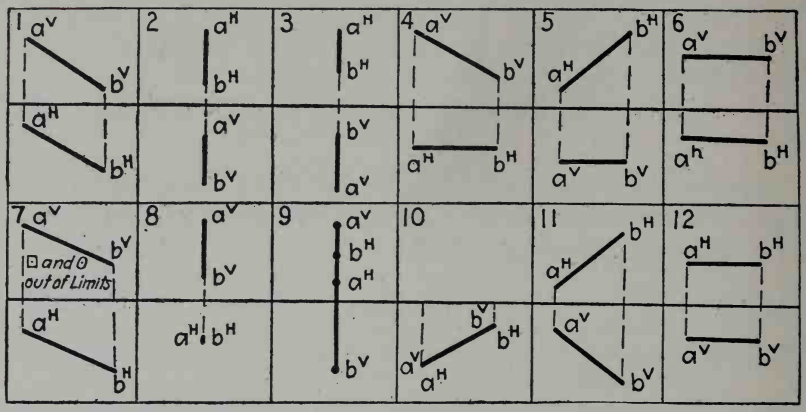
592. Through given line —— pass a plane inclined ——° to ——. Measure its angles with the other planes of projection.

593. Through given line —— pass two planes inclined ——° to ——. Measure their angles with the other planes of projection.

594. Through given line —— pass a plane inclined ——° to ——, and another ——° to ——. Measure their angles with the other planes of projection.

595. Measure the dihedral angle between the two planes required in Ex. 593 or Ex. 594.

596. Use one of the given lines as a roof ridge or rafter and draw the roof as directed by the instructor, triangular, rectangular, etc., at any inclination given, and with or without windows.



Graphic Layout No. 44.—To be used with Ex. 587–596.

105. Problem 35.—To find the traces of a plane which shall make any required angles with H and V.

Limitations.—The sum of these angles must be between the limits of 90° and 180°. A plane parallel to *H* or *V* is perpendicular to the other. A profile plane is 90° to each. Oblique planes are between these limits.

Construction.—Let it be required to draw the traces of a plane inclined 45° to *V* and 60° to *H*.

1. Draw the projections of a sphere of any diameter with its center in *GL*. (See Fig. 69.)

2. Draw a 45° cone tangent to the sphere, having its base on *V* and apex in *H*.

3. Draw a 60° cone tangent to the sphere, having its base on *H* and apex in *V*.

4. Draw VT through the apex in *V*, tangent to the base of the other cone, and HT through the apex in *H*, tangent to the base that is in *H*.

Conclusion.—This plane tangent to both cones thus fulfils the conditions of the problem

Notes.—1. Eight planes fulfilling these conditions can be drawn, all tangent to the sphere.

2. Any plane parallel to *GL* has the sum of its angles with *H* and *V* equal to 90°, hence, if the sum of the given angles is equal to 90°, the problem can be easiest solved by the use of the profile plane.

3. If it be required to pass such a plane through a given point, an auxiliary plane may be drawn anywhere, as in the foregoing, filling the conditions, and then a plane may be passed through the point parallel to the auxiliary plane. The plane S, Fig. 69, is passed through the point O, parallel to T already found. The plane S makes the same angles with *H* and *V* that T does.

Second Method.—(From Church and Bartlett.)—"If a line is perpendicular to a plane, the angles between the line and the planes of projection will be **complements** of the angles between the plane and the planes of projection. Hence if we construct

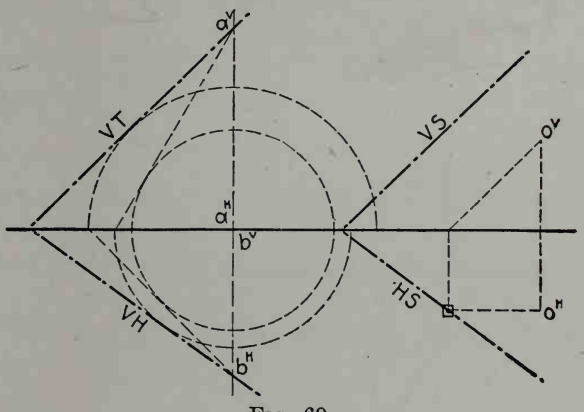


Fig. 69.

a line, making with *H* and *V* the complements of the required angles, a perpendicular plane through this point will make the required angles with *H* and *V*."

106. EXERCISES

597. Draw the traces of a plane inclined ——° to *V* and ——° to *H* through the point A(3 + 1 − 2).

598. (*a*) Draw the traces of a plane inclined ——° to *V* and ——° to *H* through an assumed point in *III*. (*b*) Assume the point in *II*. (*c*) Assume the point in *IV*.

599. Pass a plane under the conditions in Ex. 597 or 598, using the second method.

600. A line shaft making ——° with *H* and ——° with *V* carries a 12-in. pulley with 6-in. face, with its center at M(3 ft. 6 in., +2 ft. − 1 ft. 8 in.). Draw the projections of the pulley to a scale 1 in. = 1 ft.

601. A pulley 24 in. × 8 in., whose center plane is inclined ——° to H and ——° to V, runs on a shaft 8 ft. long extending from a point in the wall (V) to a point in the floor (H). The pulley is in the center of the shaft. Draw its projections.

602. A man is reading a newspaper 20 in. × 30 in. and his eye at D(10 ft. + 4 ft. − 6 ft.) is 2 ft. distant from the paper. His line of sight is inclined ——° to H and ——° to V, and travels to the geometrical center of the sheet. Draw the projections of the paper.

608. Assume a point in the —— angle, and draw the traces of all the planes that can contain the point and be inclined ——° to H and ——° to V.

SPECIAL PROBLEMS

107. Problem 36.—To find the shortest distance between two lines not in the same plane.

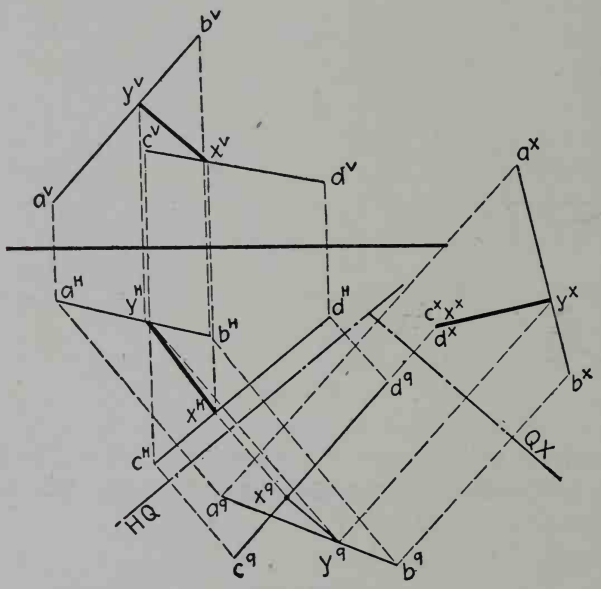


FIG. 70.

Construction.—Required the shortest distance between AB and CD (Fig. 70).

Referring to Fig. 71, we find that, when a line is perpendicular to H or V (as AB is to H), the common perpendicular is immediately drawn, without further operations on AB and CD. Why? Therefore, this problem is easiest solved by bringing one of the lines into this relation.

Return to Fig. 70. Draw Q parallel to CD.

1. Obtain the Q-projections of AB and CD.
2. Project them on X, which is perpendicular to CD.
3. Draw the perpendicular, $y^x x^x$, from $c^x d^x$ to $a^x b^x$. This is the shortest distance.

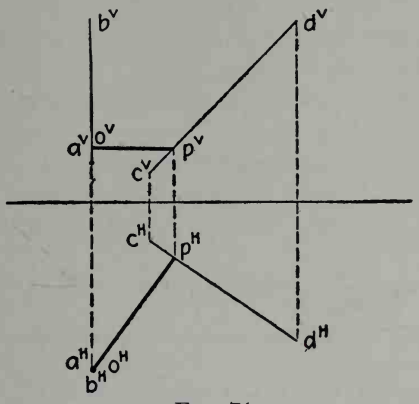


FIG. 71.

108. Supplementary Problem.—To find the shortest line, or "common perpendicular."

Construction.—Taking the X-projection of the shortest distance, $x^x y^x$, projecting it back to the Q-projections, and thence to the *H*- and *V*-projections, we obtain the *H*- and *V*-projections of the shortest line.

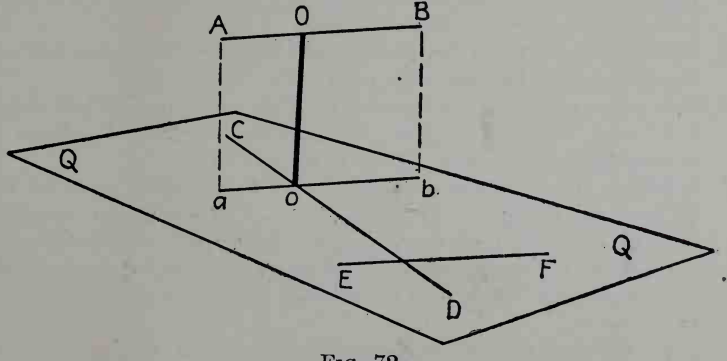


FIG. 72.

109. Second Method.

Analysis.—1. Pass a plane through one of the lines parallel to the other.

2. Project the outside line on the plane.

3. At the intersection of this projection with the given line, erect a perpendicular from the plane to the outside line. This line is the common perpendicular.

Construction.—Let AB and CD (Fig. 72) be the given lines.

1. Draw EF parallel to AB intersecting CD.
2. Pass the plane Q through EF and CD.
3. Project AB to ab on Q.
4. Where ab crosses CD at o, erect the common perpendicular oO.

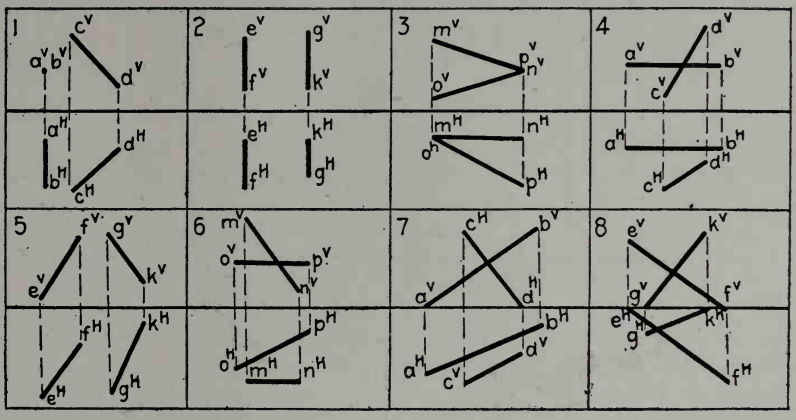
This method, the standard for many years, is still preferred by many. Let the student make the construction in orthographic projection.

110. EXERCISES

Draw the projections of the common perpendicular to the following lines:

604. A(1 + 2 − 2) B(4, 0 − 1) and C(3, 0 − 3) D(4½ + 2 − 1).
605. E(4 + 1 − 1) F(4 + 1 − 3) and G(1 − 1 − 1) K(3 − 3 − 3).
606. M(1 + 2 − 1) N(3 + 2 − 1) and O(2 + 3 − 3) P(3 + 1 − 2).
607. A(2 − 2 + 1) B(3½, 0 + 3) and C(2 − 1½, 0) D(4 − 3 + 1).
608. E(2 + 3 − ½) F(2 + 1 − 2½) and G(3 + 3 − 3) K(C, 0, 0).
609. M(2 + 3, 0) N(2, 0 − 2) and O(2½, 0 − 3) P(4, 0 + 2½).
610. X(2 + 3, 0) Y(3½, 0 − 2) and *GL*.
611. A(1 + 1 − 1) B(4 + 1 − 3) and C(2 + 3 − 1) D(4, 0 − 1).
612. E(1 + 3 − 1) F(3, 0 − 1) and G(3 + 2½ − 2) K(4, 0, 0).
613. M(1 + 2 − ½) N(2, 0 − 2) and O(3 + 3 − 3) P(3½ + 1 − ½).
614. Draw the shortest possible line parallel to —— (*H*, *V*, or *P*) intersecting both lines in Ex. —— (604–613).
615. Given a 2-in. cube. Draw the shortest line between the non-intersecting diagonals of any two adjacent faces.
616. Two wires run through a building, A (12 ft. + 23 ft., 0) B(28 ft., 0 − 6 ft.) and C(16 ft. + 12 ft., 0) D(24 ft., 0 − 16 ft.). Draw the shortest connecting wire. Scale ⅛ in. = 1 ft.
617. A 2½-in. pipe is run from a hole in a wall at E(13 ft. + 12 ft., 0) at 45° to *H* and 45° to *V*. At F(8 ft. + 6 ft., 0) a 1-in. pipe runs straight out from the wall, then bends perpendicularly and runs downward to the right at 40° to *H*. Make the shortest possible connection. How long must it be, allowing 2 in. at each end for connections? Scale ¼ in. =1 ft.
618. Draw the shortest possible horizontal pipe for Ex. 617.
619. Draw the shortest possible vertical pipe for Ex. 617.
620. Scale ¼ in. = 1 ft. In a mill there is a 6-ft. cubical bin on the floor with one edge at A(15 ft., 0, 0) B(21 ft., 0, 0). A 24-in. square chute runs its center line from C(3 ft. + 9 ft., 0) in the wall to D(16 ft., 0 − 10 ft.) in the floor. It is desired to run a 9-in. cylindrical pipe from a hole at E(18 ft. + 10 ft., 0) to F(8 ft., 0 − 13 ft.). Can the pipe pass both

the bin and the chute? If so, how much will there be to spare in each case? If not, locate the nearest point to transfer F.



Graphic Layout No. 45.—(Note.—Symbols may be transposed at option of instructor.)

621. Draw the shortest line between the two lines in Ex. —— of Layout 45.
622. Draw the shortest horizontal between the two lines in Ex. —— of Layout 45.
623. Draw the shortest vertical between the two lines in Ex. —— of Layout 45.
624. Draw the shortest line between *GL* and any oblique line in Layout 45.

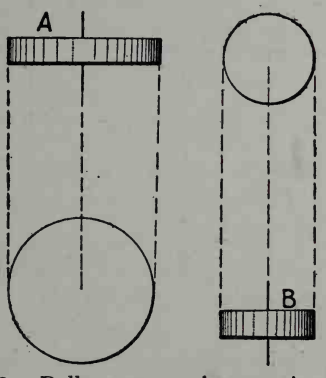


FIG. 73.—Pulleys on non-intersecting shafts.

111. The Problem of the Guide Pulley.

It is a well-known fact in machine design that a belt could be run over the pulleys A and B, shown in Fig. 73, so as to drive in one direction **or** the other, but it would be impossible to **reverse**

the direction if attached to run in one. It is possible to construct an "idler," or guide pulley, with its shaft at such an angle that the belt will not run off either pulley, running in either direction.

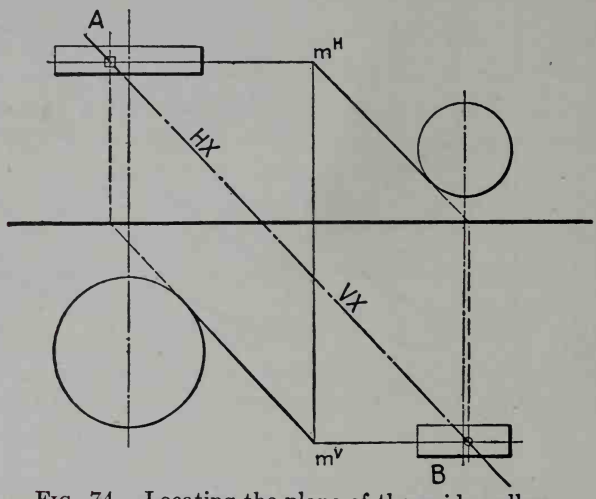


FIG. 74.—Locating the plane of the guide pulleys.

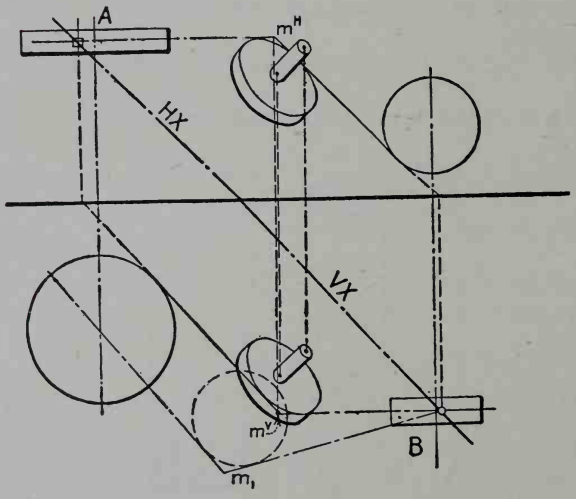


FIG. 75.—Showing revolution and counter-revolution of the guide pulley.

Construction.—1. Select a point, M (Fig. 74), between the pulleys, preferably about midway, common to the center planes of the pulleys.

2. Run a tangent from M to each of the pulleys in their center plane.

3. Draw the traces of X, the plane of the two tangents.

4. Revolve the tangents intersecting at M about VX (or HX) and construct a circle of any suitable dimensions tangent to them, and counter-revolve it to show its *H*- and *V*-projections in regular position.

112. EXERCISES

Use any combination of the following sizes of pulleys, and draw the projections of a guide pulley at a convenient point. Make all pulleys 8-in. face, and place the working pulleys in the centers of their respective shafts. Any reasonable scale.

Large pulleys, 24 in., 28 in., 32 in., 36 in., 42 in., 48 in., 54 in., 60 in

Small pulleys, 8 in., 10 in., 12 in., 14 in., 16 in., 18 in.

Guide pulleys, 40 in., 44 in., 48 in., 54 in., 60 in.

625. Draw the projections of a ——in. guide pulley to run between a ——in. pulley on shaft A(1½ − ½ + 2½) B(1½ − 1½ + 2½) and a ——in. pulley on shaft C(4 − 2 + 2) D(4 − 2 + 1).

626. Same for shafts E(1 + 2 − ¾) F(2 + 2 − ¾) and G(4 + 1½ − 2) K(4 + ½ − 2).

627. Same for shafts M(1½ + 2 − 2) N(1½ + 1 − 2) and O(4 + 1½ − ½) P(4 + 1½ − 1¼).

628. Same for shafts A(1 − 1 + 1) B(1 − 2 + 1) and C(2½ − 1 + 2) D(4½ − 1 + ½).

629. Same for shafts E(1 − 2, 0) F(3 − 2 + 3) and G(4 − ½ + 1) K(4 − 1½ + 1).

630. Same for shafts L(1 + 3 − ½) M(1 + ½ − 3) and N(4 + 1 − ½) O(4 + 3 − 3).

631. Same for shafts P(1 + 3 − ½) A(1 + ½ − 3) and X(4½ + 2 − 2½) Y(5½ + 2 − 2½).

632. Same for shafts B(½ + 1 − 1) D(1½ + 1 − 1) and E(4 + 2 − 1) K(4 + 2 − 3).

113. THE PROBLEM OF REFLECTION

Problem 38.—To find the points at which a ray is reflected from planes in passing through two points in space.

The basis of this solution is the law of physics, that the angle of incidence is equal to the angle of reflection.

Refer to Fig. 76. A ray is to pass through A, strike the plane CC, and reflect to B. To find D, the point of reflection.

Imagine CC a mirror. The observer at A looks at the mirror in the direction of B′, which is the reflection of B, and is located an equal distance on the opposite side of the mirror; *i.e.*, B′E = BE. The line of vision is a straight line from A to B′, piercing the plane (mirror) at D. The angles BDC and B′DC are equal, and equal to ADC, hence a ray from A is reflected at D to B.

If the number of planes is increased, the straight line still persists if the object is transposed symmetrically about each of the planes.

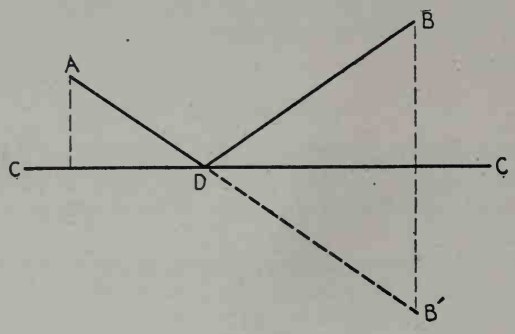


FIG. 76.

Construction.—Let it be required to find the points on *H* and *V* where a ray, through A, striking both, is reflected to B (Fig. 77).

1. Locate the projections of B at $b_1^V b_1^H$, symmetrically placed on opposite sides of *H* and *V*.
2. Draw a straight line from A to $b_1^V b_1^H$.
3. Locate the *H*- and *V*-piercing points of this line, C and D.

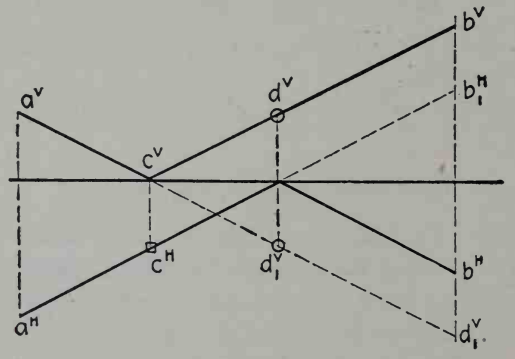


FIG. 77.

4. As C is the first point reached it is the point of reflection on *H*.
5. Transpose d_1^V to d^V, an equal distance above *H*, and it will be the point of reflection on *V*.
6. Draw the line ACDB, and it will be the path of the ray from A to B.

114. PRACTICAL EXERCISES

633. Find the length of all the oblique rafters in Fig. 78. Scale ⅛ in. = 1 ft.

634. Find the true angles between the various oblique rafters and the horizontal ridges that intersect them (Fig. 78).

635. Measure all the valley angles between the planes A and C, A and D, etc., in the roof (Fig. 78).

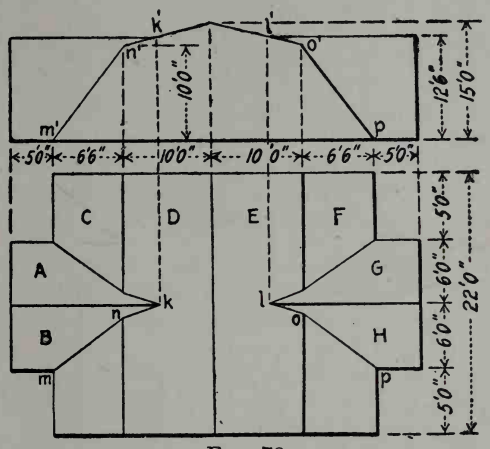


FIG. 78.

636. A rectangular chimney 3 ft. 6 in. × 5 ft. 4 in. is to be put through roof C in Fig. 78. Draw its *H*- and *V*-projections, and the true size of the opening in the roof.

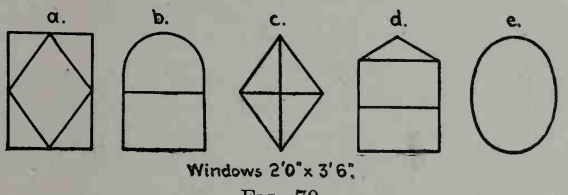


FIG. 79.

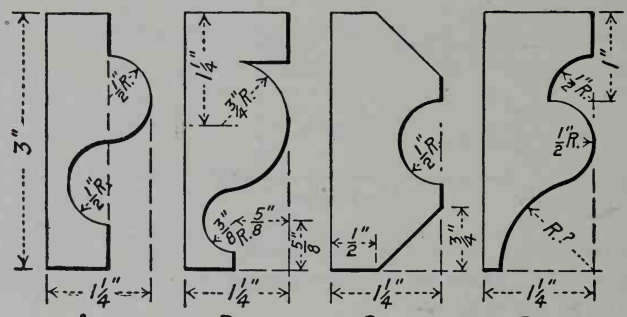
637. Roof in Fig. 78. Place window —— (Fig. 79) in the center of roof ——.

638. Cut an opening 4 ft. square in roof ——, Fig. 78.

639. An octagonal stone tower, 18 ft. in diameter, and 40 ft. high, is surmounted by a pyramidal cap, 12 ft. high. Draw its projections, and draw in the center of each of the four visible faces window —— (Fig. 79). Draw only half of the plan and the elevation of the cap. Scale ¼ in. = 1 ft.

640. A 14-in. steam feed pipe, whose center line is A(1 ft. + 3 ft. 3 in. − 3 ft.) B(4 ft., 0 − 1 ft.). The plane of the roof shed is T(5 ft. + 2 ft.) 6 in. (2 ft. − 3 ft.). Draw the projections of the opening in the roof, and its true size. Determine the dip of the roof. Scale 1 in. = 1 ft.

641. A trapeze bar, 3 ft. long, is suspended by ropes 3 ft. long. Find how high it is raised by twisting through ——° (60°, 75°, 90°, 105°, 120°).
642. What angles does the pipe in Ex. 640 make with H, V, and T. If it keeps its direction, what is the size of the hole in the floor?
643. A conical tower is 15 ft. in diameter and 14 ft. high. Draw two lines in the surface from the apex to the base, making ——° (30°, 45°, 60°) with each other.
644. Draw a cube of 1½-in. sides on H. Draw the traces of a plane that shall cut a regular hexagon from the cube. Revolve the hexagon to prove that it is regular, and calculate the angle that Q makes with H.
645. A building 28 ft. 6 in. × 41 ft. 3-in. ground plan is covered by a hip roof having a central ridge 18 ft. long, commencing 10 ft. from the front of the house. The front corner rafters are inclined 45° from the horizontal. Draw the plan and elevation of the roof. What is its height? Measure the hip angles. What are the angles made with the horizontal by the rear corner rafters?



Moulding Sections.

FIG. 80.

646. Two pulleys 24 in. × 8 in. are located on parallel horizontal shafts. The center line of one is A(4 ft. + 6 ft. 6 in. − 18 in.) B(6 ft. + 6 ft. 6 in. − 6 in.), and the other passes through C(6 ft. + 2 ft. − 5 ft.). The first pulley is midway between A and B. The belt passes through a wall whose plane is S(10 ft. + 4 ft.) 6 ft. (2 ft. − 4 ft.). Draw the projections and true size of the opening in the wall to accommodate the belt. Allow 2-in. clearance at all points. Scale ½ in. = 1 ft.
647. Inscribe a sphere in the tetrahedron A(2, 0 − 1) B(4, 0 − 1) C(3½, 0 − 3) D(3¼ + 2 − 1¾).
648. Circumscribe a sphere about the tetrahedron in Ex. 647.
649. Inscribe a sphere in the tetrahedron M(2 + 1½ − 1) N(3 + 2 −½) O(3¼ + 1½ − 1½) P(3 + 1 − 2).
650. Circumscribe a sphere about the tetrahedron in Ex. 649.
651. Pass a plane equidistant from A(2 + 3 − 2) B(3 + 1 − 1) and C(5 + 3 − ½) D(3½ + 1 − 3).
652. Draw the H- and V-projections of a 1½-in. hex nut, drilled and chamfered, on the center line M(2 + 1 − 3) N(4 + 2 − 1). Full size.

653. Determine the outline of a moulding cutter to cut section ——, Fig. 80. The cutter is mounted on an arbor, 3-in. diameter, and the center line of the arbor is 4½ in. from the table of the machine. Full size.
654. Two pipes, C(2 + 1½, 0) D(3½, 0 − 2) and E(2½, 0 − ½) F(4 + 2 − 1) are to be connected by the shortest possible **horizontal** pipe. Allow 2 in. for connections. What is its length to a scale of 1 in. = 1 ft.?
655. Draw the projections of a 2-in. square pyramid, whose hip angles are ——° (105°, 120°, 135°, 150°).
656. At a point of outcrop A(4 + 2 − 1) the strike of a body of ore is south, 50° west. The dip is 45°. A tunnel is driven from B(1 + 3 − ¼), which is on the surface, to the ore perpendicularly. Where will it strike the ore?

Note.—The "strike" of a stratum of ore is a horizontal line in the plane of the ore. The "dip" is the inclination of the ore plane to the horizontal.

657. The point of outcrop is A(200 ft. + 200 ft. − 100 ft.), and a line is run to B, a distance from A 200 ft., south, 30° west. The altitude at B is 125 ft., and a vertical bore hole strikes ore at a depth of 25 ft. Another bore hole is made at C, on the same level as B, and 150 ft. distant from A and B. At C the hole is drilled 100 ft., striking the ore. Determine the dip and strike of the vein. Scale 1 in. = 100 ft.
658. Points D, E and F are on the side hill, which dips 20° east. D is a point of outcrop of a vein of ore. From D, E bears south 45° east 400 ft. on the slope of the hill, and F bears south 15° east 600 ft. on the slope of the hill. Vertical drill holes at E and F strike ore at 100 ft. and 150 ft. respectively. Determine the dip and strike of the vein. Scale 1 in. = 100 ft.
659. At A(8 ft. + 24 ft., 0) and B(20 ft. + 24 ft., 0) in a mill wall, two parallel 6-in. pipes run downward to the right at 45° to the floor and 30° to the wall. It is proposed to run a 12-in. cylindrical chute on the center line C(30 ft. + 16 ft., 0) D(4 ft., 0 − 24 ft.). Can this be done? If not, how much could D be shifted toward the wall to effect it? Scale ⅛ in. = 1 ft.
660. A pulley, 24-in. diameter, and 12-in. face, runs on the shaft A(2 ft. + 2 ft. − 6 ft.) B(10 ft. + 7 ft. − 2 ft.). Draw the projections of the pulley. Scale ½ in. = 1 ft.
661. Draw the projections of a 1½-in. sphere tangent to the plane T(1 + 3) 4(1 − 2) at the point O(3 + ¼, x) in T.
662. Measure the various lineal and dihedral angles of a hexagonal pyramid of 2-in. base and 2½-in. altitude.
663. Draw the projections of a 10-ft. circular skylight in a roof whose plane is S(8 ft. + 24 ft.) 32 ft. (8 ft. + 16 ft.), with its center at M(20 ft. + 8 ft., x) in the roof. Scale ⅛ in. = 1 ft.
664. The eaves-line of a roof is A(16 ft., 0 − 20 ft.) B(33 ft., 0 − 5 ft.). The horizontal ridge is 13 ft. 6 in. high. The rafter at A runs perpendicularly to AB, dipping 60°. The rafter at B runs parallel to *V* up to the ridge. Calculate the cost of covering this roof at $1.50 per square yard. Scale ⅛ in. = 1 ft.

665. A tunnel through a hill runs from A(425 ft. + 275 ft. − 100 ft.) to B(2100 ft. + 150 ft. − 1300 ft.). Find its length and grade.
666. A plane T passes through *GL* and the point K(3 + 1 − 2). A plane S parallel to it cuts *V* 1¼ in. above *GL*. Find the *H*-trace of S, the distance between T and S, and their angles with *H* and *V*.
667. Pass a plane through L(1 + 2 − 1) equally distant from M(4 + 1 − ¾), N(3 + 2 − ½), and O(2 + 2 − ¼). Find a point in the plane thus found which is equally distant from M, N and O.
668. Erect a cube on *H* with its base diagonal A(3, 0 − ½) B(4, 0 − 1¼). Show its picture as seen from C(3½ + 1¾ − 3). Show the picture in its true size.

Note.—Pass the picture plane perpendicular to the line from the point of sight to the center of the cube.

669. From a hole, A(18 ft. + 10 ft., 0), in a mill wall, run a pipe that will make a straight connection with two pipes, whose center lines are B(5 ft. + 10 ft. − 3 ft. 6 in.) C(19 ft., 0 − 10 ft.) and D(3 ft., 0 − 8 ft.) E(10 ft. + 10 ft. − 8 ft.). What are the lengths of the two portions of the pipe? Scale ¼ in. = 1 ft.

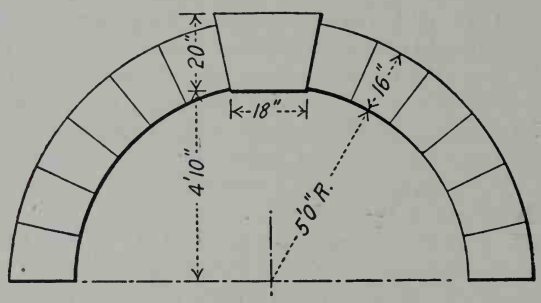


FIG. 81.—Stone arch.

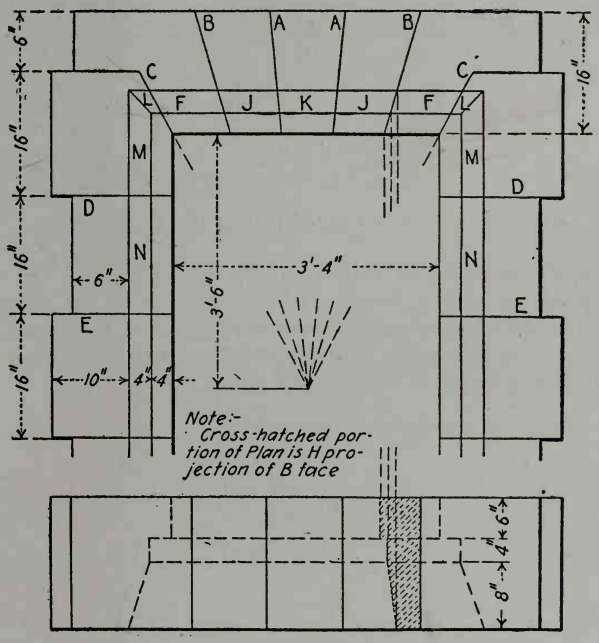
670. Draw the projections of the circle passed through A(2 + 2 − ¼), B(2½ + 1 − 1), and C(3½ + 2 − 1½).
671. The hinge of a door is M(4 + 3, 0) N(4, 0, 0). When shut, it is to the left of MN. A taut wire runs from O(3 + 2, 0) to P(5, 0 − 1). Find the outline of a slot that will allow the door to open freely through 90°.
672. Let *H* be a mirror. A ray of light passes through A(1 + 1½ − 2½) strikes *H* and passes through B(3½ + 1 − ½). Where does it reflect on *H*, and where does it strike *V*?
673. Find the path of a ray of light that passes through O(5 + 2 − 1¼) and is reflected from *H* and *V* through P(2 + 1 − 2).
674. The top of a large bin in a mill has for its corners the points A(4 ft. + 7 ft. 6 in., 0), B(20 ft. + 7 ft. 6 in., 0), C(4 ft. + 2 ft. 3 in. − 8 ft. 9 in.), and D(20 ft. + 2 ft. 3 in. − 8 ft. 9 in.). From M(16 ft. + 12 ft., 0) the center line of a 24-in. square chute runs downward to the left at 45° to *H* and 30° to *V*. Draw the projections of the hole in the bin top made by the chute, and find its true size. Scale ¼ in. = 1 ft.

675. Draw the projections of four spheres, 1¼ in., 1½ in., 1¾ in., and 2 in. in diameter, each one tangent to the other three.

676. Develop the surface of the keystone of the arch shown in Fig. 81. The masonry is 14 in. thick. Use any convenient scale.

677. M(8 in. + 7½ in. − 2 in.) N(17 in. + 4 in. − 8 in.) is the center line of a hollow tile, 4 in. × 12 in., of square section with rounded corners, ½-in. walls. The rounded corners are 1¼-in. radius. Draw its projections, scale 3 in. = 1 ft.

677. A borehole cuts a 6-ft. core from a vein, whose strike is north 60° east, and whose dip is 45° to the northwest. The borehole bears south 75° west and dips 30°. What is the real thickness of the vein?



Ex. 681.—Plate band arch.

678. At A, where the elevation is 9800 ft., a vertical borehole strikes a vein at 300 ft. B bears north, 30° east, 1500 ft. from A, and has an elevation of 9600 ft. An inclined hole is drilled, bearing north 75° east, dipping 75°, striking the vein at 525 ft. At C, whose elevation is 9900 ft., 1200 ft. from A, south 60° east, a vertical borehole cuts the ore at 375 ft. Required the dip and strike of the vein. Let *H* be at 9500 ft. level.

679. At A(1 + 3 − 3) a borehole is drilled, bearing north 30° east, and dipping 75°. It intersects a vein, whose strike is due north, and cuts an 8-ft. core. The true thickness of the vein is 5 ft. What is the dip of the vein?

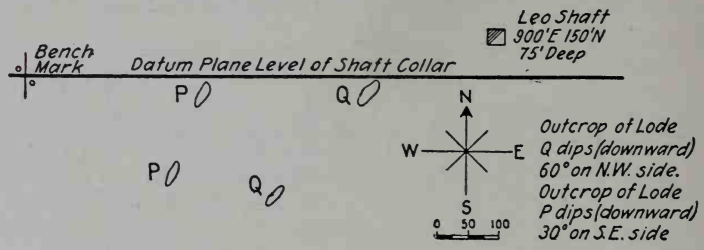
680. Vertical holes on a hill side (considered a plane) are bored 150 ft. at

A(100 ft. + 400 ft. − 300 ft.), 200 ft. at B(350 ft. + 250 ft. − 325 ft.), and 125 ft. at C(600 ft. + 325 ft. − 100 ft.), striking a vein of ore. What is the line of outcrop, and what are the dip and strike of the vein?

681. Draw the projections of the Plate Band Arch. Lay out the patterns for joints A, B, C, D, E. Scale ½ in. = 1 ft.

682. Plate Band Arch. Lay out the patterns for the slanting faces, F, J, K, L, M, and N. Scale ½ in. = 1 ft.

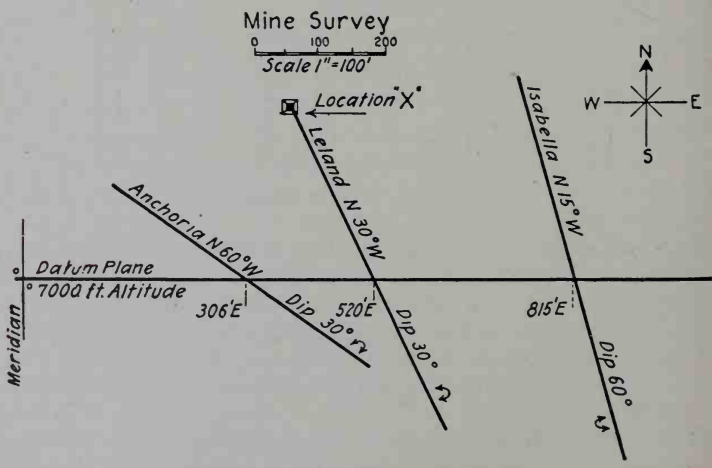
683. Plate Band Arch. Make working drawings in three views of Block —— (A, B, C, N, M). Scale 1 in. = 1 ft



Ex. 684.—Mine survey of Prince and Queen veins.

684. Given the outcrops of Prince and Queen veins, and the shaft Leo. Required the length, bearing, and inclination of the shortest cross-cut from the present bottom point of the shaft to the intersection of P and Q. Also the length, bearing, and level of the drift from the lowest point of this line to the line of the shaft.

Note.—Draw compass mark and scale.

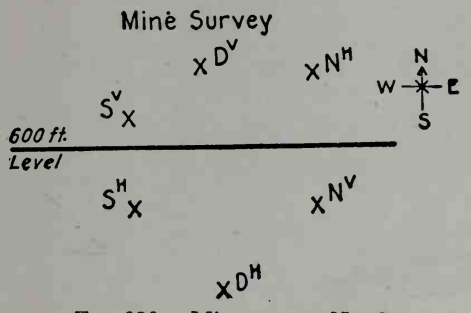


Ex. 685.—Mine survey No. 2.

685. Mine Survey of Anchoria, Leland, and Isabella lodes. Required: (1) Location and depth of a shaft to the intersection of A − L − I; (2)

length and bearing of a 10 per cent. tunnel from "location x" to the intersection of A and L; (3) length, bearing, and position of portal on the outcrop of Anchoria to the intersection A − L − I, when the tunnel has a 10 per cent. grade.

Note.—Print **Mine Survey** like plate mark. Draw compass mark and scale. Give a printed statement of the answers.



Ex. 686.—Mine survey No. 3.

686. Given the Shaft Collar of a winze "S," and a hoisting drum at "D." Required: The bearing and inclination of the shaft of a guide pulley for a mine cable at "N."

CHAPTER III

INTERSECTIONS AND DEVELOPMENTS OF PLANE SOLIDS

115. Problem 39.—To find a plane section of a pyramid or prism.

Analysis.—1. Obtain the piercing points of the various oblique edges with the given plane.

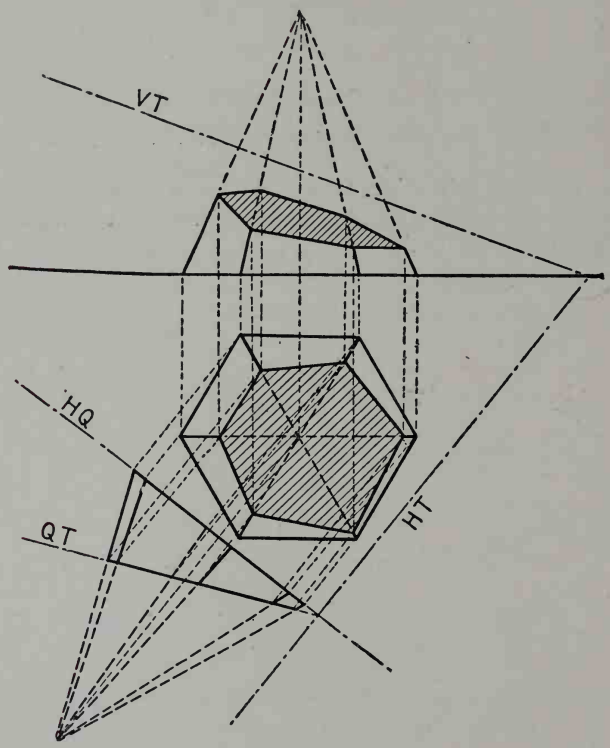


FIG. 82.

2. Connect the points two and two by straight lines in order.

Construction.—Let it be required to draw the section line of the hexagonal pyramid made by the plane T (Fig. 82).

1. Draw the Q-projection of the pyramid.

2. Draw the Q-trace of T.

3. Project the various piercing points to H and V. (The Q-projections of these points are all located on QT.)

4. Join the points in order, forming the irregular hexagon enclosing the crosshatched space.

Development

116. Definition.—To develop the surface of a solid is to bring its various faces and bases into one plane, all in their true sizes and relative positions, so that the plane figure thus outlined can be cut out and folded into the form of the solid.

Note.—In sheet metal drafting this is called "laying out the pattern."

Problem 40.—To develop the surface of a truncated prism.

Construction.—Let it be required to develop the square prism (Fig. 83), between its H-base and the T-section.

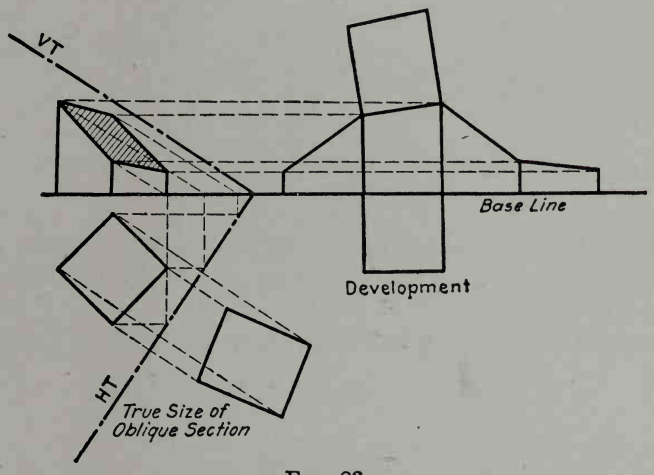


Fig. 83.

1. Lay out the "base line" on a right section of the prism. (In this case use the H-base.)

2. Lay off the true length of the base edges on this line.

3. Erect perpendiculars at these points, making them equal in length to the segments of the longitudinal edges, in order.

4. Connect the ends of these perpendiculars two and two by straight lines.

5. Attach the top and bottom bases, A and B, to their proper edges in their true sizes.

117. Problem 41.—To develop a truncated pyramid.

Construction.—Let it be required to develop the pyramid shown in Fig. 82 between the *H*-base and the T-section.

1. Obtain the true lengths of the edges, and of the segments between bases.

2. Lay out the triangles in true size and order, with a common vertex.

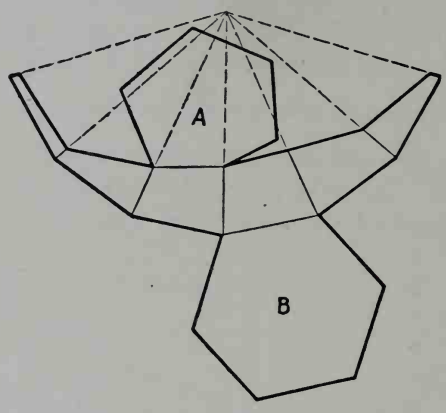
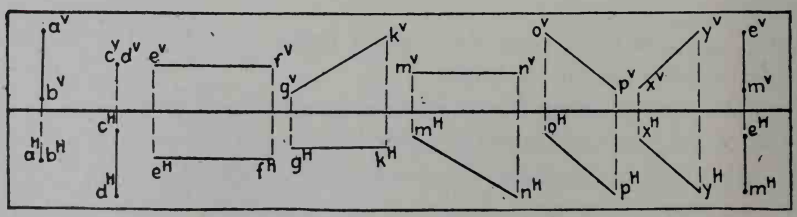


FIG. 84.—Development of truncated pyramid.

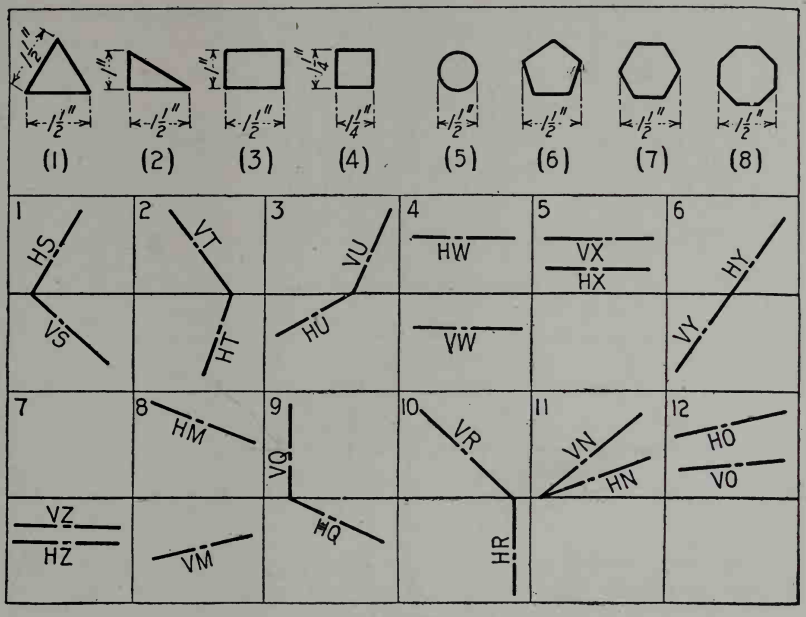
3. Lay off on the edges in this figure the true lengths of the various segments, and connect these points by straight lines, two and two.

4. Attach the bases A and B in their true sizes to the proper edges.

118. EXERCISES



Graphic Layout No. 46.—Center lines for regular solids.



Graphic Layout No. 47.—Bases for regular solids, and oblique planes for truncating. In laying out the work place the planes so that they will truncate the solids.

Note.—If desired by instructor, the layout may be drawn in III.

683. Draw the intersection of the regular prism, base ——, altitude 3 in., center line ——, made by the plane ——. Find the true size of the intersection.
684. Develop the surface of the prism in Ex. 683 between the right base and the oblique base.
685. Pass a profile plane through the prism specified in Ex. 683, and find the P-projection of the section.
686. Draw the intersection of the regular pyramid, base ——, altitude —— in. (2, 2½, 3), center line ——, made by the plane ——. Find the true size of the section.
687. Develop the surface of the pyramid in Ex. 686 between the right base and the truncated section.
688. Develop the surface of the pyramid in Ex. 686 between the apex and oblique base.
689. Make a profile section of the pyramid in Ex. 686.
690. Make a full set of details of all the glass and sheet iron pieces in the lamp shade and supports, shown in Fig. 85. Do not duplicate the details, but state the number required for each. Allow for clearance and the folding of the iron pieces.

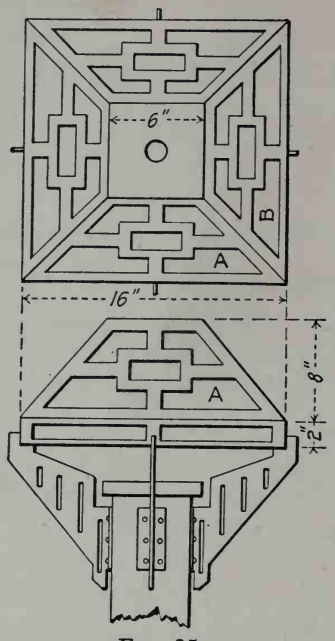


FIG. 85.

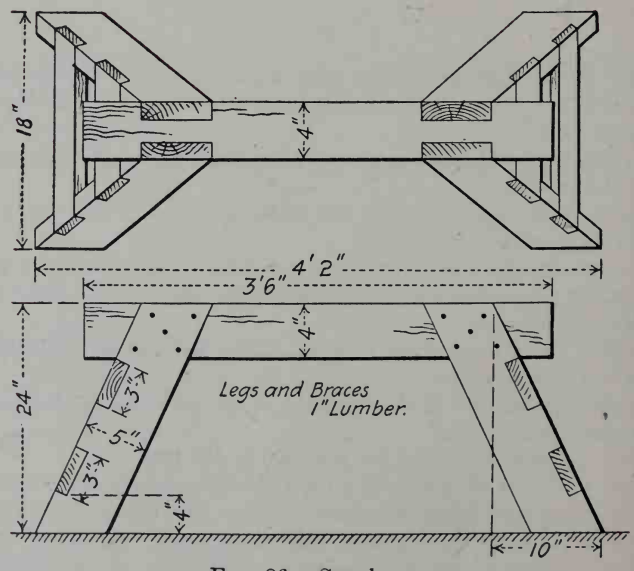


FIG. 86.—Saw horse.

691. Make a detail of the backbone of the saw horse shown in Fig. 86, showing the angles of the cuts to be made. Draw to any convenient scale.

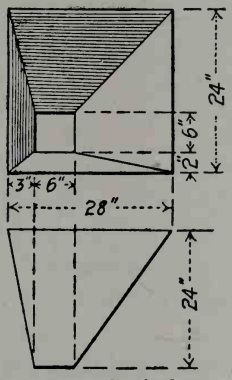


Fig. 87.—Grain hopper.

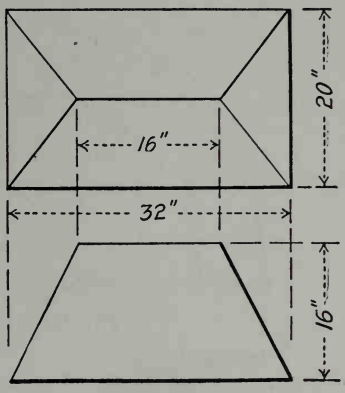


Fig. 88.—Ventilator cap.

692. Make detail drawings of the legs and braces of the saw horse, shown in Fig. 86, showing the angles of the various cuts. Draw to any convenient scale.

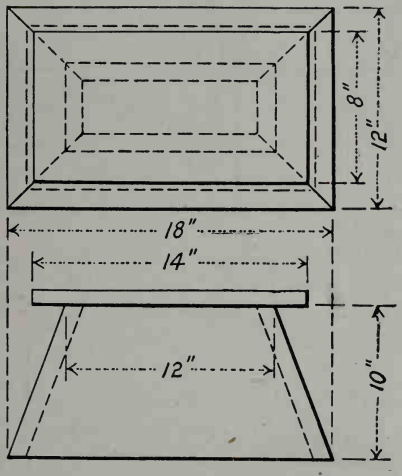


Fig. 89.—Stand.

693. Make a pattern to any convenient scale of the grain hopper, shown in Fig. 87. Use two spaces for the projections and pattern. Make a paper model of the hopper, putting on a flap for holding together.

694. Make a pattern to any convenient scale, of the ventilator cap, shown in Fig. 88.

695. Fig. 89 shows a stool, or plant stand, made of 1-in. lumber. The edges of the four inclined supporting pieces are chamfered to fit. Make a working drawing with full details to scale, showing the angles

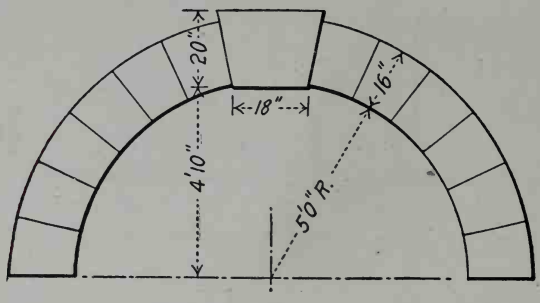


Fig. 90.—Stone arch.

of the chamfered pieces, without duplicating any of the parts. Take two spaces for the work.

696. Develop the surface of the keystone of the arch shown in Fig. 90. The masonry is 12 in. thick. Use any convenient scale.

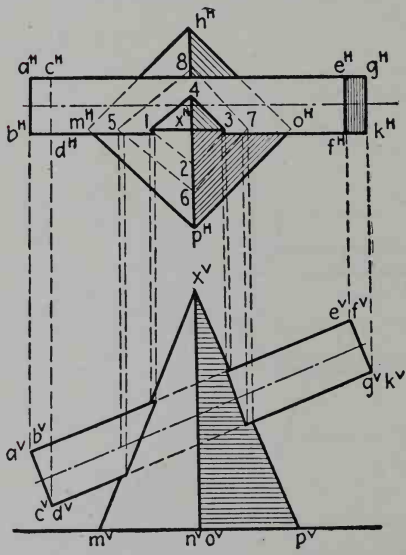


Fig. 91.

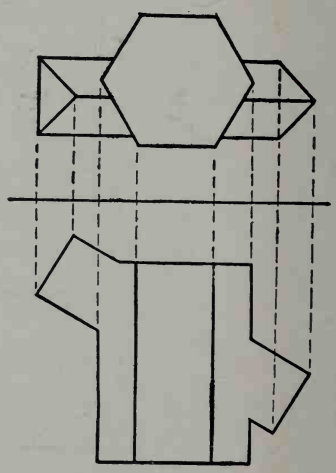


Fig. 92.

119. Problem 42.—To find the line of intersection between two plane solids.

Construction.—To find the line of intersection between the square pyramid and square prism, shown in Fig. 91.

The line of intersection between two plane solids is a broken line (sometimes two) made of the lines of intersection between the various planes bounding the solids.

1. Find the intersection of the plane ABEF (the upper face of the prism) with the pyramid. This is the quadrilateral 1234.

Note.—It is not necessary to find the traces of the plane to accomplish this.

2. Find the intersection of the plane CDGK (the lower face of the prism) with the pyramid.

3. Connect the points thus found with the limiting points on the two solids, in order, and the resulting broken line will be the line of intersection. In Fig. 91 this is the heavy line in each projection.

120. Second Method.—By projecting on an auxiliary plane.

Construction.—To find the line of intersection between the hexagonal prism and the triangular prism, Fig. 92.

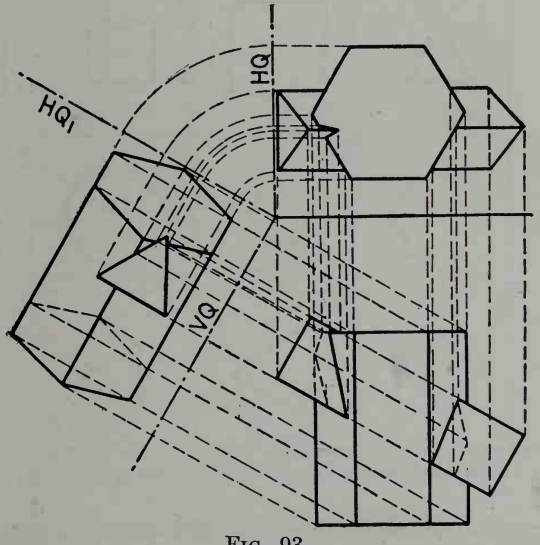
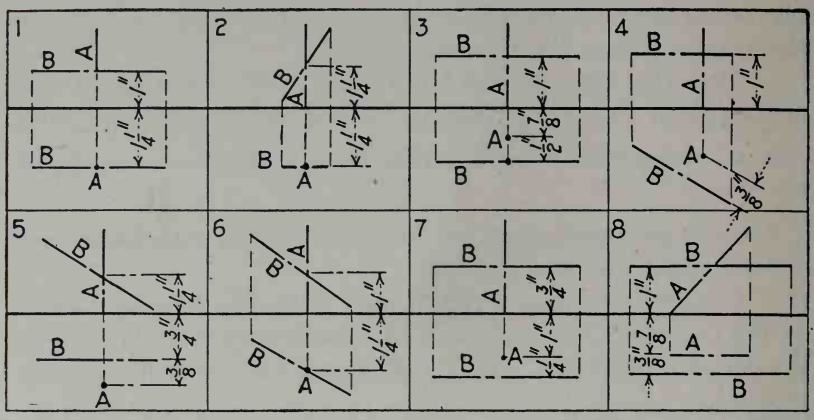


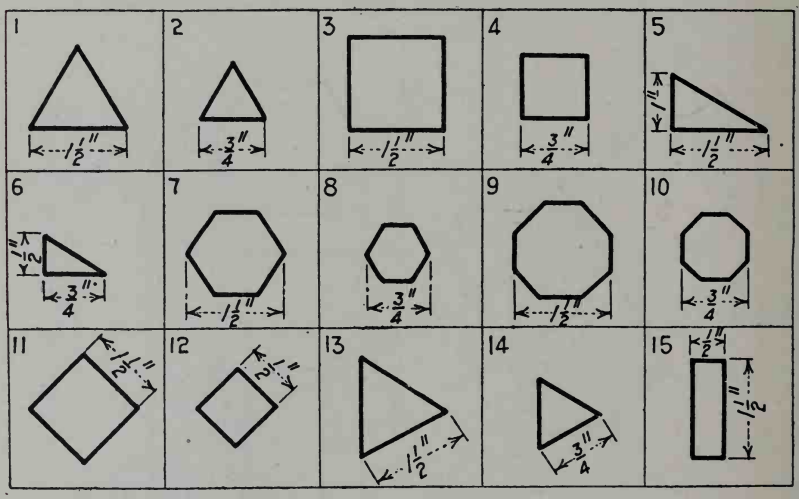
Fig. 93.

Refer to Fig. 93. Here the solids are projected on Q, which is taken perpendicular to the center line of the oblique solid. Such piercing points, as cannot be obtained from the *H*- and *V*-projections, will be obtained by direct projection from the Q-projection to the *H*-projection and *V*-projection.

121. EXERCISES



Graphic Layout No. 48.—Center lines for intersecting solids. Letter to suit, locating in *I* or *III*. Another pair (9) can be made like (8) with axes intersecting.



Graphic Layout No. 49.—Bases for solids.

698. Let A —— be the center line of a right pyramid, base ——, altitude —— in. (2, 2½, 3), and B —— be the center line of the right pyramid, base ——, altitude —— in. (2, 2½, 3).
   (*a*) Draw the line of intersection.
   (*b*) Develop the surface of pyramid A.
   (*c*) Develop the surface of pyramid B.

699. Let A —— be the center line of a right pyramid, base ——, altitude —— in. (2, 2½, 3), and B —— be the center line of the right prism, base ——, altitude 3 in.

(*a*) Draw the line of intersection.
(*b*) Develop the surface of pyramid A.
(*c*) Develop the surface of prism B.

700. Let A —— be the center line of a right prism, base ——, altitude 2 in. and B —— be the center line of a right pyramid, base ——, altitude —— in. (2, 2½, 3).
(*a*) Draw the line of intersection.
(*b*) Develop the surface of prism A.
(*c*) Develop the surface of pyramid B.

701. Let A —— be the center line of a right prism, base ——, altitude 2 in., and B —— be the center line of a right prism, base ——, altitude 3 in.
(*a*) Draw the line of intersection.
(*b*) Develop the surface of prism A.
(*c*) Develop the surface of prism B.

702. Make a paper model of the two given solids.

CONCRETE FORM PROBLEMS

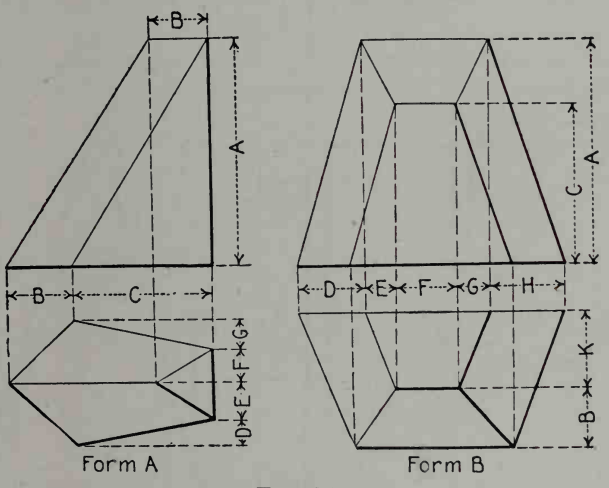


FIG. 95.

A	B	C	D	E	F	G	H	K
ft.	ft.	ft.	ft. in.	ft. in.	ft. in.	ft. in.	ft. in.	ft.
20	8	16	3	4	4 6	2 6	3	8
18	7	15	2 9	3	5	3	2 9	6
16	6	13	2 6	2 6	3	2 9	2 6	5
12	5	9	2	2	3	2	2 3	4

703. Develop the patterns for the concrete form —— (A or B), dimensions —— (1, 2, 3, or 4), constructed by the Santa Fe Railroad (dimensions altered).

704. In laying out the timbering for the gable, shown in Fig. 94, use sizes selected from the following list:

4 in. × 6 in.	6 in. × 6 in.	8 in. × 8 in.	10 in. × 10 in.
4 in. × 8 in.	6 in. × 8 in.	8 in. × 10 in.	10 in. × 12 in.
4 in. × 10 in.	6 in. × 10 in.	8 in. × 12 in.	10 in. × 14 in.
4 in. × 12 in.	6 in. × 12 in.	8 in. × 14 in.	10 in. × 16 in.

Dimensions of the gable may be altered, if desired. Detail each piece, laying off the angles of the trim, with full details of measurements. Scale to suit.

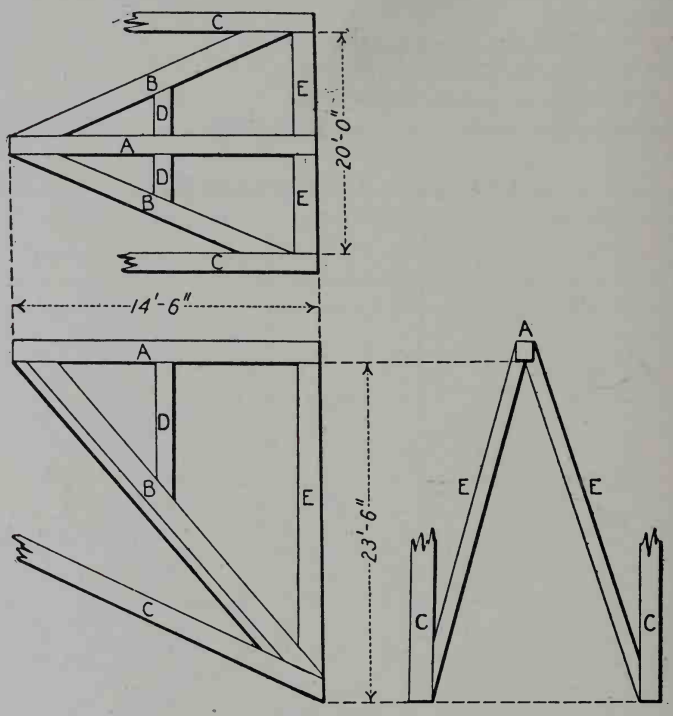
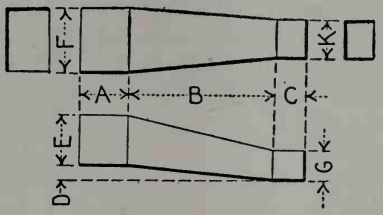


FIG. 94.—Timbering of a gable in Main Building, Armour Institute of Technology.

705. Two timbers, whose center lines are A(4 ft. + 10 ft. − 2 ft.) B(18 ft., 0 − 5 ft. 6 in.) and C(4 ft., 0 − 8 ft.) D(18 ft. + 10 ft. − 8 ft.), are connected by the shortest possible timber brace, notched to fit. Select the timber sizes from the foregoing list, and draw the projections of the three pieces in assembly and detail, with full dimensions of notches and trims. AB and CD are trimmed to fit horizontal timbers. Scale ¼ in. = 1 ft.

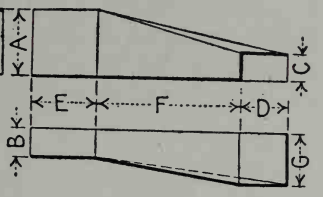
706. Using the same data as Ex. 705, make the crossbrace the shortest possible horizontal timber.

RECTANGULAR CONNECTION PIPES



A	B	C	D	E	F	G	K
in.	in.	in.	in.	in.	in.	in.	in.
6	18	6	4	10	10	4	6
5	14	5	3	8	6	3	3
4	12	4	2	6	6	3	4
4	10	3	1	6	10	2	5

Tapered Connecting Pipe.

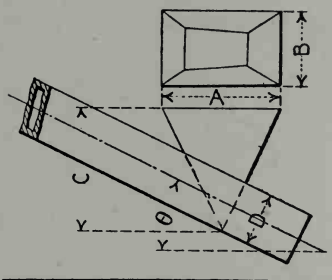


A	B	C	D	E	F	G
in.	in.	in.	in.	in.	in.	in.
8	3½	4	7	8	12	8
6	3	3	7	7	10	7
5	3	2	6	6	9	6
4	2	2	5	5	8	6

Twisted Connecting Pipe.

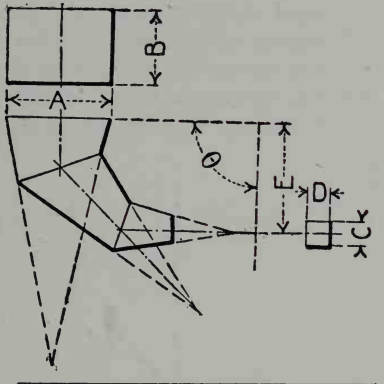
Fig. 96.

707. Develop the patterns for the tapered connection pipe —— (Fig. 96).
708. Develop the patterns for the twisted connection pipe —— (Fig. 96).
709. Develop the patterns for the hopper —— (Fig. 97). Any convenient scale.



A	B	C	D	θ
ft.in.	ft. in.	ft.	in.	degrees
4 2	2 10	5	12	15, 30, 45, 60
3 8	2 4	5	12	15, 30, 45, 60
3 2	2 0	4	14	15, 30, 45, 60
2 10	2 1	4	14	15, 30, 45, 60
2 6	1 9	4	16	15, 30, 45, 60

Hopper for Square Chute.
Fig. 97.



A	B	C	D	E	θ
in.	in.	in.	in.	in.	degrees
10	8	2½	2	12	60, 75, 90
10	6	2½	1½	12	60, 75, 90
10	4	2½	1	12	60, 75, 90
16	12	4	3	20	60, 75, 90
16	10	4	2½	20	60, 75, 90
16	8	4	2	20	60, 75, 90
24	18	8	6	30	60, 75, 90
24	15	8	5	30	60, 75, 90
24	12	8	4	30	60, 75, 90

Elbow Hopper (Pyramidal).
Fig. 98.

710. Develop the patterns for the hopper elbow ——, in —— (2, 3, 4) pieces (Fig. 98). Any convenient scale.

711. A rectangular air pipe, 10 in. × 12 in., follows the center line A(3 ft. + 6 ft., 0) B(3 ft. + 6 ft. − 2 ft.) C(7 ft. 6 in. + 2 ft. − 2 ft.) D(7 ft. 6 in., 0 − 2 ft.), coming through the wall horizontally, running downward obliquely, and then down through the floor. Keep the cross-section uniform except at the turns. Lay out the joints in projection and lay out the patterns for the piping. Scale ½ in. = 1 ft.

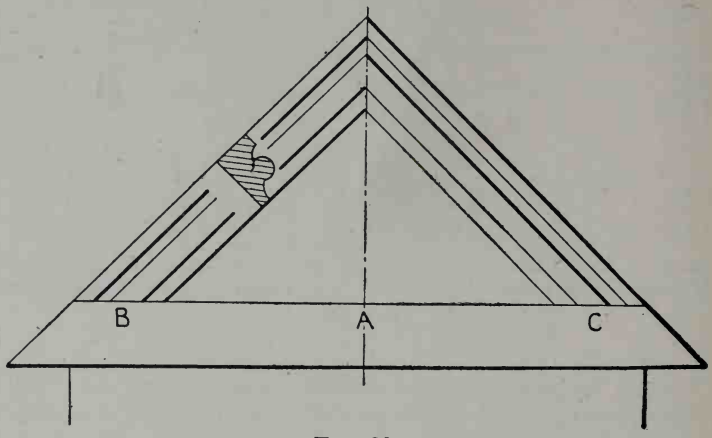
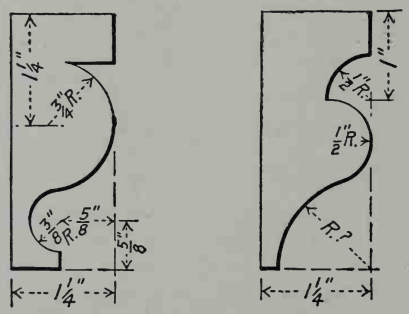


Fig. 99.

712. Fig. 99 shows a 45° gable, with "Water Table" A inclined at 30°, using moulding "D," Fig. 100, multiplying each dimension by 3. Required the line of intersection of the moulding with the water table at points B and C. Also, lay out the pattern for this moulding in sheet-metal work. (**Note.**—Develop as a cylinder, and show the shape of each end in development).



Moulding Sections

Fig. 100.

713. Fig. 101 shows a 45° gable, using moulding "B," Fig. 100, multiplying each dimension by 4, with half-round pilasters as shown at C and D. Draw the line of intersection in regions E and F. Lay out the patterns for this moulding in sheet metal. Show each end in the development. Lay out also the patterns for the pilaster in sheet metal. Any convenient scale.

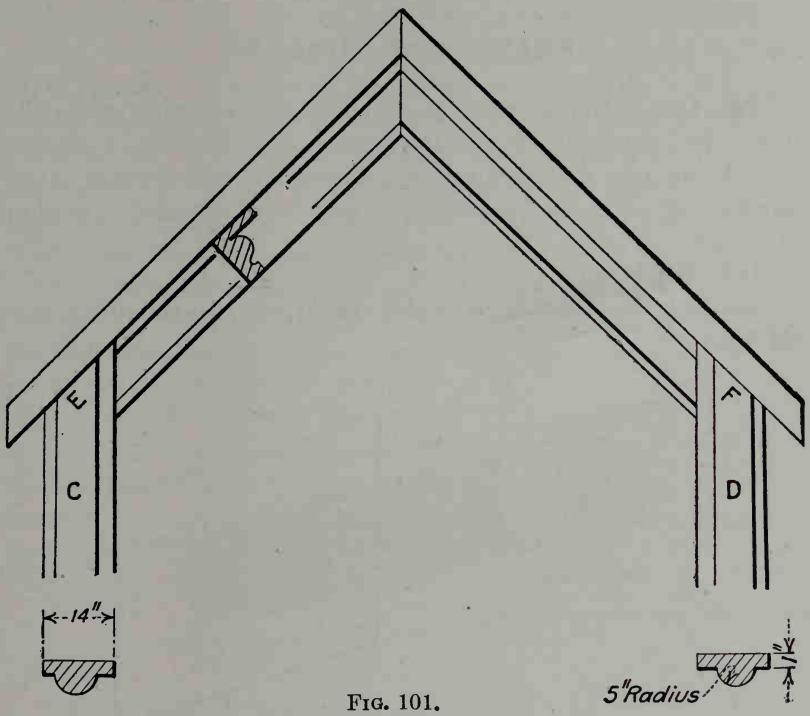


Fig. 101.

CHAPTER IV

SHADES AND SHADOWS

122. One of the interesting applications of Descriptive Geometry is the determination of the shades and shadows of objects. Catalogue cuts and architect's drawings employ it most extensively, but on many occasions it will be found useful in shop drawings.

123. Definitions.

Shade is that part of an object not exposed to the direct rays of light.

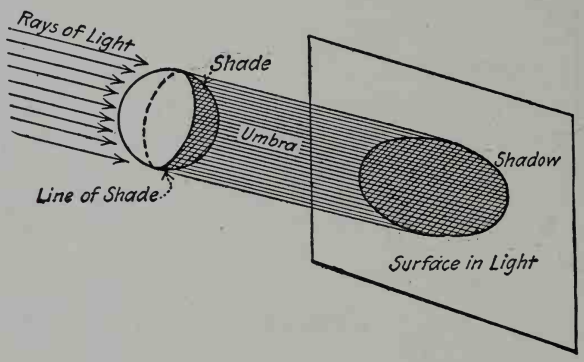


FIG. 102.

Shadow is that part of a **surface in light** from which rays of light are excluded.

Umbra is the unlighted space behind an object in light.

Note.—Shadow is the intersection of the umbra with a surface in light; that is, it is the interrupted portion of an umbra by an interposed surface.

The **Line of Shade** is the imaginary line between the light and shaded portions of an object.

124. The sun is our source of light, and its rays are assumed to be straight lines, all parallel. This assumption is very nearly true, and the deviation of the rays is so very slight that no account need be taken of the error. For convenience in conventional

shading, the sun's rays are assumed to be passing over the left shoulder of the observer in the direction of the diagonal of the conventional cube; that is, both the *H*- and *V*-projections of the rays are drawn 45° to *GL*. What angle would the rays then make with *H* and *V*?

Referring to Fig. 102, we have the rays of light depicted as coming toward a sphere in parallel lines, and either stopping (being reflected) if they strike the sphere, or passing on to the next surface in light, if they do not. One half the sphere is in **light,** the other half in **shade,** and the **line of shade** is a great circle of the sphere. This shows the **umbra** to be a cylinder of equal diameter to the sphere. The umbra, on meeting another surface, is interrupted, and its intersection therewith is the **shadow.**

125. Shadows are made up of the piercing points of the rays. From the foregoing it will appear that the composite of the tangent rays to the sphere will pierce the surface in light in the **outline of the shadow.** Hence, defining the shadow of an object resolves itself into obtaining the tangent rays and **determining their piercing points** with the various objects in their path.

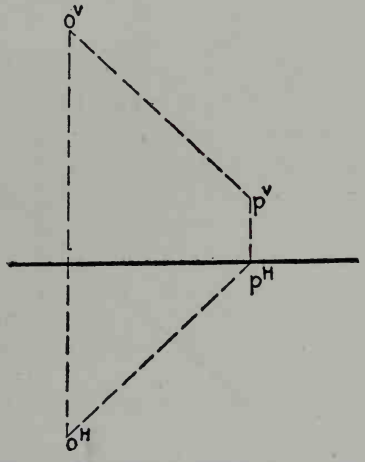


FIG. 103.—The shadow of a point.

126. LAWS

The **Shadow** of a **Point** is a **Point.**

The **Umbra** of a **Point** is a **Line.**

The **Shadow** of a **Line** is, in general, a **Line.**

The **Umbra** of a **Line** is, in general, a **Plane.**

The **Shadow** of a **Plane Figure** cast on a plane is, in general, a **Plane Figure** of similar outline.

The **Umbra** of a **Plane Figure** is, in general, a **Prism.**

THE METHOD OF FINDING SHADOWS

127. Problem 43.—To find the shadow of a point.

Construction.—Let O, Fig. 103, be the given point.

1. Draw the ray, so that $o^V p^V$ and $o^H p^H$ each make 45° with *GL*.

2. Find the piercing point of the ray with H or V, the one it meets first.

Conclusion.—O casts its shadow on V at P.

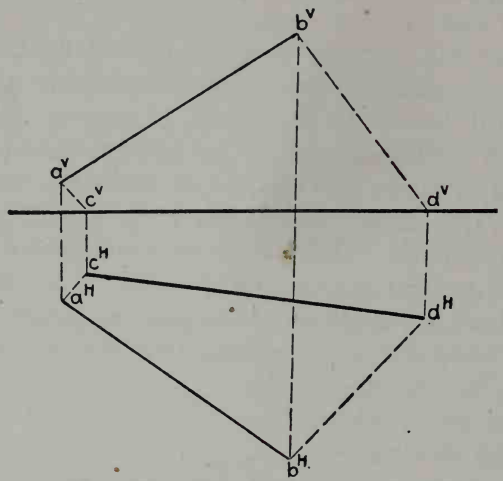


FIG. 104.—The shadow of a line.

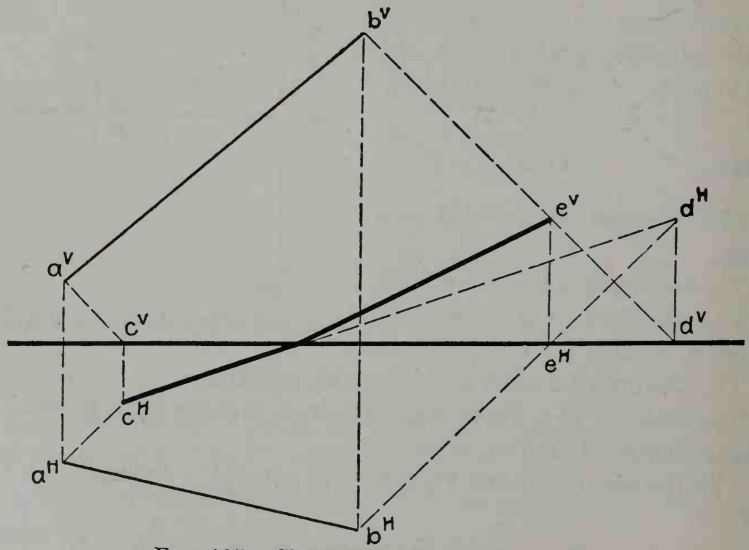


FIG. 105.—Shadow of a line on H and V.

128. Problem 44.—To find the shadow of a line.

Construction.—Let AB, Fig. 104, be the given line.

1. Find the piercing points, C and D, of A and B.

2. Draw CD; it is the shadow on H.

Special Case.—When a line casts its shadow on more than one plane.

Construction.—Let AB, Fig. 105, be the given line.

1. Find its H-piercing points, C and D, and the V-piercing point, E, of B.

2. Draw $c^H d^H$ until it meets GL, and connect it at this point with e^V.

Note.—The shadow line is the trace of the umbra plane, hence this shadow is the H-trace on H, and the V-trace on V of the plane ABCD.

129. Problem 45.—To find the shadow of a plane figure on a plane.

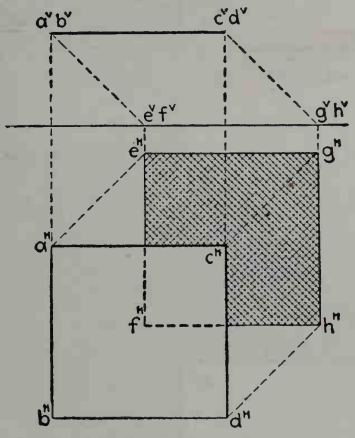


FIG. 106.—Shadow of a square.

Construction.—Let the square ABCD (Fig. 106) be the figure; to find its shadow.

1. Obtain the H-piercing points of the rays from the four corners, A, B, C, and D, at e^H, f^H, g^H, and h^H.

2. Join them in order, and fill in the enclosed space, except where it is covered (hidden) by the opaque object.

Note.—If the plane figure is parallel to the surface in light, its shadow will be an equal plane figure. Why?

130. Problem 46.—To draw the shadow of a solid on a plane.

Construction.—Let the cube shown in Fig. 107 be the given solid.

1. Draw rays from all the corners.

2. Disregard the ray through A, as it will be inside the umbra.

3. Obtain the *H*-piercing points of the rays from B, C, and D, at e^H, f^H, and g^H.

4. Connect these points with the broken line, and fill in back to the *H*-projection of the cube.

Note.—The shadows of BM and DN are the lines m^He^H and n^Hg^H.

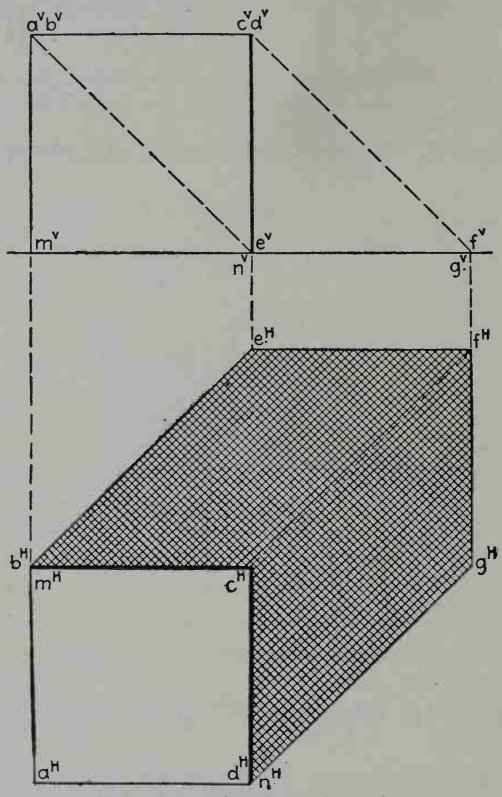


FIG. 107.—Shadow of a cube.

Special Case.—When the shadow falls on more than one plane.

Construction.—Let the hexagonal prism, Fig. 108, be the object. To find its shadow on *H* and *V*.

1. From all the exposed points draw the rays and obtain their piercing points. In this case it will be found that the point X is the only one to cast its shadow on *H*.

2. Obtain the shadow of MX on *H* and *V*, $y^H - n^V$, and of AB on *H* and *V*, $a^H - c^H - c^V$ (Problem 44, Special Case).

3. Join the remaining points in order, thus having part of the shadow on H and part on V.

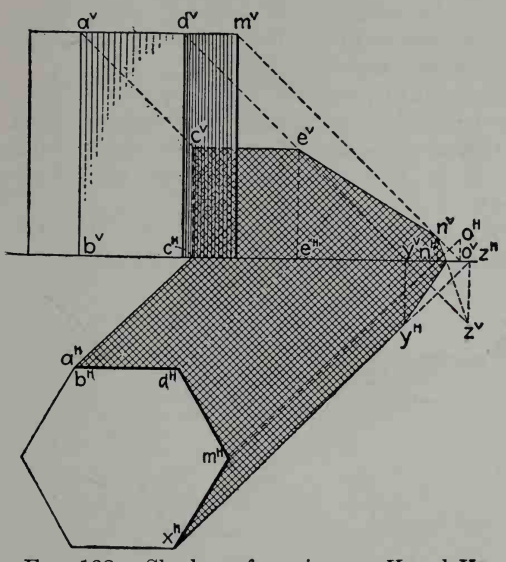


FIG. 108.—Shadow of a prism on H and V.

131. GENERAL REMARKS

The foregoing examples show the principles and rudiments of the subject of Shades and Shadows. Curved surfaces, oblique planes, and complicated objects do not alter the principles, and by applying the principles already enunciated, the most complex problems can be worked with no further instruction.

132. EXERCISES

Draw the shades and shadows of the following objects, on the specified surfaces in both projections (and profiles, if necessary). Double-crosshatch the shadows, single-crosshatch the shades, and use dotted crosshatching for the invisible parts of the shadows. For example, see Fig. 108.

Note.—When no planes are specified for the shadows, H and V are implied.

714. A cube on H, having A(3, 0 − 1) B(3, 0 − 3) for its base diagonal.
715. Twist the cube in Ex. 714, —— ° (10°, 20°, 30°, 40°).
716. Oblique cone, 1½-in. circular base in H, axis A(2, 0 − 2) B(4 + 2 − 1).
717. Hexagonal pyramid, 2-in. base, center line C(2, 0 − 1½) D(2 + 3 − 1½).
718. Pyramid in Ex. 717, center line E(1 + 1¼ − 2) F(3½ + 1¼ − 2).

719. Hexagonal prism resting on H, center line G($1\frac{1}{2}$ + $\frac{3}{4}$ − 1) K($3\frac{1}{2}$ + $\frac{3}{4}$ − 2).
720. Raise the prism in Ex. 719 $\frac{1}{2}$ in. off H.
721. Cylinder, 2-in. diameter, axis M(2, 0 − $1\frac{1}{2}$) N(2 + 3 − $1\frac{1}{2}$).
722. Surmount the cylinder in Ex. 721 with a —— (square or hexagonal) plinth, 3-in. diagonal and $\frac{1}{2}$ in. thick.
723. Cylinder tangent to H, axis O(1 + $\frac{3}{4}$ − 1) P(3 + $\frac{3}{4}$ − 2).
724. Prism —— (square or hexagonal), base diagonal 2 in., center line A(2, 0 − 2) B(2 + 2 − 2). Shadow on H and T(5 + 2) 2(5 − 2).
725. Prism —— in Ex. 724 cutting through the plane S(+ 2) ∞(− 3). Shadow on S.
726. Square prism, 1-in. base, center line A(2, 0 − 2) B(2 + 2 − 2), surmounted by a square plinth, $1\frac{1}{2}$-in. base, $\frac{1}{2}$ in. thick.

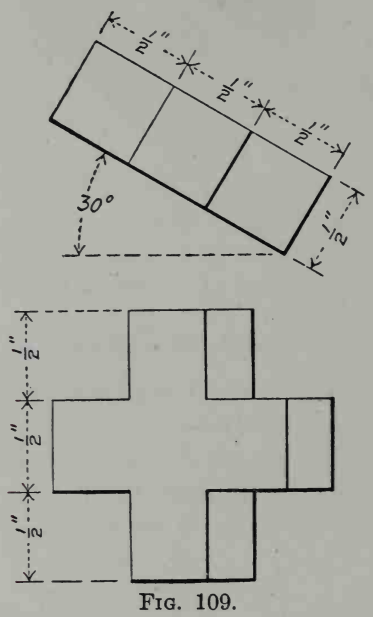


FIG. 109.

727. Prism and plinth in Ex. 726. Shadow on H, V, and T(5 + 2) 2(5 − 2).
728. Prism and plinth in Ex. 726, emerging through S(+ 2) ∞(− 3). Shadow on S.
729. Hexagonal prism, $1\frac{1}{2}$-in. base diagonal, center line A(2, 0 − 2) B(2 + 2 − 2), surmounted by a hexagonal plinth, 2-in. base diagonal, $\frac{1}{2}$ in. thick.
730. Prism and plinth, Ex. 729. Shadow on H, V, and T(5 + 2) 2(5 − 2).
731. Prism and plinth, Ex. 729, cutting through plane S(+ 2) ∞(− 3). Shadow on S.
732. Cylinder 1-in. diameter, surmounted by —— (square, round, or hexagonal) plinth, base diagonal (or diameter) 2 in., $\frac{1}{2}$ in. thick.
733. Cylinder and plinth, Ex. 732. Shadow on H, V, and T(5 + 2) 2(5 − 2).

734. Cylinder and plinth, Ex. 732, cutting through S(+ 2) ∞(− 3). Shadow on S.

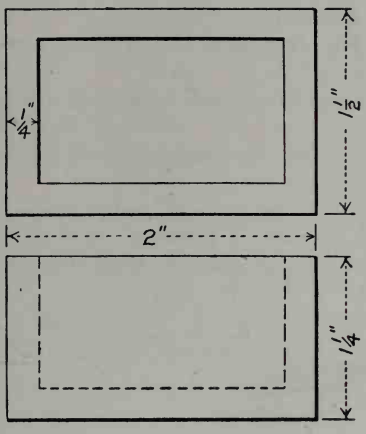


Fig. 110.

735. Cross, Fig. 109, base center at A(2, 0 − 1).

736. Twist the cross, Ex. 735, ——° (10°, 20°, 30°, 45°, 60°, 75°).

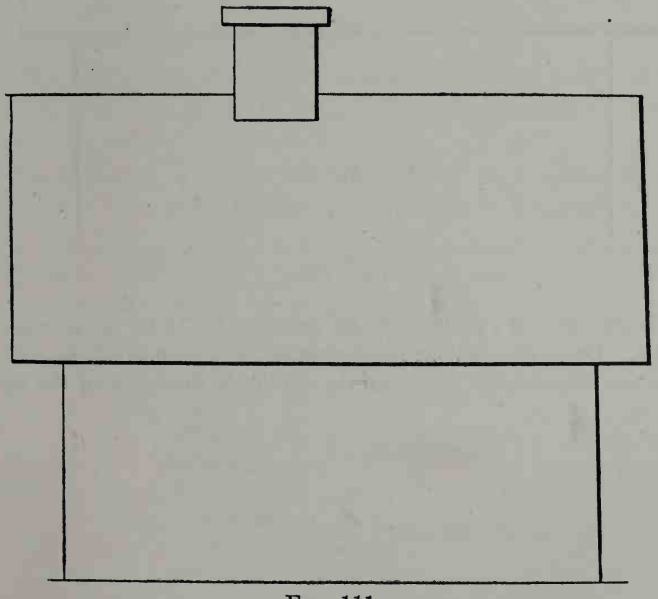


Fig. 111.

737. Cross, Ex. 736. center at B(2, 0 − 2). Shadow on *H*, *V*, and T(5 + 2) 2(5 − 2).

738. Place the cross, Ex. 736, on a stepped pyramid made up of four ¼ in. thick plinths, respectively 1¾ in., 1½ in., 1¼ in., 1 in. square.

739. Draw the box, Fig. 110, 30° from V. Shades and shadows, inside and outside, and on H and V.

740. Draw the box, Fig. 110, on S(1 + 1) 3(3 − 3). Shadows on H, V and S.

741. Draw the box, Fig. 110, on end, open front. Shadows on H, V, and itself.

742. Twist the box, Fig. 741, —— ° (10°, 20°, 30°). Shadows on H, V, and itself.

743. Side elevation of a house with 45° gable roof. Draw plan and end view. Required shadow of chimney on roof, roof edges on walls, and entire house on the ground. Dimensions and scale whatever convenient.

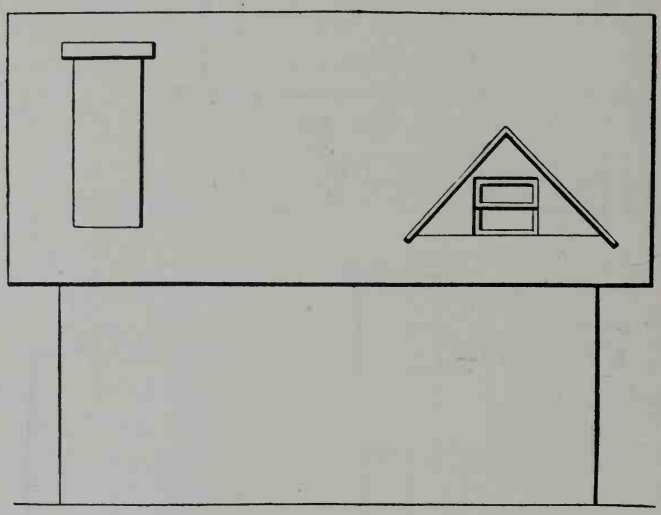


Fig. 112.

744. Side elevation of a house with 30° gable roof. Draw the plan and end view. Draw the shadow of the chimney and dormer on the roof, also the roof edges on the walls, and the shadow of the house on the ground.

CHAPTER V

CURVED LINES

133. Definitions.

A **Line** is the path of a point.

A **Straight Line** is a line in which the point moves continually in one direction.

A **Curved Line** is a line in which the point moves in a direction that is being changed constantly in obedience to some law.

A line is regarded as having one dimension only—length, without breadth or thickness. It is considered as being made up of infinitely small points, separated by infinitely small spaces. The line may, therefore, be regarded as being generated by an **immaterial** point, occupying **consecutive positions.** Two of these consecutive points determine a straight line, of infinitesimal length, but of a **determinate direction.** Such a line is called an **Element;** that is, it is one of the innumerable infinitely small straight lines that compose every line, straight or curved.

134. Classification of Lines.

Lines are **Straight** or **Curved.**

Curved Lines may be **Plane Curves,** also called **Lines of Single Curvature,** or they may be **Space Curves,** also called **Lines of Double Curvature.**

A Plane Curve is a line moving always in a plane, changing its direction in accordance with some law. Examples, circles, ellipses, etc.

A Space Curve is a curved line, no four consecutive points of which are contained in any plane. Example, the helix.

Table of Lines

Straight.	Only one kind.	
Plane Curves	Circle, Ellipse, Hyperbola, Parabola	Conic Sections.
	Involutes, Cycloidal Curves	Gearing Curves.
	Spirals, etc.	
Space Curves	Helix	
	Most Intersections of Curved Surfaces	
	Spherical Epicycloid, Spherical Hypocycloid.	

135. Projections of Curves.

A curve is projected on H, V and P in the same manner as a straight line; that is, all the points in the curve are projected on the plane of projection in lines perpendicular to the plane, and the sum of these projections of points makes up the projection of the line. As these projectors are all parallel, the surface which they compose is called the **Projecting Cylinder** of the curve. (Compare with the projecting plane of a straight line.)

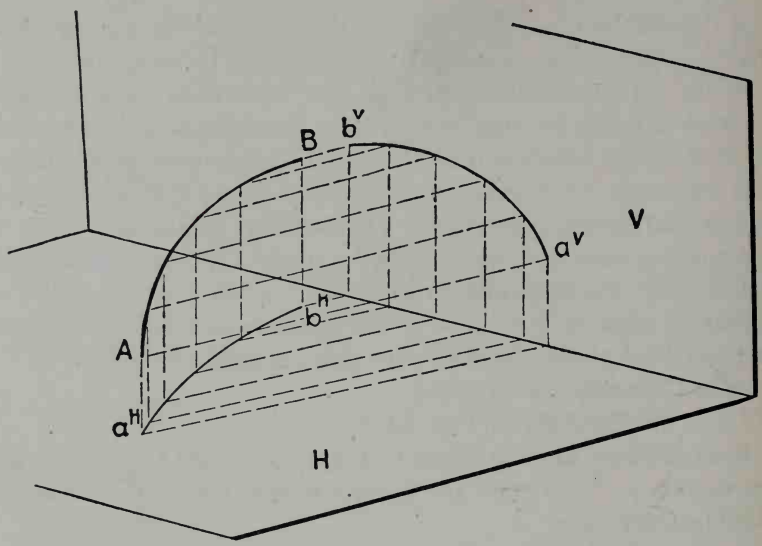


FIG. 113.—Projections of a curve.

Usually a small number (six or eight) points in space will very accurately determine a curve. Its projections are drawn by running a smooth line through the projections of the points in order, on both H and V, and on P, if necessary. (See Fig. 113.)

Note.—Plane curves, in certain positions, will project on one plane as a straight line, and the projecting cylinder is in that case a projecting plane. However, most curves do not lie so conveniently, and space curves **never** project as a straight line. Any curve, that is parallel or oblique to the plane of projection, will project as a curve on that plane. Any plane curve, lying in a plane perpendicular to the plane of projection, will project as a straight line on that plane.

136. Plane Curves Lying in a Profile Plane.

No curve, lying in a profile plane, can be shown in its true character by the H- and V-projections, because they will both be

straight lines. The P-projection will show not only the true character of the curve, but its true dimensions. The H- and V-projections of a number of points in a curve will **determine** it, but will not **show** it. (See Fig. 114.)

In a similar way, any curve, lying in a plane parallel to H, is projected in its true size on H, and as a straight line on V.

If the curve lie in a plane parallel to V, its V-projection gives its true size, and its H-projection will be a straight line.

If a curve lie in a plane perpendicular to H and oblique to V, its H-projection will be a straight line, and the V-projection will be curved, and will show the character of the curve. The curve, however, may be entirely changed by this foreshortening; *e.g.*, the projection of an oblique circle is an ellipse.

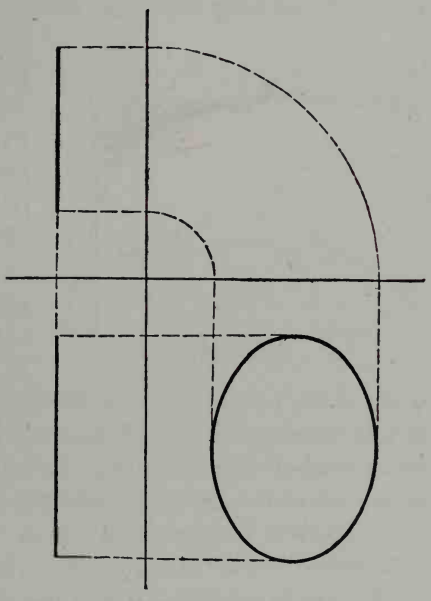


FIG. 114.—Curve in a profile plane.

The H- and V-piercing points of a curve are found by the same rule that obtains for straight lines; that is, where the V-projection crosses GL, project to the H-projection to find the H-piercing point, etc.

137. Tangents and Normals to Curves.

Definition.—A **tangent** to a curve is a line containing two consecutive points of the curve.

The tangent may be straight or curved, but, when curved, it is always especially designated. When spoken of as a tangent, the understanding is that it is a **straight line tangent.**

The tangent (from its definition) must lie in the plane of the curve at the point of tangency. Also (from its definition) it must have the same direction as the curve at the point of tangency. It is especially important to remember that the tangent must lie in the plane of the curve, as many errors occur from disregarding this fact.

Note.—The apparent discrepancy between the foregoing definition of the tangent and that given in most Plane Geometries amounts to nothing. They define a tangent as a line touching a curve at one point. Their definition answers sufficiently for beginners in Geometry, and for a section of the subject that confines its operations to a single plane, but the student can easily see how such a definition would be utterly inadequate in Space Geometry.

As the tangent is a short straight line in the curve (two consecutive points), the curve may be considered as being composed of an infinite number of infinitesimal tangents, intersecting consecutively. Fig. 115 shows a series of straight lines, A, B, C, etc., intersecting at short finite intervals. They compose so nearly a curve that it is easy to imagine that minute divisions would compose an actual curve. From the figure it can easily be seen how the tangent at any point determines the direction of the curve at that point.

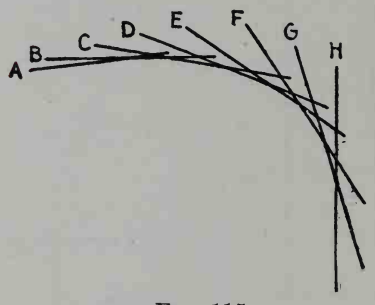


Fig. 115.

138. Projections of the Tangent.

The projections of the tangent to a curve will be tangent to the projections of the curve on the respective planes. The converse is not necessarily true, because a line lying in a different plane from that of the curve may have its projection tangent to that of the curve on one plane, **but not on any other.**

To Draw a Tangent to a Curve.

There are exact and approximate methods. In the case of the circle, the tangent is always perpendicular to the radius at the point of contact. In the case of other conic sections, the exact method will be taken up in connection with the study of those curves. For ordinary drafting work a sufficiently accurate tangent may be drawn by placing a straight edge against the curve, and laying off the tangent with the eye. A simple approximation, that can be used in nearly all cases of curves and is exact for the circle, is this: Let it be required to draw the tangent at A (Fig. 116) on the curve shown. Set the dividers at some small arbitrary span, and lay off B and C on

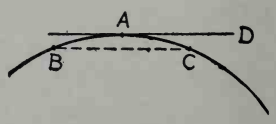


Fig. 116.

the curve at equal distances from A. Connect B and C with a straight line. Through A draw AD (the tangent) parallel to BC.

This method will not do for curves changing their radii rapidly. The eye will do close work in sharply curving lines. Numerous close approximations may be found in other text-books, and an especially good one is given in MacCord's "Elements of Descriptive Geometry," page 93, but its **practical** value is small.

139. Normals.—A normal to a curve at any point is a perpendicular to the tangent at the point of tangency. While any perpendicular is normal to the curve, it is customary to regard the normal as the perpendicular to the tangent in the plane of the curve.

In the case of a space curve, the normal may be any line perpendicular to the tangent at the point of tangency.

140. The Rectification of Curves.

To rectify a curve means to straighten it out; that is, to draw a tangent at any point equal in length to any given portion of the curve.

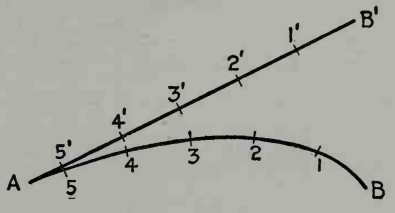


FIG. 117.—The rectification of a curved line.

First.—**To rectify a portion of any curve.**

Let the curve be AB, Fig. 117. Set the dividers conveniently small, so as to lay off an arc whose chord is approximately the length of the arc. Step off with the dividers the points 1, 2, 3, 4, etc., from B toward A, where the tangent is drawn. Lay off with a different pair of dividers the last step, 5A (which is not likely to be the same as the others), on the tangent A5′. Then, with the original dividers, step off from that point the points 4′, 3′, 2′, 1′ and B′. This rectification is fairly good, if care is taken.

Second.—**To rectify a small arc of a circle.**

If the arc be less than 60°, a convenient method is shown in Fig. 118. Let AB be the arc, and BD (of indefinite length) be the tangent. Extend the chord AB through B one-half its length to C. Using C as a center, and CA as a radius, strike the arc AD. BD will then equal (approximately) the arc BA.

Third.—**To rectify a larger arc.**

Divide the large arc into smaller ones, rectify each, and add them together.

Fourth.—**To rectify a quadrant of a circle.**

Let AB (Fig. 119) be the given quadrant. Draw the tangent at B, and from A draw a line AC, which makes 60° with CB. BC is then a very close approximation to AB. This is undoubtedly the closest approximation for such an arc. Let the

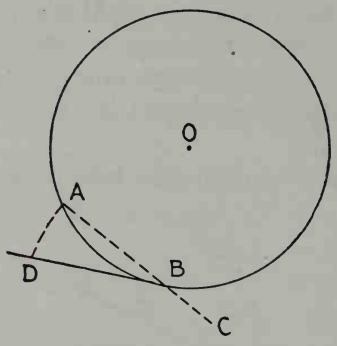


FIG. 118.—The rectification of a circular arc.

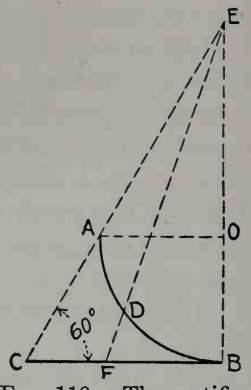


FIG. 119.—The rectification of a quadrant.

student prove by trigonometry that it is numerically equal to $\frac{\pi}{2}$R. Although there is no geometrical proof for this, your calculation will show that it is closer than the accuracy of your instruments.

A convenient method of rectifying a small arc comes with this method. To rectify BD, Fig. 119, draw the line ED through to the tangent at F. The line BF equals BD.

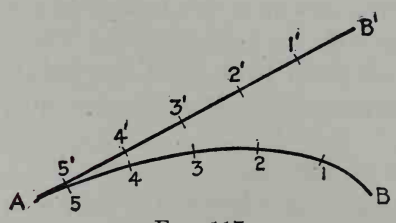


FIG. 117.

Fifth.—**To rectify an arc of a circle when its angle is known.**

Here the numerical method is best. The arc AB (angle = θ) $= 2\pi R\frac{\theta^\circ}{360^\circ} = .01745R\theta^\circ$.

141. To Lay Off Any Length of Line, Straight or Curved, on Any Curve.

Regard the line AB′ (Fig. 117) as a curved line. Lay off AB on the curve in exactly the same manner that it was laid off on the straight line.

This is often a necessary operation in practical drafting; example, in drawing cycloids for gear outlines.

142. Conic Sections.

Definitions.

Circle.—A circle is a plane curve generated by a point moving so as to be at a constant distance from another point.

Ellipse.—An ellipse is a plane curve generated by a point moving so that the **sum** of its distances from two fixed points in its plane is constant.

Hyperbola.—A hyperbola is a plane curve generated by a point moving so that the **difference** between its distances from two fixed points in its plane is constant.

Parabola.—A parabola is a plane curve generated by a point moving so that its distances from a **line and a point** in the plane are always equal.

The four foregoing curves are called "Conic Sections" from the fact that they are the possible curves that can be cut from a right circular cone by a plane. A plane can also cut a straight line from a cone. The necessary conditions for cutting these lines from a cone are:

Straight Line.—Plane through the apex.

Circle.—Plane perpendicular to the axis.

Ellipse.—Plane oblique to the axis, cutting all the elements on one side of the apex.

Hyperbola.—Plane, parallel to the axis, or a plane oblique to the axis at a smaller angle than that made by the elements of the cone.

Parabola.—Plane parallel to one element of the cone.

143. Gearing Curves.

Cycloid.—A cycloid is a plane curve generated by a point on the circumference of a circle, as the circle rolls on a straight line.

Epicycloid.—An epicycloid is a plane curve generated by a point on the circumference of a circle, as it rolls on another circle.

Hypocycloid.—A hypocycloid is a plane curve generated by a point on the circumference of a circle as it rolls on the inside of another circle.

Involute.—An involute is a plane curve generated by a point on a tangent, as the tangent rolls on a plane curve or polygon. The involute of a circle is the only one of any considerable importance, and is the curve used in gearing.

The foregoing are called "gearing curves," because they are mainly used as the outlines of teeth in gear wheels. The outlines of bevel gears are (theoretically) spherical cycloids, but their consideration is negligible in this work, and practically so in actual drafting work.

SPACE CURVES

144. Helix.—A helix is a curve generated by a point moving about a straight line at a constant distance, and in the direction of the line at a constant velocity.

Note.—This line is the line of the screw thread, twist drill, screw conveyor, worm gear, coiled spring, and many other important practical devices. It is a curve on the surface of a cylinder.

There is also a **conical helix** that sometimes appears in practical work. It is usually drawn so that it makes one complete revolution about the axis in traveling from the base to the apex, ascending the surface at a uniform rate of motion. The plan of the conical helix is the Spiral of Archimedes. By drawing the plan first and projecting the points of the curve as it intersects the various elements of the cone, the elevation is easily drawn. It is rather commonly used in sheet metal work and in spiral springs.

There is no need of defining or considering any other of the space curves.

GENERAL DEFINITIONS

145. Axis.—The line about which a point revolves, as in the case of the helix. It is also a center line for some curves, as the ellipse, parabola, and hyperbola.

Minor Axis and Major Axis.—In the ellipse there are two dimensions, sometimes called the long and short diameters, but more generally the major and minor axes. They are respectively the longest and shortest possible straight lines running through the center from one point in the curve to another. The **constant sum,** spoken of in the definition, is equal to the length of the major axis.

Focus.—The two fixed points, mentioned in the definitions of the ellipse and hyperbola, and the single one, mentioned in the definition of the parabola.

Vertex.—The nearest point in the curve to the focus. It has the smallest radius of curvature of any point in the curve, and

might be described as the center of symmetry of the curve. The circle has no vertex.

The **constant difference,** mentioned in the definition of the hyperbola, is the distance between the two vertices of the curve.

Directrix.—The straight line mentioned in the definitions of the parabola and the cycloid.

Generating Circle.—The rolling circles in the definitions of the cycloid, epicycloid and hypocycloid.

Pitch Circle, or Directing Circle.—The circle on which the generating circle rolls.

Evolute.—The curve on which the tangent generating the involute rolls.

Note.—The tangent to the evolute is normal to the involute.

Pitch.—The pitch of a helix is the distance that it travels in the direction of the axis in making one complete revolution.

PROBLEMS RELATING TO CURVED LINES

146. Problem 47.—To draw an Ellipse, having given its foci and "constant sum."

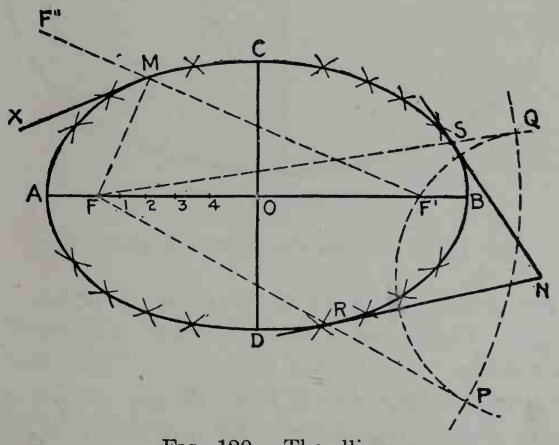


FIG. 120.—The ellipse.

Construction.—1. Draw the line AB (Fig. 120) and from a center, O, lay off F and F′ at equal distances from O, making FF′ equal to the given distance between the foci.

2. On the same line lay off A and B at equal distances from O, making AB equal to the given constant sum.

3. Spread the compasses equal to AO, and strike arcs from

F and F′ as centers. The intersections, C and D, of these arcs are the ends of the minor axis.

4. Set the compasses at arbitrary radii, A1, A2, etc., and strike arcs with them from F and F′ as centers.

5. Set the compasses at complementary radii, B1, B2, etc., and strike arcs from F and F′ as centers.

6. Through A, B, C, D, and the intersections of complementary radii, draw a smooth line. This will be the required ellipse.

Proof.—The sum of the distances of any of the points, thus found, to the foci is always equal to AB.

147. Approximate and Mechanical Methods of Drawing the Ellipse.

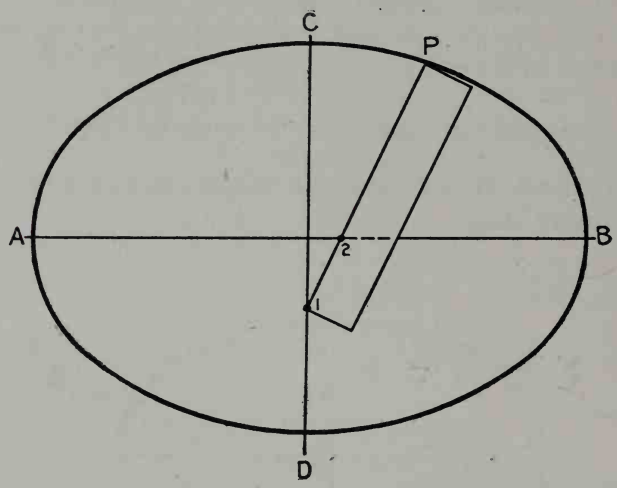


Fig. 121.—Drawing the ellipse with a trammel.

There are numerous ways of drawing an ellipse, exact and approximate. These are found in most text-books on Mechanical Drawing. The best of these is the "Trammel Method."

The Trammel Method.—1. Lay off the Major and Minor Axes, of any desired dimensions, AB and CD (Fig. 121).

2. Take a card, or any straight edge, and lay off P1 and P2, respectively equal to the semi-major and semi-minor axes, BO and CO.

3. Place the straight edge so that 2 lies on AB and 1 lies on CD, and mark the point P.

4. Do this in as many positions as desired, marking a number of points on the curve.

5. Draw a smooth curve through the points thus obtained.

Uses for the Ellipse.—In ornamental work the ellipse is extensively used, in its entirety and in portions. The ellipse is the projection of any circle in an oblique plane, and its major axis is always equal to the diameter of the circle. In machine design, the ellipse is used in certain "quick return" motions. In sheet metal work and boiler-making it is often necessary to cut elliptical holes in the plates. In designing "focusing re-reflectors" the ellipse is necessary, because all rays from one focus of an ellipse reflect from the curve to the other focus.

Limits.—When the distance between foci is zero, the ellipse is a **circle.** When it is equal to the "constant sum," the ellipse is a **straight line.**

148. Problem 48.—To draw the tangent to an Ellipse at a given point on the curve.

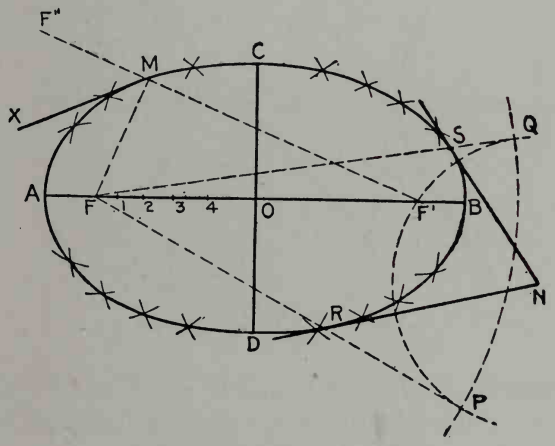


FIG. 120.—The ellipse.

Construction.—Let M (Fig. 120) be the point.

1. Draw MF and MF′, the focal lines.
2. Extend MF′ beyond M to F″.
3. Bisect the exterior focal angle FMF″.

The bisector MX will be the tangent at M.

149. Problem 49.—To draw the tangent to an Ellipse from a point outside.

Let N (Fig. 120) be the point.

1. Draw a circle with N as center and NF′ as radius.

2. Intersect the circle in points P and Q by an arc whose radius is equal to AB, and whose center is at F.

3. Connect either P or Q with F.

4. Where PF or QF intersect the ellipse at R or S will be the points to which tangents can be drawn from N.

If the axes of an ellipse are given, the foci may be determined by striking an arc from C or D as center, with a radius equal to AO. Where this arc intersects AB will be the foci.

150. Problem 50.—To draw a Hyperbola, having given its foci and the "constant difference."

Let F and F′ (Fig. 122) be the foci, and the "constant difference" be equal to the distance VV′.

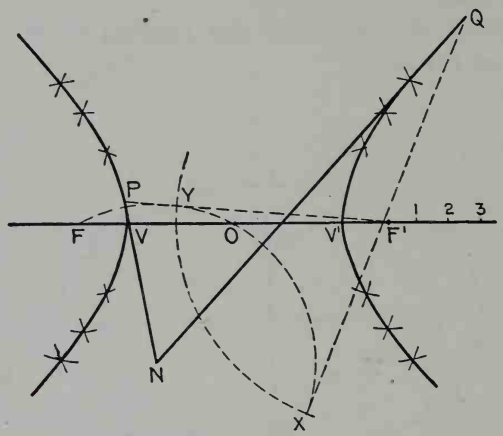


FIG. 122.—The hyperbola.

Construction.—1. Lay off F and F′ and V and V′ on the same line, at their proper distances, all symmetrically placed about a center, O. (V and V′ will be the vertices.)

2. Spread the compass at any arbitrary spans, as V1, V2, etc., and strike arcs with these radii with F and F′ as centers.

3. Using the complementary radii, V′1, V′2, etc., strike arcs with F and F′ as centers, intersecting those already drawn.

4. Draw a smooth curve through the proper intersections and V and V′. This will be the required hyperbola.

Note.—The hyperbola is in reality two curves, each having a vertex, and each unlimited in extent. The hyperbola varies in accordance with the ratio between FF′ and VV′. The most important use of the hyperbola is that it is the generating line of the pitch surface of "skew" gears.

151. Problem 51.—To draw the tangent to a Hyperbola from any point of the curve.

Construction.—Let P be the point.

1. Draw PF and PF′, the focal lines.
2. Bisect the angle FPF′.

The bisector will be the tangent.

Note.—The construction is not shown in the figure. Let the student make it for himself.

152. Problem 52.—To draw a tangent to a Hyperbola from any point outside the curve.

Let N (Fig. 122) be the given point.

Construction.—1. With N as a center, draw a circle passing through F′, the nearest focus.

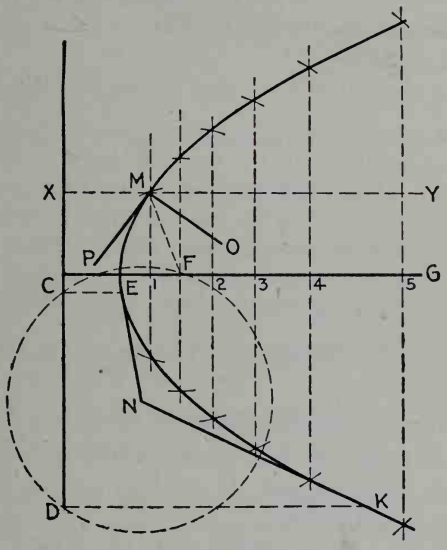


FIG. 123.—The parabola.

2. With F, the farther focus, as the center, and VV′ as the length of the radius, strike an arc, cutting the first circle in the points X and Y.

3. Draw XF and YF, extending them until they intersect the hyperbola in the points P and Q.

4. NP and NQ will be the required tangents.

153. Problem 53.—To draw a Parabola, having given the focus and the directrix.

Construction.—Let XD (Fig. 123) be the directrix and F the focus.

1. Draw a line CG through F perpendicular to XD. This is the axis of the curve.

2. Bisect the distance from F to XD at E. E will be the vertex.

3. Draw a number of lines at points, 1, 2, 3, etc., parallel to XD, at arbitrary distances.

4. Locate two points on each of the lines, equally distant from F and XD.

5. Draw a smooth line through the points thus found, and it will be the required parabola.

154. Problem 54.—To draw the Tangent to a Parabola at any point on the curve.

Construction.—Let M (Fig. 123) be the given point.

1. Draw MX, perpendicular to XD.

2. Draw the focal line MF.

3. Bisect the angle FMX. The bisector is the tangent.

Note.—Extend the line XM in the direction MY, and draw the normal MO. Notice that MY is parallel to the axis, and that MO is the bisector of the angle FMY, making OMF equal to OMY. This fact is taken advantage of in designing the reflector for the searchlight. M is a random point on the parabola, and shows that any ray from F striking the parabola at M (any point) is reflected in a line parallel to the axis, as MY. This concentrates all the reflected rays into a cylinder of light, preventing diffusion.

155. Problem 55.—To draw a Tangent to a Parabola from a point outside.

Construction.—Let N (Fig. 123) be the given point.

1. With N as a center, draw a circle through F.

2. From its intersections, C and D, with DX, draw the perpendiculars, CE and DK.

3. E and K, the intersections with the curve, are the points to which tangents can be drawn.

156. Problem 56.—To draw a Cycloid, having given the directrix and generating circle.

Construction.—Let AB be the directrix and O be the center of the generating circle, which is to be drawn tangent to the directrix, of a radius R.

1. Lay off on AB, a distance equal to $2\pi R$, and divide the circle and AB into any convenient number of equal parts, 12, 16 or 24.

2. Draw the path of the center of the circle, parallel to AB, at a distance equal to R.

3. Erect perpendiculars to the line of centers at each of the divisions of AB. The intersections will be successive positions of the center of the generating circle.

4. From the divisions, 1, 2, 3, etc., of the generating circle, run projectors parallel to AB.

5. From each of the successive positions of the center, 1′, 2′, 3′, etc., describe arcs intersecting the corresponding projectors.

6. Through these intersections draw a smooth curve, which will be the required Cycloid.

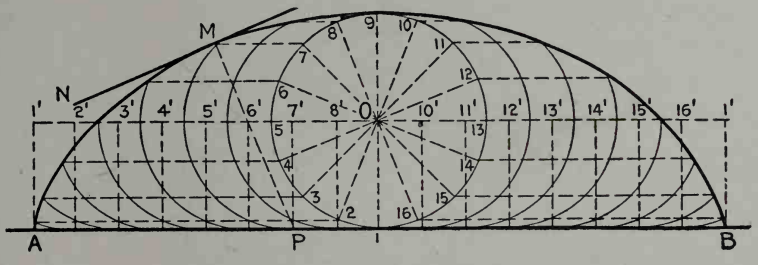


FIG. 124.—The cycloid.

157. Problem 57.—To draw the tangent to the Cycloid at any point on the curve.

Construction.—Let M (Fig. 124) be the given point, and P be the point of tangency with AB of the generating circle, when M is its intersection with the curve.

1. Draw MP. This line is the normal at M.
2. Draw the tangent perpendicular to MP.

Note.—It is owing to the fact that the normal to the cycloid at any point passes through the point P, where the generating circle and directrix are tangent, that the cycloid is a correct curve for use in gear outlines. It is this property that makes it possible to maintain a constant angular velocity ratio in gears, as will be seen when the study of Kinematics, or "Mechanism," is taken up.

158. Problem 58.—To draw an Epicycloid and a Hypocycloid. The only difference between drawing these curves and the cycloid, is that the directrix now is a circle instead of a straight line. Fig. 125 shows both curves. AB is an arc of the pitch circle, as the directrix is usually called in Machine Design, and the center of the generating circle of the Epicycloid is at C, and its path is the arc CD (center at O). The center of the Hypocycloidal circle is at E, and its path is the arc EF, center at O.

Construction.—1. Divide the generating circles into any convenient number of equal parts.

2. Step off these arcs on AB (Art. 141).

3. From the points 1, 2, 3, etc., draw arcs with their centers at O.

4. Through the divisions on AB draw radial lines intersecting CD and EF in the points occupied by the centers as the circles roll on AB.

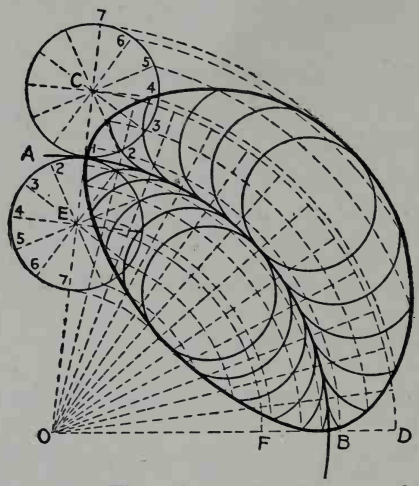


FIG. 125.—The epicycloid and hypocycloid.

5. From these center points strike arcs with radii equal to those of the generating circles, intersecting the arcs of operation (3).

6. Through the respective intersections draw the curves.

The tangents are drawn precisely as the tangents to the cycloid are drawn; that is, the normal at any point passes through the point of tangency of the generating circle and the pitch circle.

Note.—The hypocycloid generated by a circle half the size of the pitch circle is a **straight line** and is called a **radial hypocycloid.**

159. Problem 59.—To draw an Involute to a Circle.

Let AB (Fig. 126) be the directing circle, or **Evolute,** with its center at O.

Construction.—1. Divide any arc of AB, as, for example, the semicircle, into any number of equal parts.

2. At the points 1, 2, 3, etc., draw tangents in the direction of the origin, A.

3. Rectify each arc from the origin and lay off the distance on the tangent.

4. Through the extremities of the tangents draw the curve.

Note 1.—This curve is that described by the end of a taut cord as it is unwound from a post. It is not a closed curve, but a spiral that keeps growing in expanse with every revolution.

Note 2.—An involute can be described in the same way, using any curve or polygon as the evolute.

Note 3.—The **Volute**, used in architecture, is a spiral, the involute of an enlarging square.

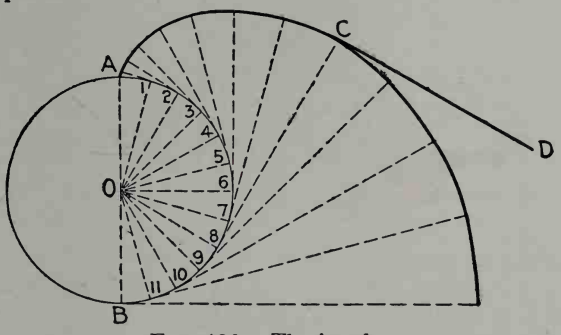


FIG. 126.—The involute.

160. Problem 60.—To draw the Tangent to an Involute at any point on the curve.

The tangents to the evolute are all normals to the involute (Art. 143), hence all that is necessary is to draw the perpendicular to the normal at the given point. Thus at the point C (Fig. 126) draw CD perpendicular to C8, the normal.

Note.—The fact that the normal to the involute is tangent to the evolute makes the curve useful in gearing in the same way as in the case of cycloidal curves.

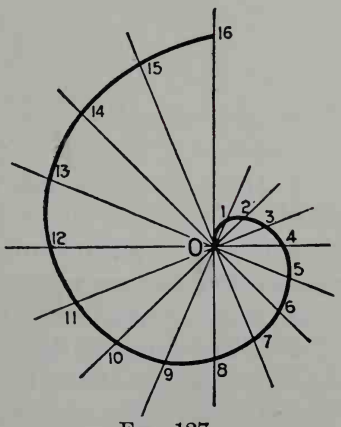


FIG. 127.

161. Problem 61.—To draw a Spiral of Archimedes.

Construction.—(See Fig. 127).

1. Draw a series of lines, radiating from O at equal angles.

2. Lay off on line 1, one unit, on line 2, two units, on line 3, three units, etc.

3. Draw the curve through these points.

162. Problem 62.—To draw the projections of a Helix, having given its Pitch and Diameter.

Let OP (Fig. 128) be the axis, O^v12 the pitch, and D the diameter.

Construction.—1. Draw the circle and divide it into any number of equal parts.

2. Divide the pitch into the same number of equal parts.

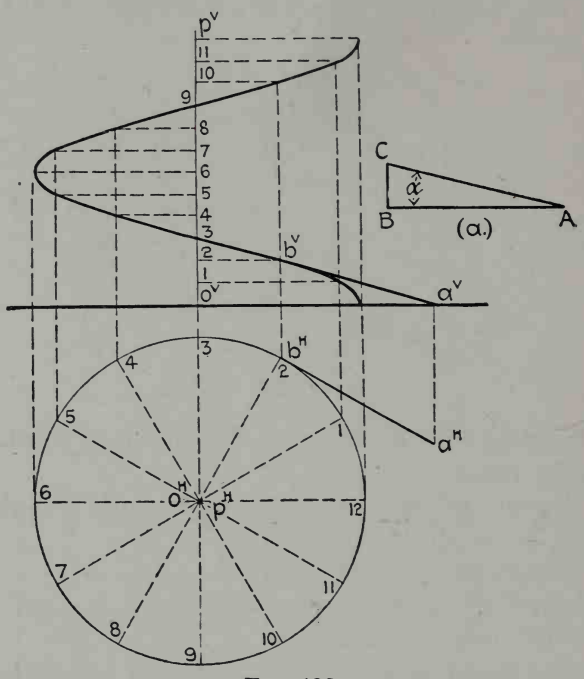


FIG. 128.

3. Draw perpendiculars to the axis from each of the divisions made in operation (2).

4. Draw projectors from each of the divisions of the circle, intersecting the corresponding perpendiculars from the axis.

5. Draw the curve through these intersections.

163. The Angle of the Helix.

From the definition (Art. 144), we see that the generating point on the helix ascends a given distance in traversing a given arc, and that the rate of ascent is constant. This means that any element of the curve makes a certain angle with the horizontal (the axis of the helix being taken in all these problems

as vertical), and that all the elements in any one helix make the **same angle** with the horizontal. Thus, in Fig. 128(*a*), if AB is the length of an elemental arc of the circle, and BC is the length of an elemental division of the pitch, the hypothenuse of the right triangle ABC will yield the inclination and the length of an element of the helix. Therefore, if a right triangle, having its base angle equal to α, be wrapped about a cylinder, the hypothenuse will become the helix. From this it is evident that

$$\tan \alpha = \frac{\text{pitch}}{\pi . \text{ diam.}}$$

164. The Tangent to the Helix.

If the right triangle, just mentioned, be unwrapped from the cylinder, its hypothenuse becomes the tangent to the helix at the point where the unwrapping ceases.

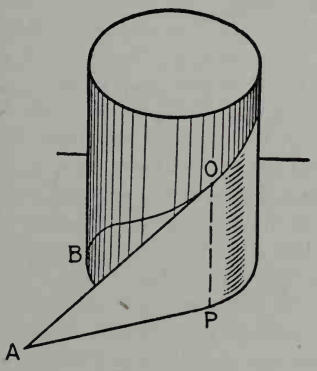


FIG. 129.—The generation of a helix.

This is so because the tangent must make the same angle with the horizontal that the helix does, because it contains two consecutive points (**one element**) of the curve.

Therefore, the tangent becomes the helix if it is rolled on the cylinder, and the length of the helix from its origin to the point of tangency is equal to the length of the tangent from the plane of the origin to the point of tangency. Referring to the picture, Fig. 129, will show that AO = BO, and further that AP = BP. AP is the *H*-projection of the tangent, and BP is the *H*-projection of the helix. From this the rule is derived: **The H-projection of the Helix is of the same length as the H-projection of the Tangent from the plane of the origin to the point of tangency.**

165. Problem 63.—To draw the Tangent to the Helix at any point on the curve.

Construction.—Let B (Fig. 128) be the given point. 1. Draw the tangent to the circle at b^H.

2. Rectify the arc b^H12, and lay it off on the tangent a^Hb^H.

3. Project a^H to a^V on *GL*.

4. a^Vb^V will be the *V*-projection, and a^Hb^H the *H*-projection of the tangent required.

166. EXERCISES IN CURVED LINES

Note.—No ground line is necessary in drawing any of the curves but the helix.

745. Draw an ellipse, AB = 5 in., FF′ = 3 in. Draw a tangent at a point on the curve 2 in. from F. Draw two tangents from a point 4 in. from F and 2 in. from F′.

746. Draw an ellipse, AB = 4½ in., and CD = 3 in. Locate F and F′, and draw a tangent at a point on the curve 1 in. from A. Draw two tangents from E, which is 3 in. from F′ and 2 in. from F.

747. A and B are two tangent ellipses of equal size. The foci of A are C(1, 0 − 2) D(3½, 0 − 2), and the more distant focus of B is at E (5, 0 + 1). Draw the two ellipses.

748. Draw a hyperbola. FF′ = 3 in. and the "constant difference" = 2 in. Draw the tangent at a point on the curve 2 in. from F. Draw a tangent from a point 2 in. from F′ and 2½ in. from F.

749. Draw a parabola with the distance between focus and directrix 1 in. Draw tangents from assumed points on and outside the curve.

750. Draw a parabola whose distance from focus to vertex is ¾ in. Draw tangents from assumed points on and outside the curve.

751. Draw a cycloid with a 1½-in. generating circle. Draw a tangent at the point on the curve at $\frac{5}{16}$ of the revolution.

752. Draw an epicycloid and a hypocycloid on a pitch circle of 6-in. radius, with 1¼-in. generating circles. Draw the normals to each curve at $\frac{5}{16}$ of a revolution from the origin.

753. Draw an involute of a 1-in. circle.

754. Draw an involute of a ½-in. square.

755. Draw an involute of a hexagon of ½-in. sides.

756. Draw an involute of a ¾-in. equilateral triangle.

757. Draw a helix of 2-in. diameter, 3-in. pitch. Draw its tangent at 60° from the origin. Also one at 90°.

758. Design a focusing reflector of diameter —— in. (4, 5, 6, 7), in which the rays from a light, placed —— in. (2, 3, 4, 5) from the vertex are concentrated at a point —— in. (8, 9, 10, 12, 14, 16) from the light.

CHAPTER VI

SINGLE-CURVED SURFACES

167. Definitions.

A line, straight or curved, moving in accordance with some law, generates a **Surface.** If the generating line be straight, and moves along two parallel or intersecting straight lines, the surface generated will be a **Plane.** There is but one kind of plane, all other kinds of surfaces are curved.

Curved Surfaces are divided into three classes: **Single-Curved, Double-Curved and Warped** surfaces.

A **Single-Curved Surface** is generated by a straight line following a curved director in such a way that any two consecutive positions of the line shall lie in a plane.

A **Warped Surface** is generated by a straight line moving so that no two of its consecutive positions shall be in the same plane.

Note.—Single-curved and warped surfaces are sometimes called **Ruled** surfaces, because they are made up of straight lines. Through every point of a ruled surface at least one straight line can be drawn in the surface.

A **Double-Curved Surface** is generated by a curve moving along a curved director, so that there are no straight lines in it.

Generatrix.—The line generating a surface.

Element.—Any position of the generatrix.

Consecutive Elements.—Two successive positions of the generatrix, having no assignable distance between them.

A **Surface of Revolution** is a surface generated by the revolution of any line, straight or curved, about a straight line axis. Surfaces of revolution may be single-curved, double-curved, or warped.

168. Classification of Surfaces.

Single-Curved Surfaces are of three varieties:

Cones, all the elements intersect in a point.

Cylinders, all the elements are parallel.

Convolutes, consecutive elements intersect **two and two,** no three elements having a common point.

Double-Curved Surfaces are mostly surfaces of revolution, spheres, ellipsoids, paraboloids, tori, and combination surfaces.

They are sometimes convex, sometimes concave, and sometimes both. The annular torus has both in continuous surface.

Warped Surfaces are numerous in their variety, but the most important are the helicoid and hyperboloid of revolution of one nappe.

169. The following table will give a convenient summary of the surfaces.

Single-Curved	Cone—all elements intersecting	Ruled Surfaces.
	Cylinder—all elements parallel	
	Convolute—consecutive elements intersecting.	
Warped	Helicoid	
	Hyperboloid of Revolution	
	Hyperbolic Paraboloid	
	Conoid	
	Cylindroid	
	Warped Cone	
	Cow's Horn.	
Double-Curved	Sphere	
	Oblate Spheroid	
	Prolate Spheroid	
	Paraboloid (both varieties)	
	Hyperboloid of Revolution of Two Nappes	
	Torus.	
Surfaces of Revolution	Right Circular Cone, Right Circular Cylinder	
	Hyperboloid of Revolution of One Nappe	
	Sphere, Spheroids, Ovoids, etc.	

170. Tangent Planes and Developments.

The most important problems in surfaces are to be found in drawing tangent planes to surfaces, the intersections of surfaces, and developing the surfaces.

A **Tangent Plane** to a surface at any point is a plane composed of all the tangent lines to all the lines of a surface passing through that point. As two intersecting lines determine a plane, the general rule for tangent planes is this: **Any two tangent lines at any point on any surface determine the tangent plane at that point.**

Any plane, passed through the point of tangency so as to intersect a surface and its tangent plane, **will cut from them a line and its tangent.**

In ruled surfaces, one and sometimes two straight lines of the surface can be drawn through each point in the surface, therefore this line (element, as it always is) can be used for one

of the tangents in the foregoing rule. This is true, because a straight line is its own tangent. On this account, the rule will read: **"In ruled surfaces, one tangent and the element at any point will determine the tangent plane at that point."**

Normal.—The normal line at any point of a curved surface is the perpendicular to the tangent plane at that point. The normal plane is any plane containing the normal line.

Development.—By **Development** is meant the rolling out of a surface into a plane figure, which can be rolled, or folded, back to make the original surface. Only single-curved surfaces are strictly developable, because they only have consecutive elements

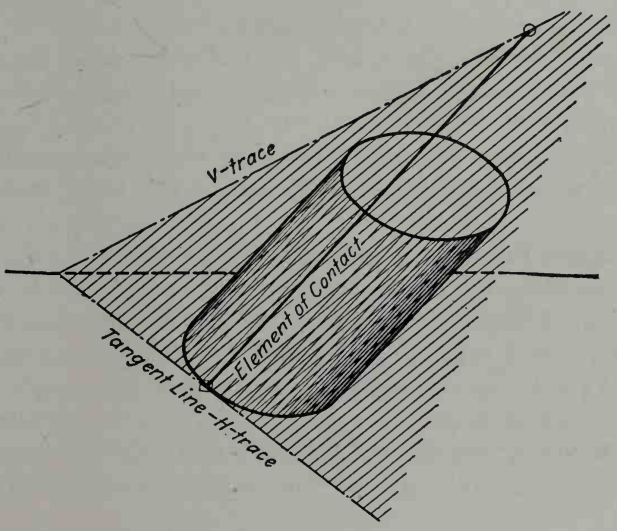


FIG. 130.—Tangent plane to a cylinder determined by two tangent lines.

lying in planes. **Approximate developments** can be made of other surfaces, as will appear when the surfaces are taken up separately.

THE CONE

171. Definitions.—A **Cone** is generated by a straight line moving so as to touch constantly a curved line directrix, while one point in the generatrix remains stationary.

Apex or **Vertex,** the stationary point on the generatrix.

Base, a limiting plane that cuts all the elements.

Axis, a straight line through the apex and the center of the base.

A **Frustum** is a portion of the surface between two parallel planes on the same side of the apex, which cut all the elements.

Discussion.—The surface of the cone that lies between the base and apex is usually all that is considered. Theoretically, however, the surface is unlimited, and extends beyond both limits to infinity. This theoretical extension creates two divisions of the surfaces, one on each side of the apex, called respectively the **Upper Nappe** and **Lower Nappe.** In practical work this condition almost never exists, and is usually ignored.

Varieties of Cones.—The different varieties of cones are distinguished by their bases and axes. The bases may be circular, elliptical, hyperbolic, etc., or combinations of curves, and the axes are either perpendicular (**right**) or oblique to the bases. Thus we designate a cone as a right circular cone, an oblique elliptical cone, etc. The base and apex, or the base and axis, are sufficient to determine the cone.

Limits.—A cone, whose apex is at infinity, becomes a **cylinder,** and one, whose apex is in the plane of the base, becomes a **plane.**

A Tangent Plane to a cone at any point on the surface is tangent all along the element through that point. This element is called the **Element of Contact.** As all elements pass through the apex, **every tangent plane must contain the apex.** This fact is of great importance in passing tangent planes. It means that **any line through the apex,** except lines inside the cone, **will lie in one or two tangent planes.** From this fact, and from Art. 155, we derive the special rule for passing tangent planes to cones.

Rule.—One tangent line and the apex are sufficient to determine a tangent plane to any cone.

Axiom.—If the base of a cone is in H, the H-trace of any tangent plane will be tangent to the base.

172. Development of Cones.—When the surface of a cone is rolled out, we find that there is one point common to all elements, the apex, and this point will remain a point in the development. From this point all the elements radiate in fan shape, making up an integral of all the minute triangles that form the surface of the cone. In the case of a right circular cone, the elements are all of equal length, and the base rolls out into an arc of a circle, whose radius is the length of the elements (sometimes called "**slant height**"). The length of the arc thus derived is

equal to the circumference of the base. All other cones have elements of varying length, and their bases will not roll out in circular paths. The development of all cones, except the right circular, is accomplished by **"triangulation;"** that is, dividing

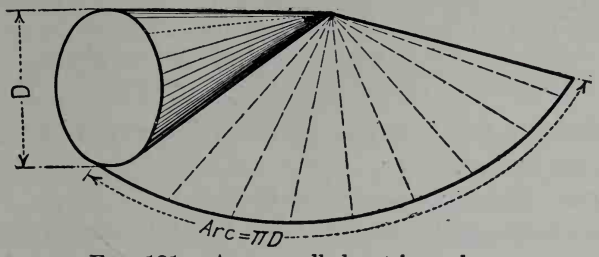


FIG. 131.—A cone rolled out in a plane.

the surface into small triangles, and laying them out flat in order and (approximately) true size. An exception to this may be found in an oblique elliptical cone that may have a circular right section. If such be found, the circular method is better than triangulation.

173. Problem 64.—To locate a point on the surface of a cone.

Construction.—Let the *V*-projection of a point (o^V in Fig. 132) be given.

1. Draw the element of the surface through the point.

2. Project o^V to o^H on the *H*-projection of the element.

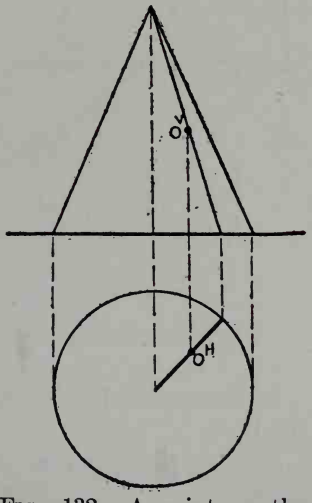


FIG. 132.—A point on the surface of a cone.

Note.—Two points are possible in this case, unless front or rear surface is specified.

174. Problem 65.—To obtain the line of intersection of a Plane and a Cone.

Construction.—Exactly the same as for the intersection of a plane and pyramid, taking a reasonable number of elements of the cone, instead of edges.

175. Problem 66.—To obtain the piercing points of a Cone by a given Line.

Analysis.—1. Pass a plane through the given line and the apex of the cone.

2. Obtain the elements of the cone cut by this plane.

3. Where these elements intersect the given line will be located its piercing points with the cone.

Let the student make the construction.

Note.—There are occasions when a plane intersecting a number of elements can be used to better advantage than a plane through the apex. Such a plane will cut a curve from the cone surface, and the piercing points will be at the intersections of the given line with the curve.

176. Problem 66.—To pass a plane Tangent to a Cone at a point on the surface.

Analysis.—Read the rule given in Art. 171.

Construction.—Refer to Fig. 133. Let O be the given point.

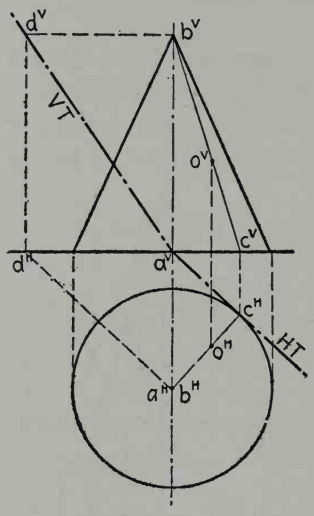


Fig. 133.

1. Draw the element through O. This is the **Element of Contact,** and one line of the plane.

2. Where the element of contact intersects the base, draw the tangent to the base.

This is the *H*-trace of the required plane, and the *V*-trace may be obtained in several ways known to the student.

177. Problem 67.—To pass a plane Tangent to a Cone through a Point outside the surface.

Analysis.—1. Draw a line through the apex and the given point. (See Art. 171.)

2. Find the piercing point of this line with the plane of the base.

3. From this point draw one of the two possible tangents to the base.

4. Pass the required plane through the tangent line and the apex.

Construction.—Let the student make it.

Note 1.—If the line through the given point and the apex is parallel to the base, the tangent line in the plane of the base will be parallel to said line, and may be drawn on either side of the base.

Note 2.—If the line through the apex is too nearly parallel to the plane of the base to obtain the piercing point within the limits of the drawing, a plane may be passed parallel to the base, nearer the apex, and a tangent

may be drawn to the new base, or a second cone similar to the first may be drawn with its apex in the line at another point, and the tangent plane drawn to both cones.

178. Problem 68.—To pass a plane Tangent to a Cone parallel to a given line.

Analysis.—1. Draw a line through the apex parallel to the given line.

2. Having this line, finish as in Problem 67.

Construction.—Let the student make it.

Impossible Conditions.—When the line through the apex parallel to the given line is inside the cone, the problem is impossible. In such a case the line will pierce the plane of the base inside the base. Otherwise the problem is possible.

179. Problem 69.—To pass a plane Tangent to a Cone, making a given angle with *H*.

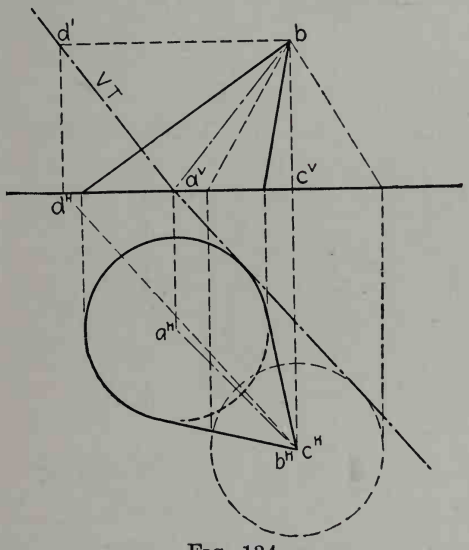


Fig. 134.

Limitations.—Only certain conditions will allow a solution of this problem.

1. If the cone be a right circular cone, with its base on *H*, there is only one angle possible, and that is the angle that its elements make with *H*.

2. If the base be in some plane not parallel to *H*, there are many angles at which tangent planes may be passed, though not all.

3. If the base be in H, and the axis oblique, different angles, but not all, may be assigned to the problem.

4. If the base be in H, and a different curve from the circle, even though the axis be perpendicular, there will be some variety to the angles that may be required.

Each problem will fix its own limitations.

Construction.—Let it be required to pass a plane tangent to the cone AB (Fig. 134) making 60° with H.

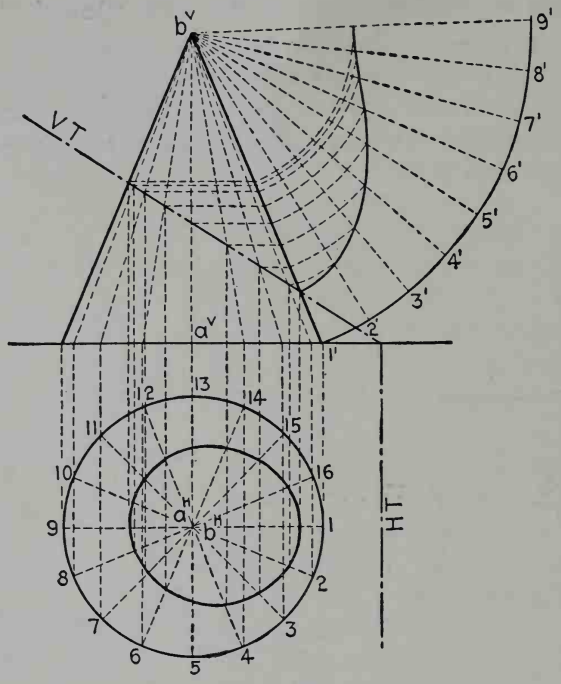


FIG. 135.

1. With B as its apex, draw the projections of a 60° cone, axis BC.

2. Draw the H-trace of T, tangent to the bases of the two cones. This may be drawn on either side of the cones.

3. Locate the V-trace of T by any lines through B, lying in T.

Note.—Under certain conditions four planes can be passed making the same angle with H, all tangent to the cone.

180. Problem 70.—To develop the surface of a right circular cone.

Analysis.—Read Art. 172.

Construction.—Let it be required to develop the truncated cone in Fig. 135; that is, the surface included between the base and the plane T.

1. Divide the base into any convenient number of equal parts, and draw the elements.

2. From b^v, the apex, on the *V*-projection, describe an arc of a radius to the true length of the elements.

3. Make this arc equal in length to the circumference of the base of the cone, and divide it into the same number of equal parts. (See Note below.)

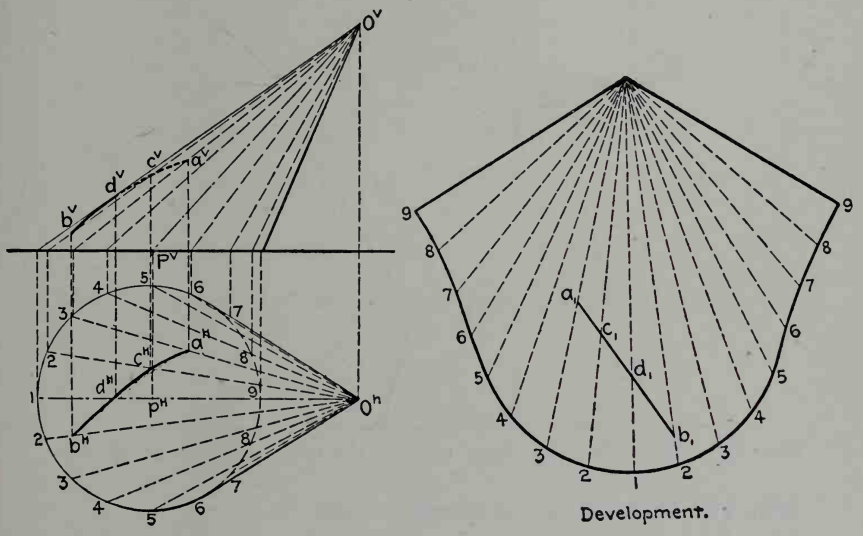


FIG. 136.

4. Measure the true length of the truncated elements, and lay them off on the corresponding radii of the **Pattern** (development).

5. Draw a smooth curve through these points.

Note.—The accurate way to lay off this arc equal in length to the circumference is by this equation: $\frac{R}{r} = \frac{360°}{\theta°}$, or $\theta° = \frac{r}{R}360°$, where R equals the slant height of the cone, r the radius of the base, and θ is the number of degrees in the angle of the development. It can be done more roughly by stepping off small arcs with the dividers.

181. Problem 71.—To develop the surface of an oblique cone. This is accomplished by triangulation, and it is as nearly exact as the chord of a small arc is to the length of the arc.

Construction.—The cone PO (Fig. 136).

1. Divide the base into a number of equal arcs.
2. Draw the elements of these divisions, dividing the cone into small (approximate) triangles.
3. Measure the true length of all the elements.

Note.—By a little care in the division this can be done in pairs.

4. Rectify one of the equal arcs.
5. Beginning (preferably) with the longest element, O1, construct triangles equal to the (approximate) true size of those in the cone, as shown in the development at the right.

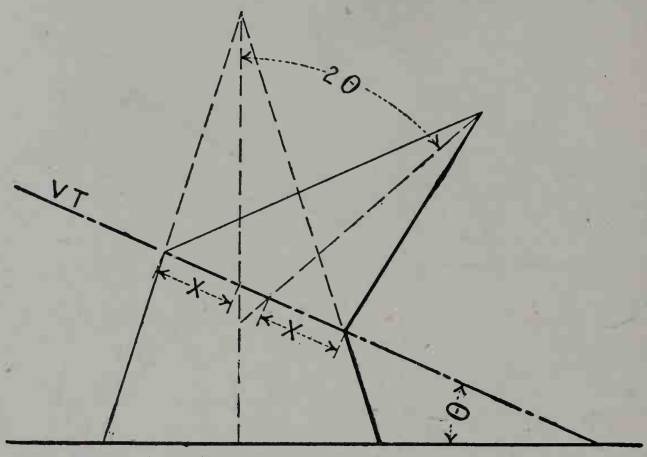


Fig. 137.—Reversing the upper part of the cone to make an elbow.

182. The Problem of the Shortest Path.

The shortest line that can be drawn on any surface from one point to another is a straight line on the development. Let A and B (Fig. 136) be two points on the surface of the cone.

1. Locate them on the development, see a_1 and b_1.
2. Draw a straight line from a_1 to b_1, and locate its intersections, c_1, d_1, etc., with the intervening elements.
3. Return all these points to their original positions in the projections of the cone, and draw the curve through them.

CONICAL ELBOWS

183. Principle.—A plane section, through all the elements of a cone, as in Fig. 137, is an ellipse, which is a symmetrical curve. A symmetrical curve can be inverted upon itself.

Construction.—1. If it is desired to have a conical elbow of two pieces to turn through an axial angle of 2θ, pass a plane through the cone at an angle of θ with the base.

2. Turn the truncated portion on the lower part, and the required elbow is made.

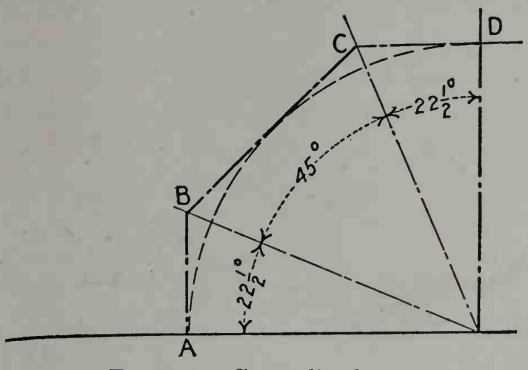


FIG. 138.—Center line layout.

MULTI-JOINT ELBOWS

183. The construction of this is essentially that of the single-joint elbow.

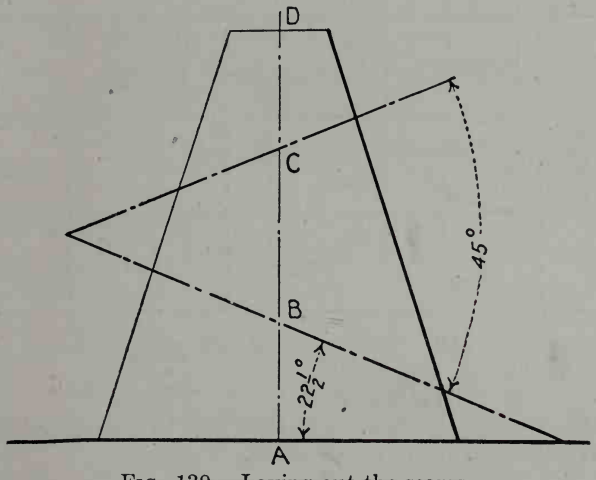


FIG. 139.—Laying out the seams.

Example.—Required to reduce a 6-in. pipe to a 2-in. pipe, through a 90° bend, in three pieces.

1. Layout the center lines according to Fig. 138.

2. Lay off the total length of the center lines as the altitude of the frustum of a cone of 6-in. base and 2-in. top.

3. Cut the frustum by planes through B and C at the angles shown in Fig. 139.

4. Reverse these truncated portions as shown in Fig. 140.

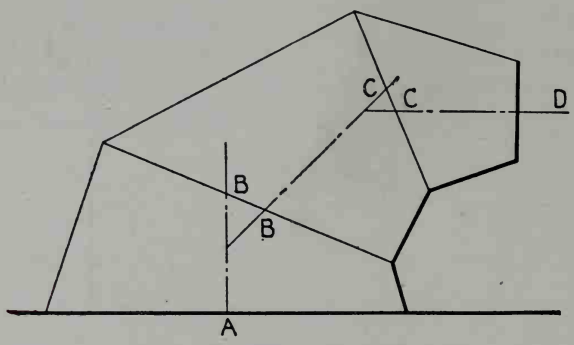
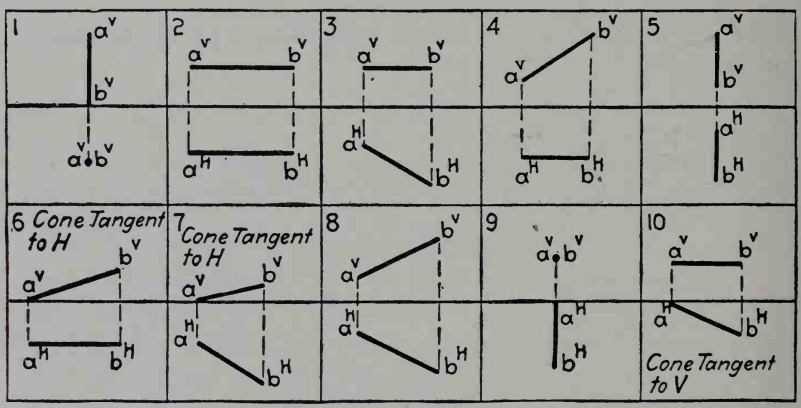


FIG. 140.—Three-piece elbow.

184. **EXERCISES**



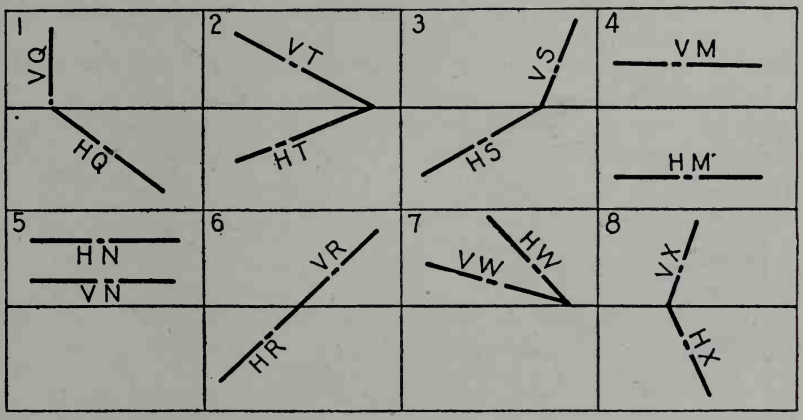
Graphic Layout No. 50.—Center lines of right circular cones.
Base —— in. (1, 1½, 1¾, 2, 2½).
Altitude —— in. (1½, 2, 2½, 3, 3½).
Note.—All these axes are in *I*. The signs may be reversed, and the lines drawn in *III* at the option of the instructor.

Dimensioned Layout.—Center lines. Let A or B be the center of the base.

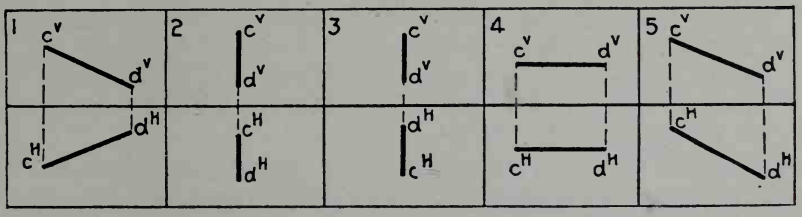
(1) A(3 + 3 − 1½) B(3, 0 − 1½).
(2) A(1 + 1 − 1½) B(3½ + 1 − 1½).

(3) A(2 + 1 − 1) B(4 + 1 − 2).
(4) A(2 + 1 − 1½) B(4 + 2½ − 1½).
(5) A(3 + 2½ − 1) B(3 + 1 − 2).
(6) A(3 + 2½ − 1) B(3 + 1 − 2).
(7) A(2, 0 − 1) B(4 + 1 − 1). (Cone tangent to *H*.)
(8) A(2, 0 − 1) B(4 + 1 − 2). (Cone tangent to *H*.)
(9) A(2 + 1 − 1) B(4 + 2 − 2½).
(10) A(3 + 1½, 0) B(3 + 1½ − 3).
(11) A(2 + 1, 0) B(4 + 1 − 1¼). (Cone tangent to *V*.)

Note.—The above center lines may be drawn in *III*, reversing the signs + and −, if preferred.



Graphic Layout No. 50A.—Cutting planes. (**Note.**—The signs on these planes may be reversed, at the option of the instructor.)



Graphic Layout No. 51.—Piercing lines. (**Note.**—If the cone is drawn in *III*, these lines should be drawn in *III*. Draw them so that they are likely to pierce the cone.)

Cutting Planes.—Traces may be reversed, if desired.
(1) Q(1½ + 3) 1½(5 − 2).
(2) S(5 + 3) 1(5 − 2).
(3) T(1 + 3) 2(5 − 2).
(4) U(+ 2) ∞(− 2).

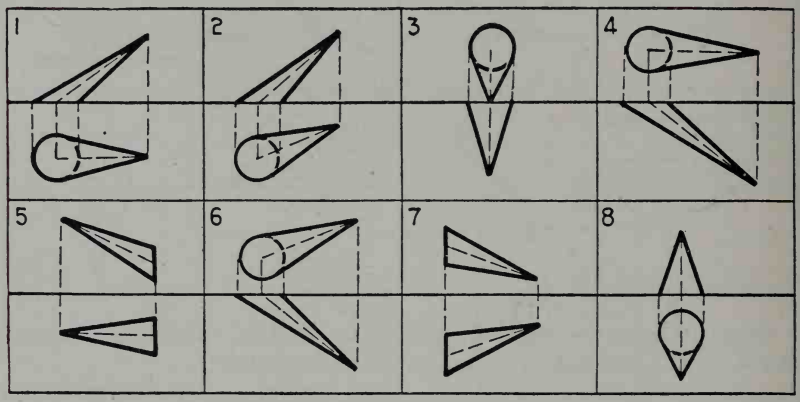
(5) W(− 1) ∞ (− ½).
(6) X(1 + 3) 3(5 − 3).
(7) Y(1 + 1) 5(1 + 2½).
(8) Z(2 + 3) 3(2 − 2).
(9) P(3½ + 3) 3½(3½ − 3).

Piercing Lines.—Signs may be reversed, if desired.

(1) A(2, 0 − 1¼) B(4 + 2 − 2).
(2) C(2, 0 − 2) B(4 + 2 − ½).
(3) E(3 + 2 − ½) F(3 + 1 − 2½).
(4) G(3 + 2 − 2) K(3 + ½ − 1).
(5) M(1 + 1 − 1½) N(4 + 1 − 1½).

759. Locate a point on the —— (front or rear) surface of cone ——, —— in. away from *H*.

760. Cut the surface of cone —— by plane ——. Find the true size of the section.



Graphic Layout No. 52.—Oblique cones with circular bases, diameters 1½ in., 1¾ in., 2 in., 2½ in., and axes 1½ in., 2 in. 2½ in., 3 in. long.

Oblique cones with elliptical bases, 1 in. × 1½ in., 1 in. × 2 in., 1½ in. × 2 in., 1½ in. × 2½ in., 2 in. × 2½ in., and axes 1½, 2, 2½, 3 in. long.

Use this Layout in Ex. 770–781.

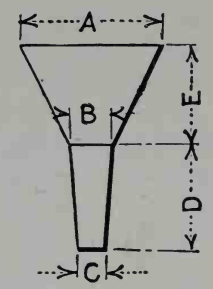
761. Develop the surface between the base and cutting plane, in Ex. 760.
762. Develop the surface between the apex and cutting plane, in Ex. 760.
763. Find the points in which the surface of cone —— is pierced by line ——.
764. Find the shortest line on the surface of cone —— between the piercing points of line ——.
765. Assume a point on the surface of cone ——, and draw the traces of a plane tangent at that point.
766. Assume a point in space outside the cone, and draw the traces of a plane (or two planes, if desired) tangent to the cone ——.
767. Draw the traces of one of the tangent planes to cone ——, inclined ——° to —— (*H*, *V*, or *P*).

768. Draw the traces of the tangent plane (or planes) to the cone ——, parallel to line —— (one of the given piercing lines).
769. What are the limitations of the specifications of Ex. 766 and 767, if any?

Dimensioned Layout.—Let the axes be the following lines, with the circular or elliptical base —— at A. Use in Ex. 770–781.

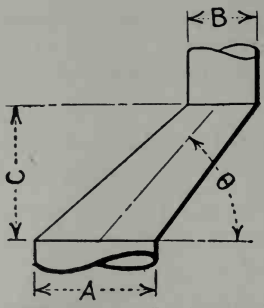
(1) A (2, 0 − 1½) B(4 + 2 − 1½).
(2) A(2, 0 − 1) B(4 + 2 − 2).
(3) A(3 + 1, 0) B(3 + 2½ − 1½).
(4) A(2 + 1½, 0) B(3½ + 1½ − 2).
(5) A(2 + 1, 0) B(3½ + 2 − 2).
(6) A(4 + 1 − 1½) B(2 + 2 − 1½).
(7) A(4 + 2 − 2) B(2 + 1 − ½).
(8) A(3, 0 − 1) B(3 + 2 − 2).

770. Cone ——, base ——, axis ——; locate a point on the —— (front or rear) surface, —— in. away from *H*.
771. Cone ——, base ——, axis ——; draw the intersection, and its true size, made by plane ——.
772. Develop the surface of the base frustum in Ex. 770.
773. Develop the surface of the apex frustum in Ex. 770.
774. Cone ——, base ——, axis ——. Find the points in which it is pierced by line ——.
775. Draw the shortest line on the surface of the cone between the piercing points in Ex. 774.
776. Cone ——, base ——, axis ——; assume a point on the surface, and draw the traces of a plane tangent at that point.
777. Assume a point in space, outside of cone ——, base ——, axis ——; draw the traces of a plane (or both planes, if desired) tangent to the cone through the assumed point.
778. Cone ——, base ——, axis ——; draw the traces of a plane tangent, and inclined ——° to —— (*H*, *V*, or *P*).
779. Cone ——, base ——, axis ——; draw the traces of a plane tangent to the cone and parallel to line ——.
780. What are the limitations of the specifications of Ex. 778 and 779, if any?
781. Cone ——, base ——, axis ——; draw the projections and true size of its right section at the middle point of its axis.
782. Lay out the patterns for Funnel No. ——. Graphic Layout No. 53.
783. Lay out the patterns for Motor Boat Funnel No. ——.
784. Lay out the patterns for Ventilator Hood No. ——.
785. Lay out the patterns for Sink Drainer No. ——.
786. Lay out the patterns for Offset Reducing Pipe No. ——, C = ——, θ = ——.
787. Lay out the patterns for the Furnace Hood No. ——. Make the plan of the conical helix a Spiral of Archimedes, making one complete revolution. Omit lap and rivet holes.
788. Lay out the patterns for Chimney Transition Piece No.——. Graphic Layout No. 54.



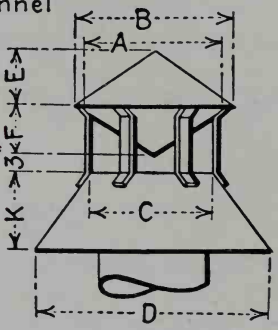
	A	B	C	D	E
1	4	1	½	4	4
2	6	1¼	¾	4	5
3	8	2½	1	6	8
4	10	3	1¼	5	7

Funnel



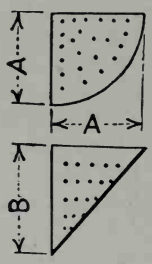
	A	B	C	θ
1	8	3	10,12,16	30°,45°,60°,67½°
2	8	4	"	"
3	10	5	12,15,18	"
4	10	6	"	"
5	12	6	"	"

Offset Reducing Pipe



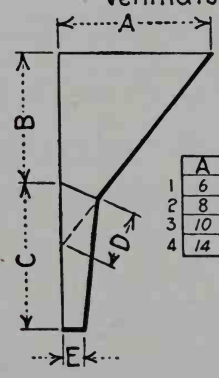
	A	B	C	D	E	F	K
1	27	24	20	36	6	5	8
2	32	28	26	40	7	6	9
3	36	32	28	44	8	7	10
4	40	35	30	48	9	8	12
5	45	40	36	54	10	9	14

Ventilator Hood



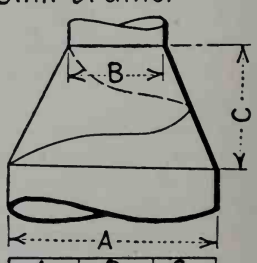
	A	B
1	6	8
2	6	10
3	8	10
4	8	12
5	10	12

Sink Drainer



	A	B	C	D	E
1	6	8	8	4	¾
2	8	10	10	5	⅞
3	10	12	14	6	1
4	14	18	16	10	1¼

Motor Boat Funnel



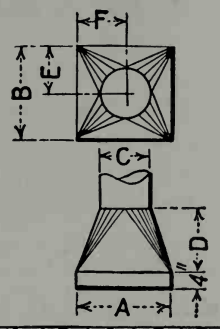
	A	B	C
1	24	12	20
2	28	12	24
3	30	13½	26
4	32	15	28
5	40	18	30

Furnace Hood

Graphic Layout No. 53.—Dimensions in tables are in inches.

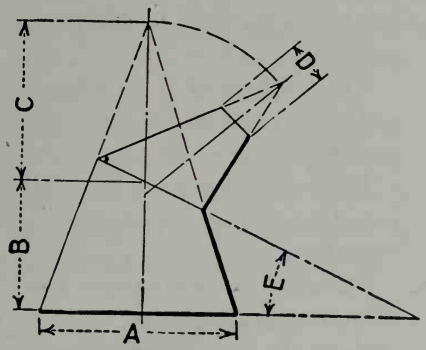
789. Lay out the patterns for Conical Elbow No. ——, E = ——° (30, 37½, 45).

790. Lay out the patterns for Multi-joint Conical Elbow No. ——, C = ——° (75, 90, 105, 120, 135), number of pieces = —— (3, 4, 5, 6).



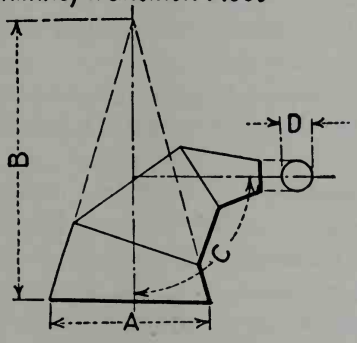
	A	B	C	D	E	F
1	10	10	4	12	5	5
2	10	12	4	14	5	4
3	12	12	4	12	6	6
4	12	12	5	14	6	5
5	12	14	5	16	7	4
6	14	14	6	18	6	8
7	14	14	6	20	7	7
8	14	18	7	20	8	7

Chimney Transition Piece



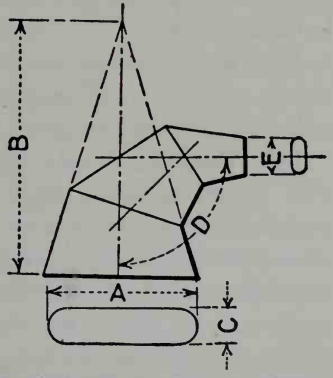
	A	B	C	D	E
1	10	12	8	2	30°,37½°,45°
2	10	14	10	3	"
3	12	14	10	4	"
4	12	16	12	3	"
5	14	16	12	4	"
6	14	18	14	4	"
7	16	20	12	4	"
8	16	24	16	5	"

Conical Elbow



	A	B	C (Degrees)	D	Number of Pieces
1	4	8	75,90,105,120,135	1	3,4,5,6
2	4	10	"	1	"
3	4	12	"	1	"
4	6	12	"	1½	"
5	6	14	"	1½	"
6	6	16	"	1½	"
7	8	16	"	2	"
8	8	20	"	2	"
9	8	24	"	2	"

Multi-Joint Conical Elbow



	A	B	C	D	E	Number
1	8	16	3	75°,90°,105°,120°	2	3,4,5,6
2	8	20	4	"	2	"
3	10	20	5	"	3	"
4	10	24	6	"	3	"

Multi-Joint Oblong Conical Elbow

Graphic Layout No. 54.—Dimensions given in inches, angles in degrees.

791. Lay out the patterns for Multi-joint Oblong Conical Elbow No. ——, D = ——° (75, 90, 105, 120, 135), number of pieces = —— (3, 4, 5, 6).

Note.—The dimensions for A and C in Ex. 791 may be transposed.

THE CYLINDER

185. Definition.—A cylinder is a single-curved surface generated by a line moving along any curved-line directrix, always remaining parallel to a given line. It is a **cone with its apex at infinity.**

The **Surface** of a cylinder is usually taken to be that portion which is included between two plane bases. The bases may be parallel or not. Theoretically the cylinder is infinite in extent.

Axis.—The axis is a line through the center of a right section, parallel to all the elements. It is a useful line in defining the cylinder in space.

Varieties of Cylinders.—The characteristic distinction between cylinders is the curve of their **right section.** This is at some variance to the cone distinctions, which are founded on base and axial peculiarities. The reason for this is that it is absolutely necessary to know the right section of a cylinder in most of the problems arising; as, for instance, in its development, the right section always rolls out in a straight line. Also, in most tangent problems, the right section has to be considered. So, when a cylinder is spoken of as circular, elliptical, etc., **a right section is understood.**

Limits.—When the directrix becomes a straight line, the cylinder becomes a plane, and when the directrix becomes a point the cylinder becomes a line.

186. Tangent Planes to Cylinders.—A plane which contains two consecutive elements is tangent. **The Tangent Plane is parallel to the axis, and to every element.** The two consecutive elements contained in the tangent plane form the **Element of Contact.** This element of contact is one of the two tangent lines necessary to determine the tangent plane. From this fact we derive the special rule for passing planes tangent to cylinders.

Rule.—A tangent line and a line parallel to the axis, intersecting the tangent, determine a tangent plane to a cylinder.

Axiom.—When the base of a cylinder is in H, the H-trace of any plane, tangent to the cylinder, will be tangent to the base curve of the cylinder.

Axiom.—A plane cannot in general be tangent to two cylinders unless the axes are parallel. The exceptions, which are not frequent, will only be found when two outside elements of the two cylinders are in the same plane. That plane will be the

tangent plane. If two cylinders have parallel axes, four planes tangent to both can usually be passed. Two cylinders of equal diameter, with axes intersecting, can have two common tangent planes outside.

187. Development.—As was before mentioned, the right section of a cylinder rolls out into a straight line. This is true, because the right section is perpendicular to all the elements, and when they are rolled out they remain parallel, and the plane perpendicular to them cuts their plane in a straight line perpendicular to them. Therefore, the first thing to do is to **rectify a right section.** Then lay off perpendiculars, properly spaced, with the true lengths of the various elements laid off in order.

PROBLEMS RELATING TO CYLINDERS

188. Problem 72.—To locate a point on the surface.

Analysis.—1. Assume one projection of the point.

2. Draw a line through that projection parallel to the axis. (This will be the element on which it lies.)

3. Find the intersection of this element with the base.

4. Project this intersection to the other projection of the base and draw the other projection of the element.

5. Project the point to the second projection of the element.

Let the student make the construction.

189. Problem 73.—To find the intersection of a plane and a cylinder.

Analysis.—1. Draw a sufficient number of elements.

2. Obtain the piercing points of these elements with the plane.

3. Draw a smooth curve through these piercing points in order.

190. Problem 74.—To find the points in which a straight line pierces a cylinder.

There are two methods: 1. to use when the base of the cylinder lies in one of the planes of projection; 2. to use when the base is in space.

Analysis. First Method.—When the base is in H, V or P.

1. Pass a plane through the line parallel to the axis.

2. At the points in which the trace of this plane crosses the base, draw elements of the cylinder.

3. The points in which these elements are intersected by the given line will be the required points.

Second Method.—When the base is in space.

1. Draw a sufficient number of elements.
2. Pass a plane through the line, preferably a projecting plane.
3. Find the intersections of this plane with enough of the elements to yield a curve that will be intersected twice by the given line.
4. The required points will then be where the given line crosses the curve.

191. Problem 75.—To pass a plane tangent to a cylinder through a given point on the surface.

Analysis.—1. Through the given point draw a line parallel to the axis.

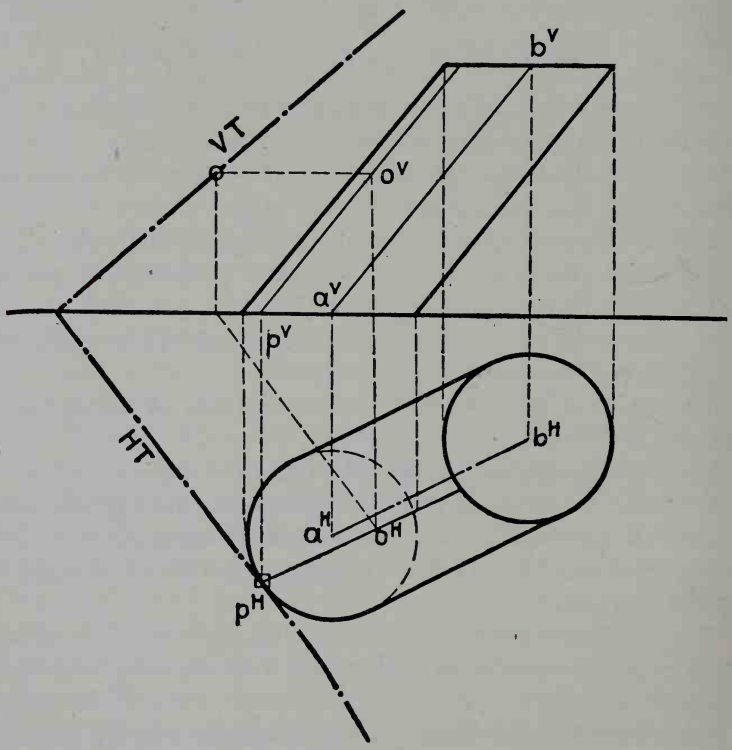


FIG. 141.

(This will be one line of the required plane, the element of contact.)

2. At the point where the element of contact crosses any curve of the surface (preferably one of the bases), draw the tangent to that curve.

3. Pass the plane through the element and tangent.

Construction.—Let O (Fig. 141) be the given point on the surface of the elliptical cylinder whose axis is AB.

1. Draw the element OP.
2. Draw the tangent to the base at P.
3. This tangent will be the *H*-trace of the tangent plane.
4. Draw the *V*-trace by any of the known means.

Note.—This construction, as illustrated in Fig. 141, is only possible when the base of the cylinder is in *H*, *V* or *P*. If the base is in an oblique plane, the tangent will not be one of the traces of the plane. The **analysis** will apply to **any** situation of the cylinder.

192. Problem 76.—To pass a plane tangent to a cylinder through a point outside the surface.

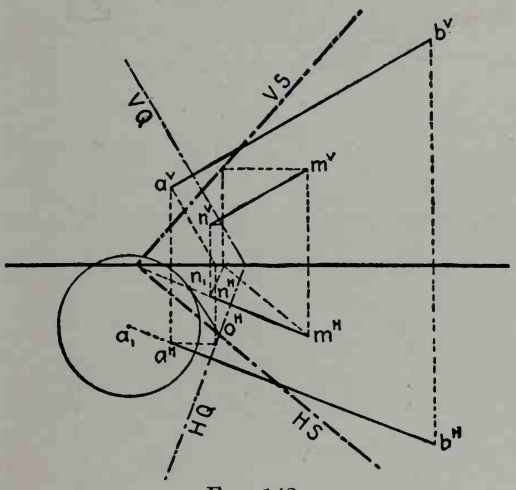


Fig. 142.

Analysis.—1. Draw a line through the point parallel to the axis.

2. Find the point in which this line pierces the plane of the base.

3. From this piercing point draw a tangent to the base.

4. The parallel line and the tangent line will determine the required tangent plane.

Let the student make his own construction for a cylinder similar to Fig. 141. The construction here given is for a difficult case.

Note.—The projections of the cylinder are not drawn. This is done so as not to complicate the figure unnecessarily.

Construction.—Let the cylinder be a circular cylinder of ¾-in.

diameter, whose axis is AB (Fig. 142). Let it be required to pass a tangent plane through M, a point outside the surface.

1. Draw the line MN, parallel to AB through M.
2. Pass a plane Q through A, perpendicular to AB. (This will be the plane of the base, cutting a circle from the surface.)
3. Find the piercing point, N, of MN with Q.
4. Revolve A and N into *H* about HQ.
5. Draw a ¾-in. circle about a_1 as a center.
6. From n_1 draw a tangent to the circle.
7. Counter-revolve the tangent n_1o into the plane Q.
8. Pass the plane S through NO and MN. It will be the required tangent plane.

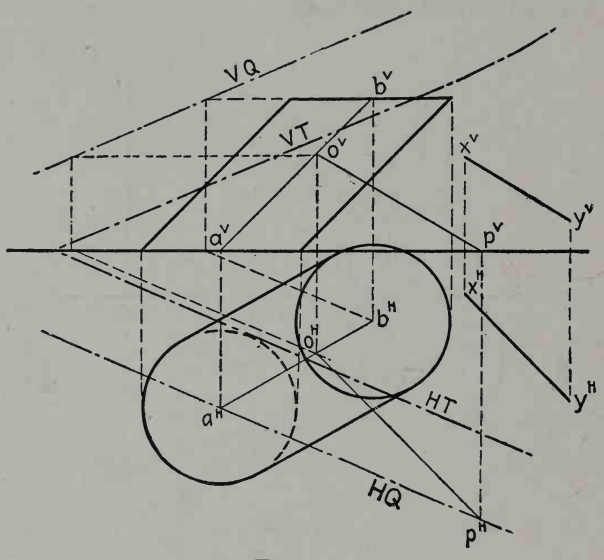


FIG. 143.

193. Problem 77.—To pass a plane tangent to a cylinder and parallel to a given line.

Analysis.—1. Pass a plane parallel to the axis and the given line. Why?

2. Find the intersection of this plane and the plane of the base.
3. Draw a tangent to the base parallel to the intersection obtained in operation (2).
4. Pass the required tangent plane through this tangent line, parallel to the auxiliary plane, passed in operation (1).

Construction.—Let AB, Fig. 143, be the axis of the cylinder, and let it be required to pass the tangent plane parallel to XY.

1. Pass the auxiliary plane Q through AB parallel to XY.

2. Draw the required plane T, tangent to the cylinder and parallel to Q. The H-trace of T will be tangent to the base in H, and parallel to HQ, and the V-trace of T will be parallel to VQ.

194. If the cylinder be circular and its axis oblique, the best method for the foregoing problem is the **"Normal" method.** Its analysis is as follows:

1. Pass a plane through the axis parallel to the given line.

2. Pass a plane parallel to this plane at a distance equal to the radius of the cylinder.

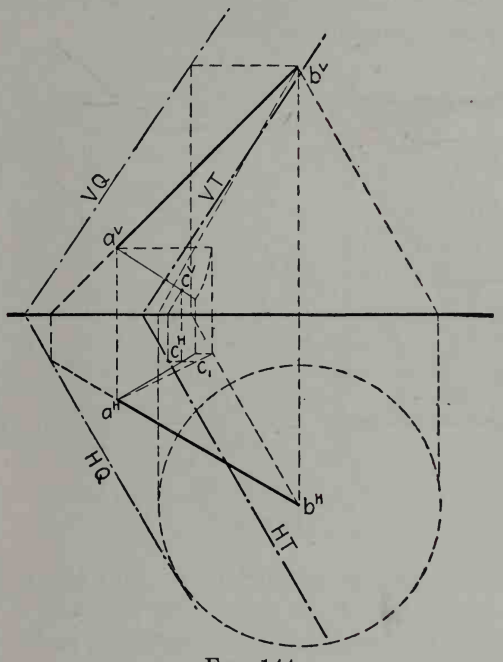


FIG. 144.

The problem may be done in this way without drawing any elements or bases of the cylinder. Let the student prove that the "normal" method is correct.

195. Problem 78.—To pass a plane, inclined at any given angle to H or V, tangent to a cylinder.

Limitations.—The limits of this angle must be between 90° and the angle between the elements and the stipulated plane.

Analysis.—1. Pass an auxiliary plane through the axis making the given angle with H or V, as required; see Problem 40.

2. If the base of the cylinder is in H, draw the H-trace of the required plane, tangent to the base and parallel to the H-trace of the auxiliary plane.

3. Draw the V-trace of the required plane parallel to the V-trace of the auxiliary plane.

196. Normal Method.—When this problem is required of a circular cylinder, oblique to H and V, the normal method, like that in Art. 194, is the easiest and best.

Fig. 144 shows the construction of such a problem, in which AB is the axis and AC the radius of the given cylinder. The elements of the auxiliary cone make the given angle with H. T is one of the four possible required planes.

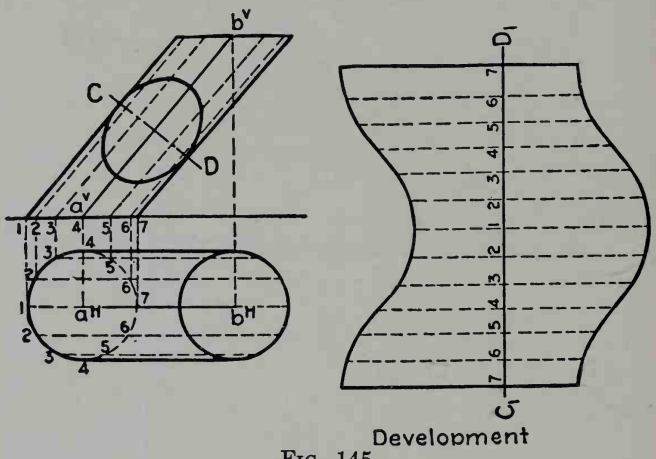


FIG. 145.

197. When the axis is parallel to H, and the angle between the tangent plane and H is given, the problem is very easily solved by projecting on a plane perpendicular to the axis.

Note.—Four planes can be passed tangent to any cylinder, making the given angle, if the problem is possible.

198. Problem 79.—To develop the surface of a cylinder.

Construction.—To develop the elliptical cylinder, whose axis is AB (Fig. 145).

1. Draw a sufficient number of elements.

2. Pass a plane perpendicular to the axis, as, for example, along the line CD.

3. Revolve this right section to show its true size.

4. Lay off the base line, C_1D_1, equal in length to the length of

the ellipse, with divisions equal to the respective arcs, as shown by the corresponding numerals.

5. Through all these divisions draw perpendiculars to C_1D_1.

6. Lay off on these perpendiculars the true lengths of their respective elements, measured above and below the right section.

7. Draw a smooth curve through the extremities.

Note.—It is not always necessary to show both projections of the right section. In this case the *H*-projection is unnecessary.

PIPE ELBOWS

199. The object of a pipe elbow is to make a turn through any desired angle and preserve the pipe diameter through the turn.

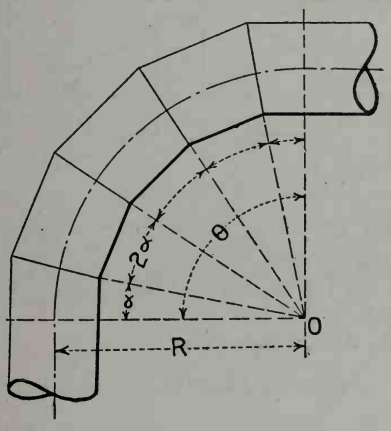


FIG. 146.—Five-piece pipe elbow.

The surface thus approximated is that of the **torus**, and, in case of a double bend, the **serpentine.**

The elbow is laid out as follows:

Divide the center-angle (θ) by $2n - 2$ (n being the number of pieces in the turn). This division will yield α, the angle of the joint, *i.e.*, $\alpha \frac{\alpha}{2n - 2}$.

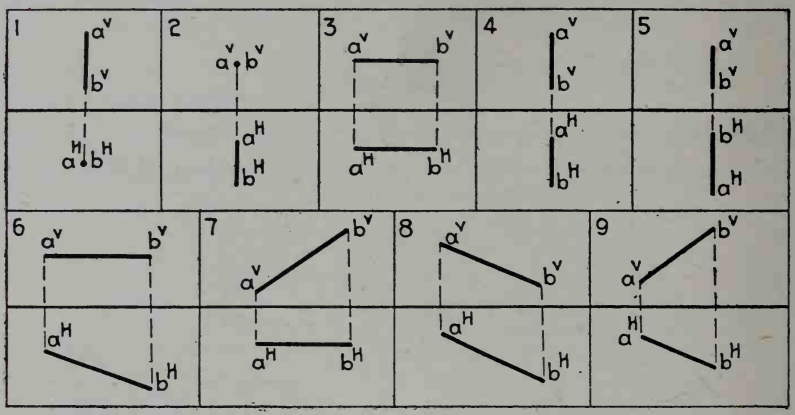
Example.—Required to design a 5-piece elbow for a 90° turn (axial angle), on 10-in. radius.

The center-angle θ is the supplement of the axial angle; $\therefore \theta = 90°$.

$$2n - 2 = 8. \quad \therefore \alpha = 11\tfrac{1}{4}°.$$

(See Fig. 146.) Lay off a 90° arc on a 10-in. radius, and divide the arc into arcs of 11¼°. The first of these radii will be the projection of the first joint, and the **odd** radii, the third, fifth, and seventh, will be the projections of the other joints. Pipes laid out perpendicularly to the **even** radii will give a pipe bend of uniform right section.

200. EXERCISES



Graphic Layout No. 55.—Axes for circular or elliptical cylinders, with bases perpendicular to the axes.

Bases.—Circles 1 in., 1½ in., 2 in.; ellipses 1 in. × 1½ in., 1 in. × 2 in. 1½ in. × 2 in., 1½.,n. × 2½ in.

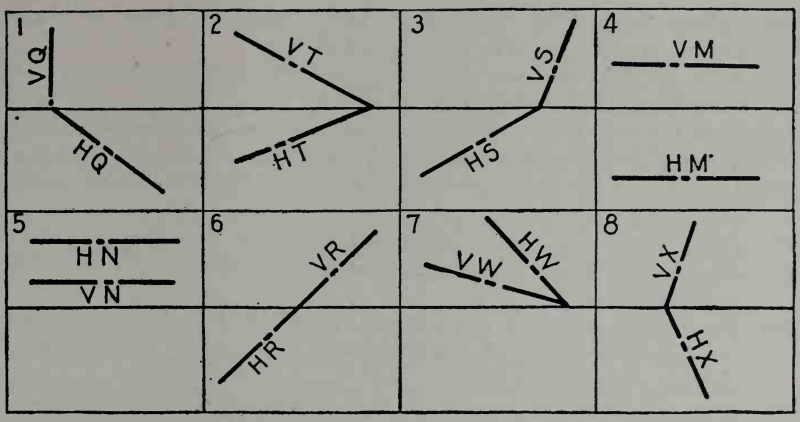
Altitudes.—1½ in., 2 in., 2½ in., 3 in.

Note.—The signs for these lines may be reversed, and the cylinders drawn in *III*, if desired.

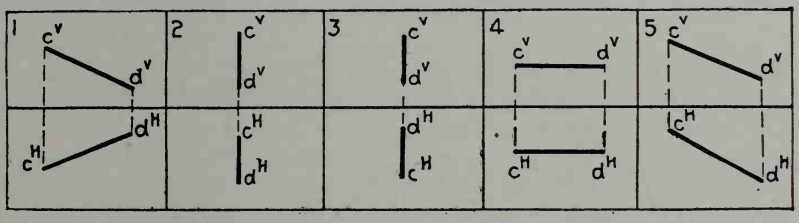
Dimensioned Layout.—Center lines for circular cylinders or elliptical cylinders, with bases perpendicular to the axes.

(1) A(3, 0 − 1½) B(3 + 3 − 1½).
(2) A(3 + 1, 0) B(3 + 1 − 3).
(3) A(2 + 1½ − 1) B(4 + 1½ − 1).
(4) A(2 + 1 − 1) B(4 + 1 − 2).
(5) A(2 + 2 − 1) B(4 + 1 − 1).
(6) A(3 + 2 − ½) B(3 + 1 − 2).
(7) A(3 + 2½ − 2) B(3 + 1 − 1).
(8) A(1 + 2½ − 1) B(3½ + 1 − 2).
(9) A(2 + 2 − 2½) B(4 + 1 − 1).

Note.—The foregoing axes may be drawn in *III* by reversing the signs + and −. In this event, the signs on the plane traces and piercing lines should be reversed, if they enter the exercise.



Graphic Layout No. 56.—Cutting planes. (**Note.**—If the cylinders are drawn in *III*, reverse the lettering of these traces.)



Graphic Layout No. 57.—Piercing lines. (**Note.**—If the axes are drawn in *III*, draw these lines in *III*, and in any case, draw them so that they are likely to pierce the cylinder.

Cutting Planes.

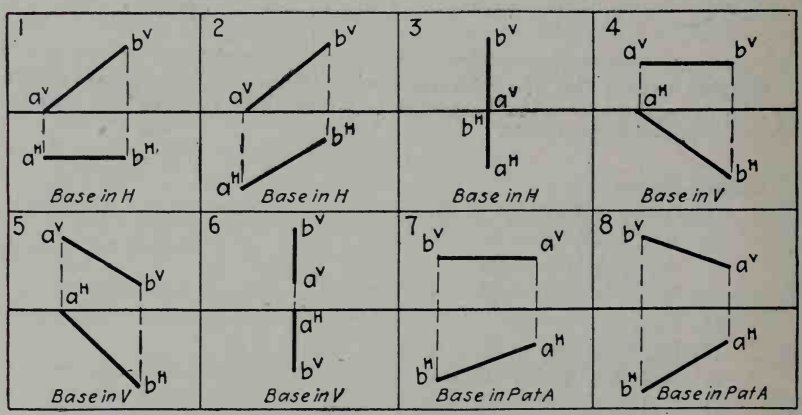
(1) Q(1½ + 3) 1½(5 − 2).
(2) R(1 + 2) 4(4 − 3).
(3) S(5 + 3) 1(5 − 2).
(4) T(1 + 3) 2(5 − 2).
(5) U(+ 2) ∞ (− 1).
(6) W(+ ½) ∞ (+ 3).
(7) X(1 + 1) 5(1 + 2½).
(8) Y(1 + 3) 3(5 − 3).
(9) Z(4 + 3) 3(4 − 2).
(10) P(3½ + 3) 3½(3½ − 3).

Piercing Lines.

(1) A(2, 0 − 1¼) B(4 + 2 − 2).
(2) C(2, 0 − 2) D(4 + 2 − ½).
(3) E(3 + 2 − ½) F(3 + 1 − 2½)
(4) G(3 + 2 − 2) K(3 + ½ − 1).
(5) M(1 + 1 − 1½) N(4 + 1 − 1½).

792. Cylinder ——, axis ——, base, or right section ——; locate the projections of a point on the —— (front or rear) surface.

793. Cylinder ——, axis ——, base, or right section——; draw the intersection, and its true size, made by plane ——.
794. Draw the development of the cylinder in Ex. 793, included between one of the bases and the cutting plane.
795. Cylinder ——, axis ——, base, or right section ——; find the points in which it is pierced by line ——.
796. Draw the shortest line on the surface between the two piercing points found in Ex. 795.
797. Draw the H-, V-, and P-intersections of cylinder ——, axis ——, base, or right section ——, continuing the cylinder to meet the planes, if necessary.
798. Draw the traces of the plane tangent to cylinder ——, axis ——, base, or right section ——, at a point assumed on the —— (front or rear) surface, —— in. from H.
799. Draw the traces of —— (one or two) tangent planes to cylinder ——, axis ——, base, or right section ——, through a point without the surface, in —— (I, II, III or IV).
800. Draw the traces of —— (one or two) tangent planes to cylinder ——, axis ——, base, or right section ——, parallel to A( ) B( ).
801. Draw the traces of —— (one, two, three, four) tangent planes to cylinder ——, axis ——, base, or right section, inclined ——° to —— (H, V, or P).



Graphic Layout No. 58.—Axes for elliptical cylinders, with circular or elliptical bases in H, V, or P, as given.

Bases.—1 in., 1½ in., 1¾ in., 2 in., circles; 1 in. × 1½ in., 1 in. × 2 in., 1½ in. × 2 in., ellipses.

Dimensioned Layout for Axes of Foregoing Cylinders.

(1) A(1½, 0 − 1½) B(3½ + 2 − 1½). Base in H.
(2) A(2 + 1½, 0) B(4 + ½ − 2). Base in V.
(3) A(3 + 2, 0) B(3, 0 − 1½). Base in H.
(4) A(3 + 1¾, 0) B(3, 0 − 2). Base in V.

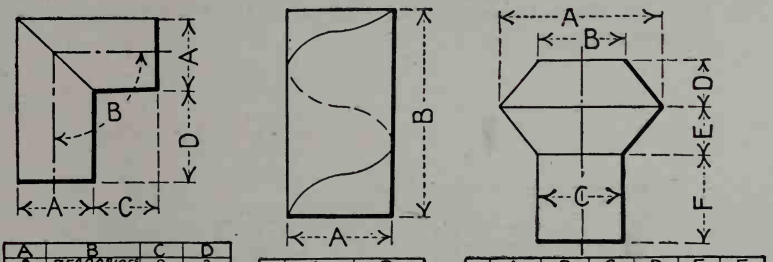
(5) A(1 + 2 − 1) B(3½ + 1 − 1). Base in *P*, at B.
(6) A(1½, 0 − 1) B(3 + 2 − 2). Base in *H*.
(7) A(1 + 2½ − 2) B(3½ + 1 − 1). Base in *P*, at B.

Note.—The axes in both layouts may be located in *III* by reversing the + and − signs, at the option of the instructor.

Note 2.—The cutting planes and piercing lines for the following exercises are found in Graphic Layouts Nos. 56 and 57.

802. Cylinder, axis ——, base ——; assume a point on the —— (front or rear) surface —— in. from *H*.
803. Cylinder, axis ——, base ——; draw the projections and true size of the section made by plane ——.
804. Cylinder, axis ——, base ——; draw the projections and true size of its right section at the middle point of the axis.
805. Cylinder, axis ——, base ——; locate the points in which it is pierced by line ——.
806. Draw the shortest line on the surface of the cylinder between the points found in Ex. 805.
807. Draw the *H*-, *V*-, and *P*-sections of cylinder, base ——, axis ——. (Continue the surface until the section is made.)
808. Cylinder, axis ——, base ——; draw the traces of the plane tangent at a point on the —— (front or rear) surface, assumed.
809. Cylinder, axis ——, base ——; draw the traces of —— (one, two) planes tangent through a point outside the surface, assumed in —— (*I*, *II*, *III*, *IV*).
810. Cylinder, axis ——, base ——; draw the traces of —— (one, two, three, four) planes tangent, inclined ——° to —— (*H*, *V*, or *P*).
811. Cylinder, axis ——, base ——; draw the traces of —— (one, two) planes tangent to it and parallel to X( ) Y( ).

PRACTICAL EXERCISES



	A	B	C	D
1	2	75° 90° 105°	2	3
2	4		4	5
3	6		6	7
4	8		8	9
5	10		10	11

2-Piece Pipe Elbow

	A	B
1	1½	2
2	2	3
3	2½	4
4	1	3

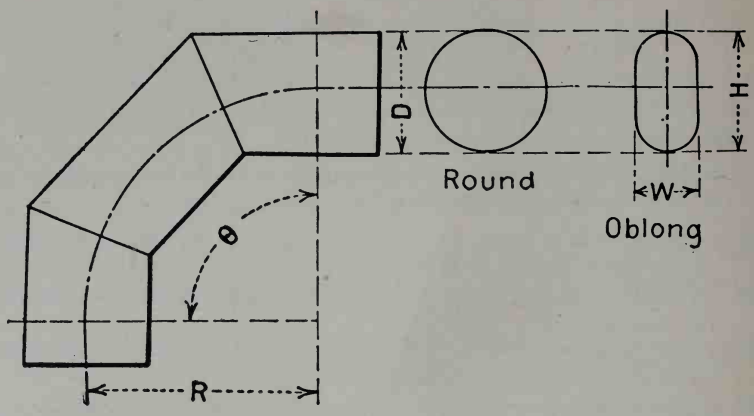
Helical Tubing

	A	B	C	D	E	F
1	22	10	12	6	8	12
2	24	12	12	6	8	12
3	26	14	14	7	10	12
4	28	16	14	7	10	12

Locomotive Stack

Graphic Layout No. 59.—Dimensions in inches.

812. Lay out the patterns for Two-piece Pipe Elbow ——; θ = ——°.
813. Lay out the patterns for Helical Tubing ——.
814. Lay out the patterns for Locomotive Stack ——.



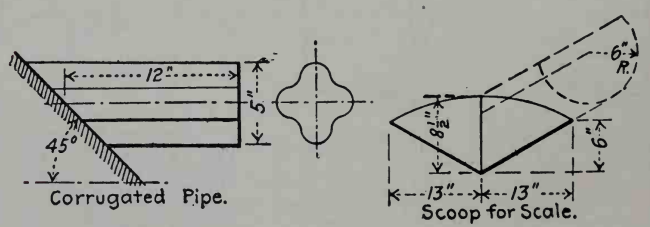
	Number of Pieces	θ	R	D	W	H
1	3, 4, 5, 6, 7	60°, 75°, 90°, 105°, 120°	8	4	2	4
2	"	"	10	5	2	6
3	"	"	12	6	2	8
4	"	"	16	8	3	6
5	"	"	20	10	5	8
6	"	"	7	4	2	5
7	"	"	9	5	2	7
8	"	"	11	6	2	9
9	"	"	14	8	3	9
10	"	"	18	10	4	8

Multi-Joint Pipe Elbow

Graphic Layout No. 60.—Dimensions in inches.

815. Lay out the patterns for the Multi-joint Pipe Elbow ——, θ = ——°, number of pieces ——.

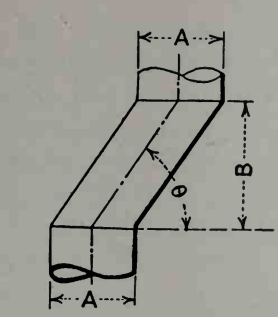
816. Lay out the patterns for the Oblong Pipe Elbow ——, θ = ——°, number of pieces ——.



Graphic Layout No. 61.

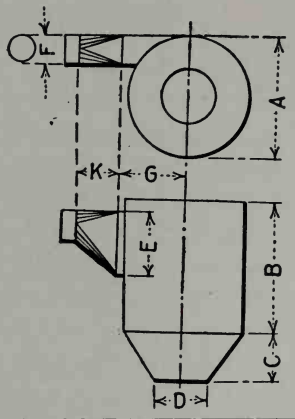
817. Lay out the patterns for the Corrugated Pipe.

818. Lay out the patterns for the Scoop.



	A	B	θ
1	4	6	45°,60°,67½°,75°
2	4	7	"
3	4	8	"
4	6	8	"
5	6	10	"
6	6	12	"
7	8	12	"
8	8	14	"
9	8	16	"

Oblique Connector for Circular Pipe

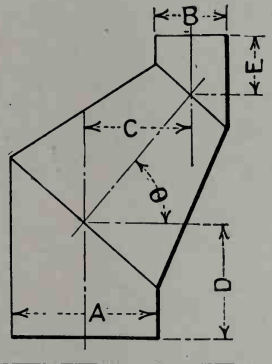


	A	B	C	D	E	F	G	K
1	89½	109	80	8	64	20	50	36
2	73½	89½	64	8	53½	16	43	32
3	61½	74	53	7	44	13	38	28
4	41	49	35	5	29	8	24	24
5	21	23	17½	3	13½	3¼	17	16

Morse Rarefied Dust Collector

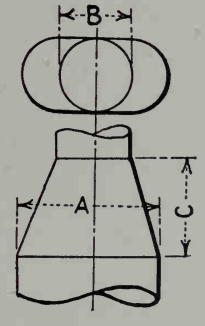
Graphic Layout No. 62.—Dimensions in inches.

819. Lay out the patterns for Oblique Connector ——, θ = ——°.
820. Lay out the patterns for the Dust Collector ——.



	A	B	C	D	E	θ
1	8	4	6	10	5	30°,45°,52½°,60°
2	8	3	8	10	5	
3	8	5	10	10	5	
4	10	4	8	10	5	
5	10	6	10	10	5	
6	12	4	10	10	5	
7	12	6	12	10	5	
8	12	5	14	10	5	

Offset Pipe Reducer



	A	B	C
1	16	10	10
2	18	10	10
3	20	12	12
4	24	12	12
5	26	12	14
6	28	14	14
7	30	14	16
8	32	14	16
9	36	16	18
10	40	16	18

Oblong Hood

Graphic Layout No. 63.—Dimensions in inches.

820. Lay out the patterns for Offset Pipe Reducer ——, θ = ——°.
821. Lay out the patterns for Oblong Hood ——.

THE CONVOLUTE

201. The third class of single-curved surfaces, the **Convolute,** is generated by the tangent line to any space curve, as it moves along the curve. The tangent line to a curve contains two consecutive points of the curve. This means that two consecutive tangents have one point in common; thus, A, B, and C are consecutive points in a curve, and AB and BC are therefore two consecutive tangents with B as their intersection. Thus all **consecutive tangents** to any space curve intersect two and two, and therefore become elements of a single-curved surface.

202. Helical Convolute.—The only convolute of importance is that generated by the tangent to the cylindrical helix. Fig. 147 shows both projections.

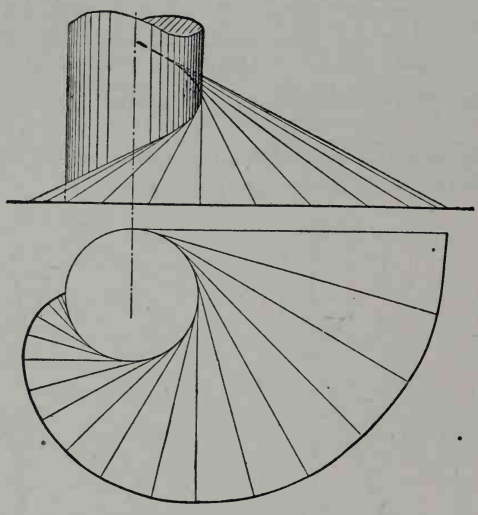


FIG. 147.—Projections of a convolute.

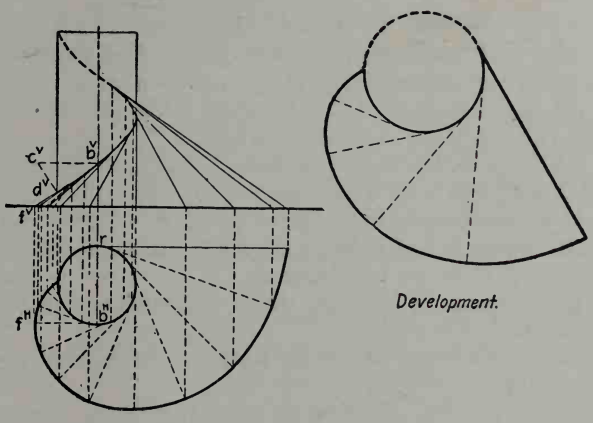
It is sometimes called the "Developable Helicoid," and its most frequent application is in screw conveyors.

Theoretically, the surface is unlimited, the portion shown here being a "three-quarter turn." In practical design it is always limited by a cylinder and upper and lower bases, or by two cylinders. The *H*-trace of the surface is an involute. (See Fig. 147.)

203. Problems.—The problems that relate to this surface do not differ materially from those of other single-curved surfaces; therefore the only problem given a solution here is the problem of the development of the surface.

204. Problem 80.—To develop the surface of a convolute.

Discussion.—If the helical directrix be rolled into a plane, element by element, it will be seen to have a **constant radius of curvature,** and will therefore develop as a circle. This radius is the normal line at any point of the helix that intersects the axis, because all such normals will have equal lengths. The elements of the convolute, being tangent to the helix, are perpendicular to the respective normals, and when the rolling out occurs, these elements will be consequently tangent to the circle made by developing the helix. As the lengths of the tangents are proportional to their *H*-projections, the developed curve of their extremities will be the involute to the developed helix.



The Convolute and its Development.

FIG. 148.

This is so because each tangent is equal in length to the helix from the origin to the point of tangency, and when the helix is developed into a circle, the tangents are equal to the arc from the origin to the point of tangency. This corresponds to the generation of the involute.

Analysis.—The entire analysis is practically confined to the determination of the radius of the circle of the developed helix.

Graphical Solution.—To determine the **development radius** graphically, erect a perpendicular $c^v d^v$ (Fig. 148) to the tangent $b^v f^v$ at the point where $b^v f^v$ crosses the outside element of the cylinder. The horizontal line $b^v c^v$ included between this perpendicular and the point of tangency is the desired radius.

Construction of the Development.—Draw the circle with the radius $b^v c^v$ and draw an involute to it. Make the arc of the developed involute equal to the arc of the helix, since the helix from any point to the origin is equal to the tangent from that point to the plane of the origin. This, then, is accomplished by measuring the tangent and laying off its length on the involute in the development.

Mathematical Value.—The mathematical value for the radius of the development is the expression $\frac{R}{\cos^2.\theta}$, in which R is the radius of the helical cylinder, and θ is the inclination of the elements to the horizontal.

EXERCISES

822. Draw a 270° convolute to the helix, whose axis is A(1½, 0 − 1¼) B(1½ + 2 − 1¼), 1-in. diameter, 2-in. pitch. Find the traces of the tangent plane containing C(2½, x − 2) on the surface.
823. Draw the convolute in Ex. 822. Draw the traces of the tangent plane through D(1¾ + ¼, y) on the front surface.
824. Draw a 270° convolute to the "left-hand" helix, whose axis is E(4½, 0 − 1) F(4½ + 3 − 1), 1⅛-in. diameter, 2½-in. pitch. Draw the traces of a plane tangent at the point G(4¾, x − 1¾) on the surface.
825. Draw the convolute in Ex. 824. Draw the traces of the plane tangent at K(3½ + 1, y) on the front surface.
826. Draw the convolute in Ex. 822. Draw the traces of a plane tangent containing M(1 + 2 − 2½).
827. Convolute in Ex. 824. Draw the traces of the tangent plane containing N(4¼ + 1 − 2½).
828. Convolute in Ex. 822. Find its intersection with the plane S(1 + 1) 4(1 − 3).
829. Convolute in Ex. 824. Find its intersection with the plane T(+ 1½) ∞(− 3).
830. Develop the surface of the convolute in Ex. 822.
831. Develop the surface of the convolute in Ex. 824.
832. Find the shortest path on the surface of the convolute in Ex. 822 from O(1¾, x − 2) to P(3, y − 1).
833. Find the shortest path on the surface of the convolute in Ex. 824 from C(5, x − 1¾) to D(3, y − ½).

CHAPTER VII

WARPED SURFACES

204. Discussion.—According to the definition in Art. 167, a warped surface is generated by moving a straight line so that no two consecutive positions shall be in the same plane. It thus falls into the classification with single-curved surfaces as a **ruled surface.** Planes may be passed through any point of a warped surface that will cut straight lines from the surface, always one, and, in the case of several surfaces, two straight lines. Other planes cut curves from them.

Warped surfaces are of many varieties and are common in practical work. In many cases, when they occur in practical work, it is not necessary for the draftsman to recognize their peculiarities, but very often, as in arches, sheet metal work, turbine blades, propeller blades, skew gears, conveyors, etc., it is highly important that engineers and draftsmen know their properties and methods of generation.

205. Methods of Generation.—Warped surfaces may be generated:

1. By revolving one line about another not in the same plane.
2. By moving a straight line, the **generatrix,** along two lines, **directrices,** either straight, curved, or both, keeping parallel to some plane, called the **Plane Director.**
3. By moving a straight line along three directrices, straight, curved, or both.
4. By moving a straight line along an axis at a constant angle with the axis, following a space curve as a directrix.

Other methods might be invented, but these cover all the cases met in practical work.

206. Tangent Planes.—In Art. 170 we find the general rule for tangent planes to all **Ruled Surfaces,** as follows: **One tangent line and the element at any point determine the tangent plane at that point.**

In general, a tangent plane is only tangent **at one point** of a warped surface. There are certain points, as in conoids and warped cones, where the tangent plane will be tangent all along the element, but such exceptions will be readily recognized.

All planes tangent to warped surfaces (except in the cases just noted) intersect the surface, because they are not tangent all along the element, and, conversely, **any plane that intersects a warped surface in an element will,** in general, **be tangent to the surface at some point.**

207. Double-Ruled Surfaces.—Two surfaces, the **Hyperboloid of Revolution of One Nappe** and the **Hyperbolic Paraboloid,** are called double-ruled surfaces from the fact, as will be shown later, that they are capable of generation by **two different straight lines.** This yields the phenomenon that **through every point in the surface two straight lines** (elements) **can be drawn lying wholly within the surface.** From this fact we have the special rule for tangent planes to these surfaces: **The two elements of a double-ruled surface through any point of the surface determine the tangent plane at that point.**

208. Representation.—Some warped surfaces can be shown **in outline** in orthographic projection with sufficient clarity, and others must be shown by drawing a sufficient number of elements. In practical drafting, where the surface is always limited, the outlines are usually sufficient.

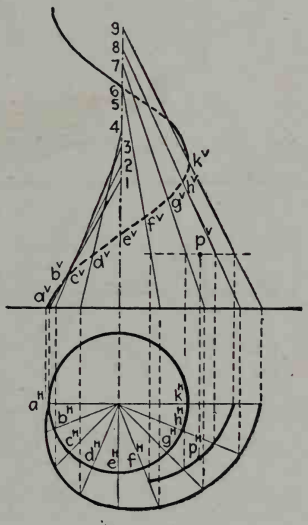
209. Location of Points.—To locate a point on any warped surface, one projection being given, find the piercing point of the **projector** of the point with the surface. This is done by passing a plane through the projector, and finding its intersection with the surface. Where the projector intersects this line will be the location of the point. In many cases it will be easy to draw the element through the point, simplifying the operation.

210. Developments.—Strictly speaking, a warped surface **cannot be developed,** because no two consecutive elements lie in the same plane, and cannot, therefore, be rolled into a plane without "warping" them; that is, destroying the relation between them. However, there are **approximate** developments that give very satisfactory results, almost as good as the development of a cone by triangulation. These warped developments also are obtained by triangulation. A sufficient number of elements are drawn, dividing the surface into minute warped quadrilaterals, in which the amount of warp is so slight as to be negligible. Diagonals are then drawn across each of these quadrilaterals, dividing them into triangles. These triangles are then measured and placed in order in their (approximate)

true size. The resulting surface, obtained by bending back this development, cannot be the exact surface, as shown in the drawing, but will be near enough for practical purposes. In making single-curved surfaces from flat patterns there need be no distortion of the metal, but in making up a warped surface, there will be some distortion.

THE HELICOID

211. The most frequent warped surface encountered in drafting is the **Helicoid.** It is generated by Method 4, Art. 205, by moving a straight line along a helix and its axis, keeping a constant angle with the axis. The helix and its axis are the directrices. The surface is theoretically unlimited, but in practice is usually included between two helices. Any circular cylinder, having its axis coincident with that of the helical directrix, will intersect the helicoid in a **helix.** Any plane perpendicular to the axis will cut the helicoid in a **spiral of Archimedes.** Any plane passed through the axis will intersect the helicoid in an **element.**



Helicoid.
FIG. 149.

Fig. 149 shows a helicoid, with its axial angle 30°. In drawing the helix a certain number of points were laid out, and through these points lines intersecting the axis are drawn, making the given angle with the axis. As the helix is everywhere equally distant from the axis, this is simple, as follows.

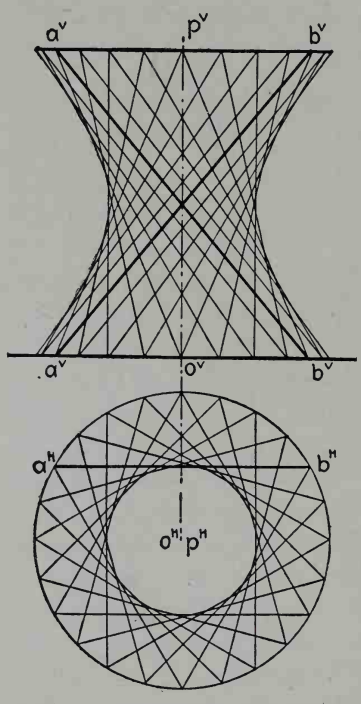
1. Draw one element parallel to V, which will be drawn in its true angle with the axis.

2. As the generatrix is moved, it climbs both axis and helix in exactly the same upward distances. Therefore, from any point on the helix, an element may be drawn to a point on the axis that is the same distance up, measured on the V-projection. Thus in Fig. 149, the point b^v is the same distance above a^v as the point 2 is above 1 on the axis.

The point P is located on the surface by drawing the element

through the point and locating the other projection on the element. If the V-projection of the point is given, pass a plane through the point parallel to H. This will cut the elements in a curve on which the H-projection will be located.

The helicoid is the surface of the V-thread, the square thread, spiral springs, screw conveyors, certain fire-escapes, the twist drill, the spiral runway, helicoidal arch, and numerous other surfaces frequent in machine and structural design.



Hyperboloid of Revolution.

FIG. 150.

Helicoidal Arch.—In the design of skew arches, that is, arches with their axes oblique to one or both of the openings, warped surfaces are often encountered, **cylindroids** or **cow's horns** being the usual surfaces in such arches. The arch called the **helicoidal arch** takes its name not from the character of its surface, as might be supposed, but from the surface of the "coursing joints." The line of the joint is laid out as a straight line on the developed surface of the arch, and, when brought into its actual position, it becomes a helix. The surface of the joint is generated by a line normal to the surface of the arch moving along the helix, radial to the axis of the helix. This surface is therefore identified as a helicoid. Inasmuch as the developed helix is a straight line the work of laying out the blocks is very simple. The seam, being helicoidal, must be laid out with care, to ensure good-fitting blocks.

THE HYPERBOLOID OF REVOLUTION OF ONE NAPPE

212. This peculiar surface is the only warped surface that is **a surface of revolution.** It is generated by Method 1, Art. 205, by revolving a straight line about an axis not in the same plane. Fig. 150 shows this surface represented by its elements. Either projection of all the elements could be inverted, and there would

be no change in the surface. If AB be considered an element, its V-projection might be either of the lines marked $a^v b^v$, and the same surface would be generated by revolving either line about the axis. This makes it a **double-ruled surface,** and a tangent plane is determined at any point by two elements through that point.

213. To Draw the Elements.—All points in the generatrix move in circles, hence there is a vertex to the curve of the outline, a smallest circle, called the **Gorge Circle,** generated by the point nearest the axis. All elements intersect the gorge circle and their H-projections are all tangent to its H-projection. Two limiting planes, **bases,** are always necessary, always taken perpendicular

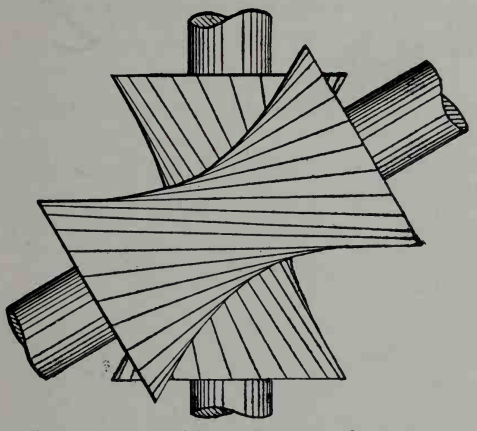


FIG. 151.—Rolling hyperboloids. (Both surfaces are generated by revolving the same straight line about two different axes, and they thus have one element in common, and are tangent to each other along this element.)

to the axis, and usually at equal distances from the gorge circle. Being perpendicular to the axis, these bases are circular. Why? One base is usually in H, and the elements terminate in the bases. Hence, 1. Draw the gorge and base circles. 2. Draw as many elements as desirable, 16 or 24, the H-projections tangent to the gorge circle and terminating in the base circles. 3. Project the terminals of the elements to the V-traces of the respective base planes.

Note.—When the bases are symmetrically placed, there will be only one circle projected on H, as in Fig. 150.

214. To Pass a Plane Tangent at any Point on the Surface.—Draw the two elements through the given point. They will determine the tangent plane.

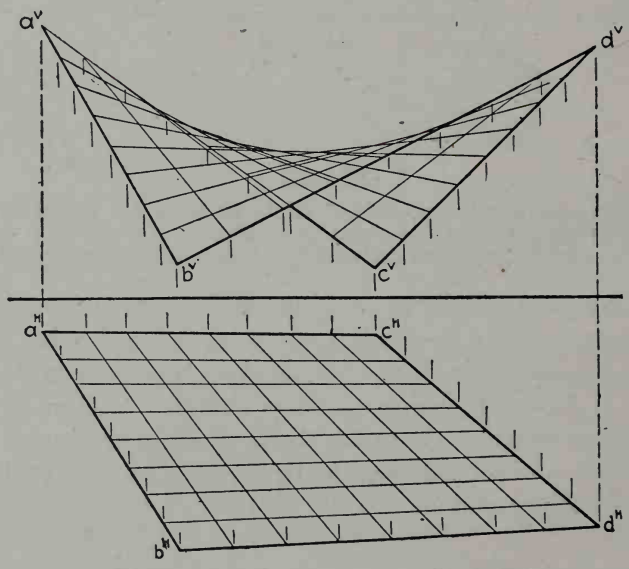
Tangent planes to this surface, as to all surfaces of revolution, are perpendicular to the plane of the axis and the point of tangency, called the **Meridian Plane.**

The **Meridian Section** of this surface is a hyperbola, and the same surface can be generated by revolving a hyperbola about its vertical axis. If proof of this is desired, it can be found in Church and Bartlett, page 150.

The most important function of the hyperboloid is found in the pitch surfaces of skew gears. If one line be successively revolved about two axes, not in its plane, the line will be simultaneously an element of both hyperboloids; that is, the two hyperboloids generated will be tangent. These, if used as the basis of gears, form an ideal transmission medium between shafts not in the same plane.

THE HYPERBOLIC PARABOLOID

215. This surface is generated by moving a straight line along two straight lines, not in the same plane, keeping it always par-



Hyperbolic Paraboloid, Showing Double Ruling

FIG. 152.

allel to a plane director. The elements are all parallel to this plane, but not to each other.

Representation.—If a definite position of the two directrices is given, the elements may be drawn without the assistance of the plane director, by dividing each directrix into the same number of equal parts and connecting the points in order. This is so, because a series of parallel planes will cut any two lines in proportional segments.

216. Second Generation.—If any two elements be taken, as BD and AC (Fig. 152) and divided as were the directrices AB and CD, and joined in order, the surface thus generated would be identical with the first surface. In the two generations, the directrices of the first become elements of the second, and the elements of the first become directrices. This makes it a double-ruled surface, and the rule for tangents is the same as for the hyperboloid.

217. It is named the Hyperbolic Paraboloid because certain planes cut hyperbolas from the surface, others parabolas, while tangent planes cut straight lines. It is chiefly of interest to the mathematician, but it is occasionally met in practical work, and is then usually called a "Warped Quadrilateral."

THE CYLINDROID

218. The Cylindroid is a warped surface with two curved-line directrices and a plane director. It differs from a cylinder in that its elements, while parallel to one plane are not parallel to each other. It is found in practical work in arches and sheet metal work.

THE CONOID

219. The Conoid has one curved-line directrix, one straight-line directrix, and a plane director.

A helicoid, whose elements are at right angles to the axis, may be regarded as a conoid.

THE COW'S HORN

220. This surface has three directrices, two being circles, and the third a straight line. The circles are usually in parallel planes with their centers in a plane perpendicular to these parallel planes. The straight line directrix usually lies in the plane of the centers. The Cow's Horn is frequently encountered in stone arch designs, and occasionally in sheet metal hoods.

THE WARPED CONE

221. This surface, often found in reducing hoods and pipe connections, has three directrices; as follows, two circles or ellipses (or one of each) in planes not parallel, and the line joining their centers, called the axis.

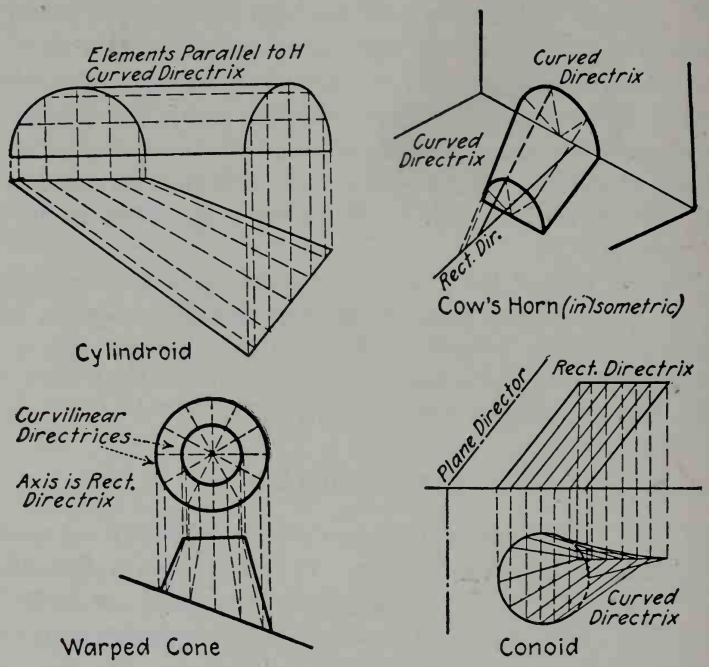


FIG. 153.—Minor warped surfaces and their methods of generation.

222. Other Warped Surfaces.—Besides the foregoing, there are many forms, which have no specific names. They may have three straight directrices, three curved directrices, or combinations of these with plane directors. Whenever they are met, they will present no difficulties to the student, who is familiar with those described.

223. The Development of a Specimen Warped Surface.—Develop the helicoid shown in Fig. 154, for a half turn, between *H* and the helical directrix.

1. Draw the elements A1, B2, etc.
2. Draw diagonals A2, B3, etc., of the quadrilaterals.
3. Measure the true lengths of the various elements and diagonals. The construction on the left shows a simple arrange-

ment for accomplishing this. Lay off the heights of the various points, a′, b′, etc., on a vertical line, and lay off the lengths of the various *H*-projections on a horizontal line, elements on one

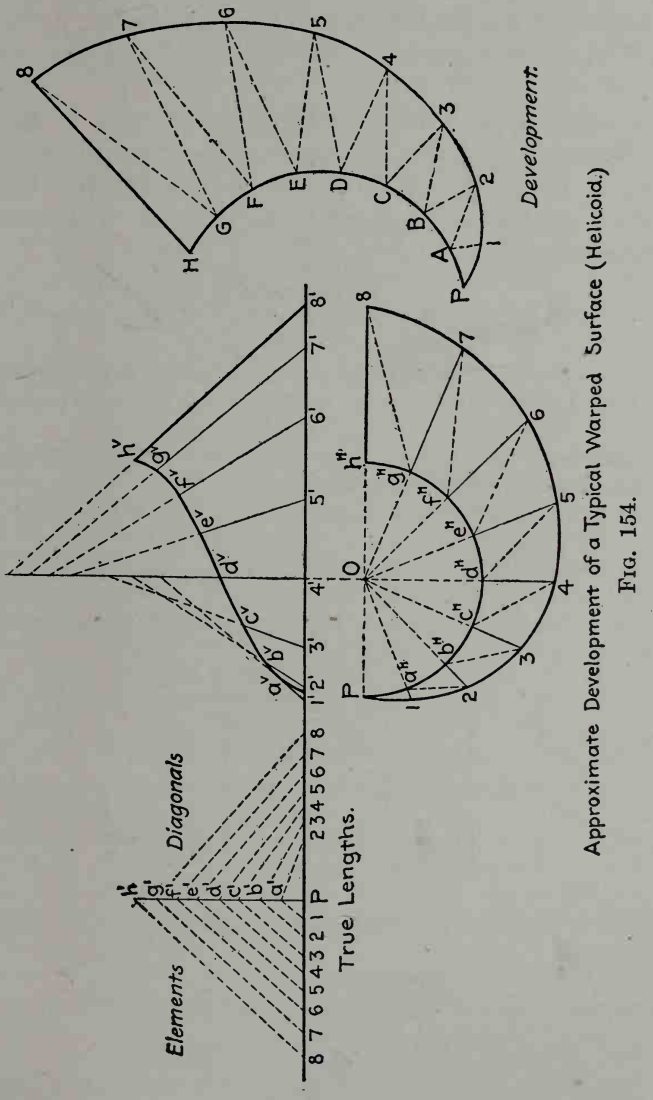


FIG. 154.

side, diagonals on the other. Their true lengths will be the hypothenuses, a′1 or a′2, etc.

4. Rectify the various arcs of the helix and spiral. The arcs

of the helix are equal in length to corresponding arcs of a circle whose radius $= \frac{OP}{\cos^2 \theta}$. (See the graphical solution given in Art. 204.) The lengths of the spiral arcs are obtained by stepping off small chords with the dividers.

5. With these materials, construct the triangles in order, as shown on the right.

This method, with such modifications as may be necessary, is employed in the development of all warped surfaces. It is laborious, but there is no shorter method. The important operation in this, as in all problems relating to surfaces, is to **get the elements.**

224. EXERCISES

Helicoid

Helicoid dimensions: pitch of helix, 1 in., 2 in., 2½ in., 3 in.; diameter of helix, 1 in., 1¼ in., 1½ in. Angle of elements with axis, 30°, 45°, 60°, 75°, 90°.

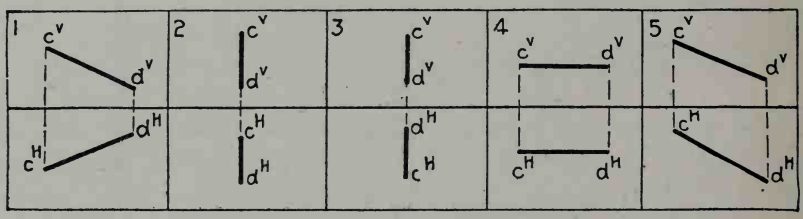
834. **Right-hand helix,** axis A(2, 0 − 1½) B(2 + 3 − 1½), pitch ——, diameter ——, elemental angle ——°.

(*a*) Draw the projections of as much of the helicoid as possible, showing its *H*-trace.

(*b*) Locate E(1¼, x − 2) and F(2½ + 1, y) on the surface.

835. Helicoid in Ex. 834. Draw the traces of the plane tangent at E or F.

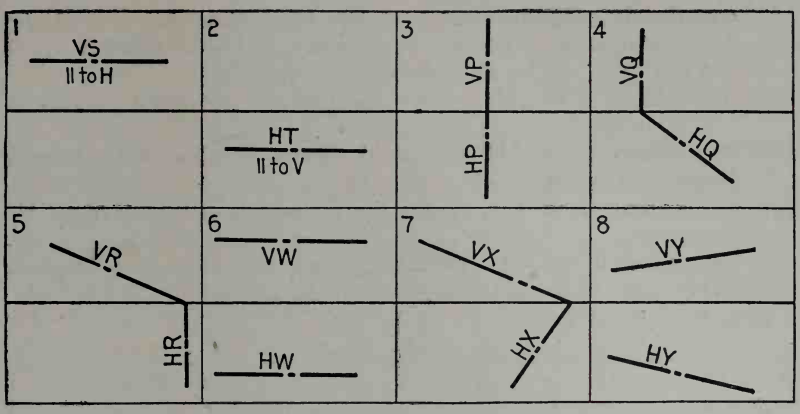
836. Helicoid in Ex. 834. Locate the points in which the surface is pierced by line ——.



Graphic Layout No. 64.—Piercing lines to use for all the given surfaces in these exercises.

Dimensioned Layout for the Same.

(1) A(2, 0 − 1¼) B(4 + 2 − 2).
(2) C(2, 0 − 2) B(4 + 2 − ½).
(3) E(3 + 2 − ½) F(3 + 1 − 2½).
(4) G(3 + 2 − 2) J(3 + ½ − 1).
(5) M(1 + 1 − 1½) N(4 + 1 − 1½).



Graphic Layout No. 65.—Cutting planes for the surfaces in these exercises.

Dimensioned Layout for the Same.

(1) S(+ 1) parallel to *H*.
(2) T(− 1¾) parallel to V.
(3) P(3 + 3) 3(3 − 3).
(4) Q(1 + 3) 1(5 − 2).
(5) R(5 + 1½) (1 − 3).
(6) W(+ 1) ∞ (− 3).
(7) X(1 + 3) 5½(4 − 3).

837. Helicoid in Ex. 834. Draw the projections of the curve cut from the surface by plane No. ——.

838. Develop the surface of Helicoid, Ex. 834, through a revolution of 180°. (**Note.**—If the elemental angle is more than 45°, the surface must be limited by a cylinder of 4-in. diameter.)

839. **Left-hand helix;** axis M(4, 0 − 1¼) N(4 + 3 − 1¼), pitch ——, diameter ——, elemental angle ——°.
(*a*) Draw the projections of as much of the helicoid as possible, showing *H*-trace.
(*b*) Locate E(4½ + ½, x) and K(3, y − 2) on the surface.

840. Helicoid in Ex. 839. Draw the traces of the plane tangent at —— (E or K).

841. Helicoid in Ex. 839. Locate the points in which the surface is pierced by the line ——.

842. Helicoid in Ex. 839. Draw the projections of the curve cut from the surface by the plane ——.

843. Helicoid in Ex. 839. Develop its surface through a 180° generation. (**Note.**—If the elemental angle is more than 45°, the surface should be limited by a cylinder of 4-in. diameter.)

Hyperboloid of Revolution of One Nappe

Data for Hyperboloid: Axis, A(3, 0 − 1½) B(3 + 3 − 1½). Gorge Circle Diameter: ½ in., ¾ in., 1 in., 1¼ in., 1½ in. Inclination of Elements to *H:* 30°, 37½°, 45°, 52½°, 60°.

844. **Hyperboloid**; gorge diameter ——, elemental inclination ——°. Draw the projections of its surface (24 elements), and locate C(2½ + ¾, x) and D(3½, y − 2) on its surface.

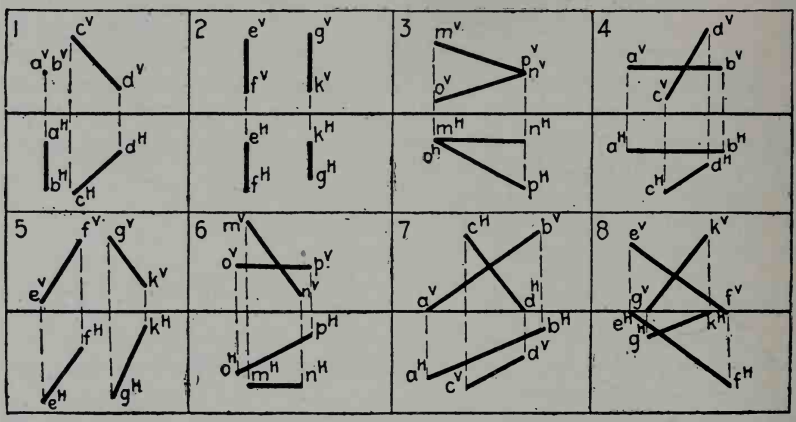
845. Hyperboloid in Ex. 844. Draw the traces of the plane tangent at —— (C or D).

846. Hyperboloid in Ex. 844. Locate the points in which the surface is pierced by the line ——.

847. Hyperboloid in Ex. 844. Draw the projections of the curve cut by plane ——.

848. Hyperboloid in Ex. 844. Develop the surface of the truncated portion between the lower base and the section made by plane——.

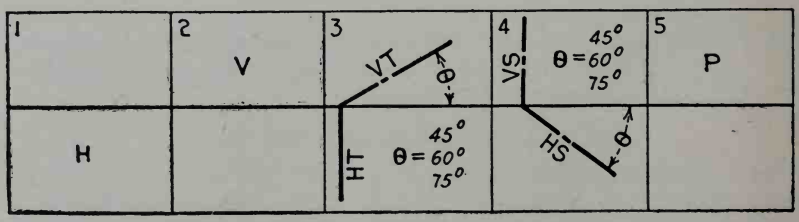
Hyperbolic Paraboloid.



Graphic Layout No. 66.—Directrices for paraboloids.

Dimensioned Layout for Same.

(1) A(1 + 2 − 2) B(4, 0 − 1) and C(3, 0 − 3) D(4½ + 2 − 1).
(2) E(4 + 1 − 1) F(4 + 1 − 3) and G(1 − 1 − 1) K(3 − 3 − 3).
(3) M(1 + 2 − 1) N(3 + 2 − 1) and O(2 + 3 − 3) P(3 + 1 − 2).
(4) O(1½, 0 − 2) P(1½ + 1½, 0) and X(3½ + 2, 0) Y(3½, 0 − 1).
(5) A(1½, 0 − 2) B(1½ + 2, 0) and C(4, 0, 0) D(4 + 1½ − 2).
(6) E(1½ + 2 − ½) F(4 + 2 − 2) and G(4 + 2, 0) K(2, 0 − 1).
(7) M(1½ + 3 − 1) N(1½, 0 − 1) and O(4 + 2, 0) P(4, 0 − 2½).
(8) X(1 + 2½ − ½) Y(3 + 1 − 2) and A(3½ + ½ − ½) B(5 + 2 − 2½)



Graphic Layout No. 67.—Plane directors.

849. **Hyperbolic Paraboloid.**—Directrices No. ——.
(*a*) Draw the projections of the surface within the limits of the given directrices, without using a plane director.
(*b*) Draw the traces of its plane director.
(*c*) Locate the projections of the points C(2, x − 1½) and D(3 + 1, y) on the surface.

850. Hyperbolic Paraboloid in Ex. 849. Draw the traces of a plane tangent to the surface at —— (C or D).

851. Hyperbolic Paraboloid in Ex. 849. Locate the point or points in which it is pierced by the line ——.

852. Hyperbolic Paraboloid in Ex. 849. Draw the projections of the curve cut from the surface by plane ——.

853. Hyperbolic Paraboloid in Ex. 849. Develop the surface included between the given directrices.

854. Hyperbolic Paraboloid in Ex. 852. Develop the surface included between the directrix —— and the curve cut by the given plane.

855. **Hyperbolic Paraboloid.**—Directrices No ——, Plane Director ——.
(*a*) Draw eight elements of the surface.
(*b*) Locate the projections of the points E(2 + 1, x) and F(3, y − 1¼) on the surface.

856. Hyperbolic Paraboloid in Ex. 855. Draw the traces of a plane tangent to it at —— (E or F).

857. Hyperbolic Paraboloid in Ex. 855. Find the point or points in which it is pierced by line ——.

858. Hyperbolic Paraboloid in Ex. 855. Draw the projections of the curve cut by the plane ——.

859. Hyperbolic Paraboloid in Ex. 855. Develop the surface included between the directrices.

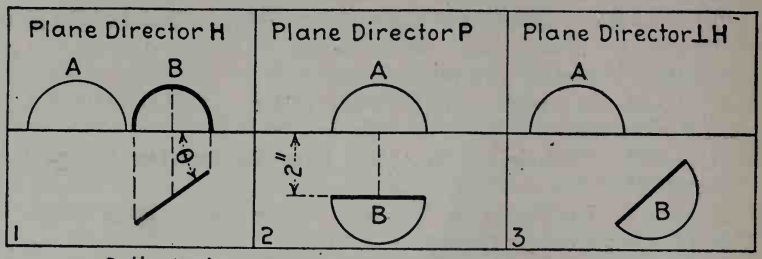
860. Hyperbolic Paraboloid in Ex. 855. Develop the surface included between directrix —— and the cutting plane ——.

MINOR WARPED SURFACES

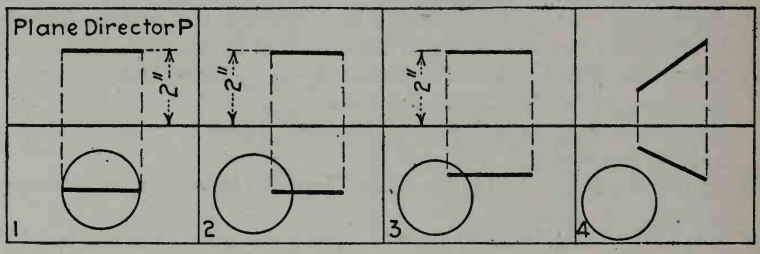
Conoids, Directrices, one a Straight Line, as given, and either a Circle in *H*, of 1 in., 1½ in., 2 in., or 2½ in. diam., or an Ellipse in *H*, 1 in. × 1½ in., 1 in. × 2 in., or 1½ in. × 2 in. Plane directors for (2), (3), or (4), to be chosen from (3) in Graphic Layout No. 67.

Cow's Horn. Circular Directrices, 1 in., 1¼ in., 1½ in., 1¾ in., 2 in., 2½ in., 3 in., 4 in. diams. (Any combination may be used.) Straight Directrix, CD, lies in *H*.

Warped Cone. Diams. A, 1¼ in., 1½ in., 1¾ in., 2 in., 2½ in. Diams. B, ½ in., ¾ in., 1 in., 1⅛ in. $\theta = 22\frac{1}{2}°, 30°, 37\frac{1}{2}°, 45°$.



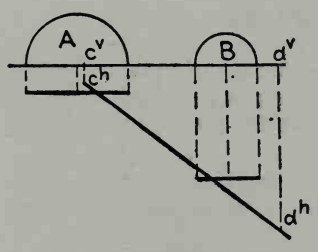
Cylindroids; *Directrices are Equal Semi-Circles*



'Conoids; *Rectilinear Directrix Given*

Curved Directrix: Circle in H, *Diameter* $1\frac{1}{2}$", $1\frac{3}{4}$", 2", $2\frac{1}{2}$" *or*
Ellipse in H, $1" \times 1\frac{1}{2}$", $1" \times 2$", $1\frac{1}{2}" \times 2"$

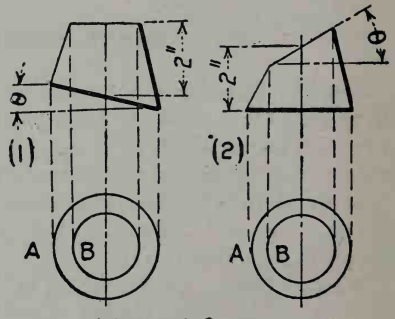
Plane Director for 2, 3 or 4, Selected from Graphic Layout #67, Plane #3



Cow's Horn

Circular Directrices
Diams, 1", $1\frac{1}{2}$", $1\frac{3}{4}$", 2", $2\frac{1}{2}$", 3", 4"

Straight Directrix, CD *in* H



Warped Cone

Diams, A - $1\frac{1}{4}$", $1\frac{1}{2}$", $1\frac{3}{4}$", 2, $2\frac{1}{2}$"
" B - $\frac{1}{2}$", $\frac{3}{4}$", 1", $1\frac{1}{8}$"
$\theta = 22\frac{1}{2}°, 30°, 37\frac{1}{2}°, 45°$

Graphic Layout No. 68.—Minor warped surfaces.

861. **Cylindroid No. ——**; diameters A and B, —— (2 in., 2½ in., 3 in., 4 in.), θ = —— (30°, 45°, 60°, 75°, 90°).
(*a*) Draw its projections (12 elements).
(*b*) Locate A(2 + ¾, x) and B(3½, y − 1½) on the surface.
(*c*) Draw the traces of a plane tangent at —— (A or B).
(*d*) Locate the points in which it is pierced by the line ——.
(*e*) Draw the projections of the line cut from its surface by the plane ——.
(*f*) Develop the surface included between the given bases.

862. **Conoid No. ——**; diameter of circle in *H* ——, plane director ——. Same requirements as for Ex. 861.

863. **Conoid No. ——**; elliptical base in *H* ——, plane director ——. Same requirements as for Ex. 861.

864. **Cow's Horn**; diameter A ——, diameter B ——. Same requirements as for Ex. 861.

865. **Warped Cone, No. ——**; diameter A ——, diameter B ——, θ = ——. Same requirements as for Ex. 861.

PRACTICAL EXERCISES

In all the development exercises use two spaces.

866. Draw the projections of a helical spring made of ¼-in. square steel, outside diameter 3 in., pitch 1¼ in., 2 coils.

867. Draw the projections of a left-handed square thread, 2 in. diameter, ¾-in. pitch (single thread), and 3 in. long.

868. Draw the projections of a helical spring made of ¼-in. wire, outside diameter 2½ in., pitch 1 in., and 3 coils.

869. Draw the projections of a V-thread, 2½ in. diameter, ½-in. pitch, 3 in. long. Show a section made by a plane perpendicular to the axis.

870. Draw the projections of a twist drill, 1 in. diameter, 5½ in. long, made from a flat bar ¼ in. thick. Pitch of helix 3½ in. Sharpen point at 30°. See Fig. 155. Do not use a ground line.

871. Design a screw conveyor to scale, 12-in. diameter, capable of delivering 6000 cu. ft. of grain per hour, making 100 revolutions per minute. Make the diameter of the shaft 2 in. See Fig. 155.

872. Draw a conical-helical spring, ¼-in. round wire, large diameter 2½ in., small diameter 1¼ in., 3 turns, 1-in. pitch.

873. Draw the projections of the pitch surfaces of two hyperboloidal gears, operating at 45°, having a gorge circle ratio of 5 to 3.

Note.—Draw one axis perpendicular to *H*, and one inclined 45° to *H*. Take a line for the common element, at relative distances of 5 and 3 from the axes, intersecting their common perpendicular.

874. Show the surface of a cow's horn arch, which has one opening a 12-ft. semicircle at A(2, 0, 0) and the other an 8-ft. semicircle at B(3, 0 − 1). Draw the projections of a 2-ft. square stone on the interior of the arch, its center being at C(2¾ + ¼, x). Scale ¼ in. = 1 ft.

875. Design a conoidal top piece for the hip roof, as shown in Fig. 155. Make the three roofs 45° to *H*, and make the top piece a right conoid of suitable proportions.

876. Develop the surface of the automobile fender shown in Fig. 155.
877. Develop the surface of the reducing hood (warped cone), Fig. 155.
878. Develop the surface of Edison horn for talking machine (cow's horn), Fig. 155.

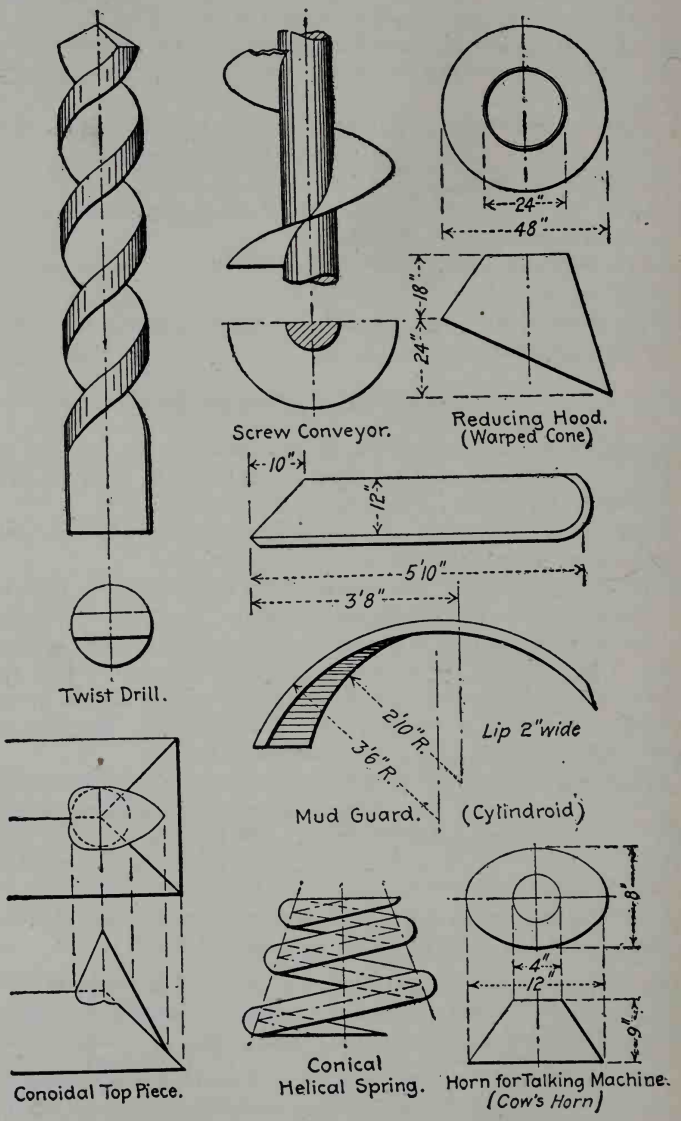


FIG. 155.

879. What is the surface of concrete form B, Fig. 95, Art. 121? Develop the surface.
880. What is the surface of concrete form A, Fig. 95, Art. 121? Develop the surface.

CHAPTER VIII

DOUBLE-CURVED SURFACES AND SURFACES OF REVOLUTION

DOUBLE-CURVED SURFACES

225. A double-curved surface is generated by a curve following a curve in such a way as to generate a surface that is neither single-curved or warped. A double-curved surface contains **no straight lines.** It may be either double convex, like a sphere or ellipsoid, or it may be concavo-convex, like a torus, or like the turned surfaces of numberless articles.

As all important double-curved surfaces are also **surfaces of revolution,** they will be considered under that head.

SURFACES OF REVOLUTION

226. Any line revolving about an axis generates a surface of revolution.

Single-Curved Surfaces of Revolution.—If the generatrix is a straight line and in the plane of the axis, the surface will be single-curved. The **Cylinder of Revolution** and the **Cone of Revolution** are the only examples.

Warped Surfaces of Revolution.—The only example in this class is the **Hyperboloid of Revolution of One Nappe.**

Double-Curved Surfaces of Revolution.—Any plane curve revolving about an axis generates this surface, except in the case of the Hyperbola.

Sphere.—Generated by a circle revolving about one of its diameters.

Ellipsoids. 1. Prolate Spheroid.—Generated by revolving an ellipse about its major axis. Its shape resembles a foot ball.

2. Oblate Spheroid.—Generated by revolving an ellipse about its minor axis. Its shape resembles a door knob.

Paraboloids.—Generated by revolving a parabola about either axis. When revolved about the transverse axis, it generates the surface used in search-light reflectors. Why?

Torus.—Generated by revolving a circle about an axis that is not a diameter, usually completely outside the circle. In this case it is called a **ring,** or **annular torus.** As a matter of fact, a torus is often generated by other curves than the circle, or by a combination of curves. The architectural detail is of this character. In this work the word torus will be used to mean the annular torus.

Hyperboloid of Revolution of Two Nappes.—Generated by revolving a hyperbola about its transverse axis. It is not often encountered in drafting.

Many surfaces generated by combination curves will be found, some with names and some without.

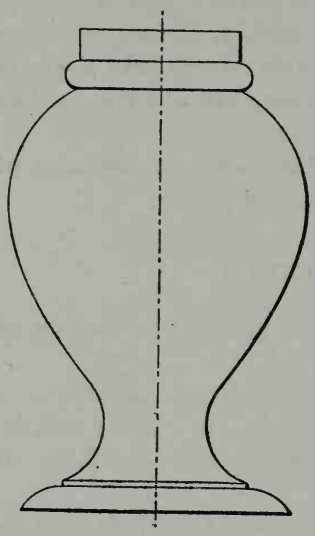


Fig. 156.

227. Meridian and Right Sections.—Every plane perpendicular to the axis cuts a surface of revolution in a circle. Hence, we may even conceive a surface to be generated by a circle moving in the direction of the axis, and changing its diameter according to some law. Any surface turned in a lathe is generated in this way. (See Fig. 156.)

Every plane through the axis, called a **Meridian Plane,** cuts the surface in a **Meridian Line,** which has the identical form of the line used to generate the surface.

Any two surfaces of revolution having a common axis will intersect in a circle or circles, perpendicular to the axis. When the axis is parallel to the plane of projection these intersections appear as straight lines. (See Fig. 156.)

228. Representation.—Two views of double-curved surfaces of revolution are all that are necessary, and usually one is sufficient, and that in outline. A center line on V, perpendicular to GL, is almost always drawn, and the meridian section parallel to V constitutes the V-projection. The H-projection is merely the circle of a diameter equal to the largest dimension of the generatrix, with as many other circles, visible or invisible, as may be necessary to show the various intersections of the surface. Fig. 157 shows a torus, which will fairly represent all such surfaces.

Any view of a sphere is a circle, so both projections of any sphere will be circles of equal diameter.

229. Problem 81.—To locate a point on any Surface of Revolution.

Let it be required to locate O on the torus in Fig. 157, assuming either projection.

Analysis.—Any plane parallel to *H* will cut the torus in a circle.

Construction, assuming the *V*-projection, o^V.

1. Through o^V pass a plane parallel to *H*. It will cut a circle of a radius equal to OP ($o^V{}_1 p^V$).

2. Draw the *H*-projection of that circle, with its center at p^H.

3. Project o^V to o^H on the circle.

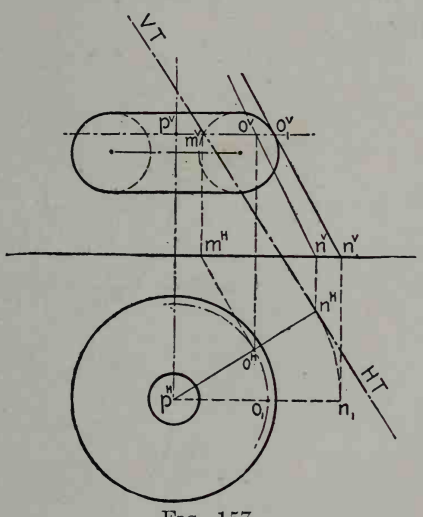


Fig. 157.

Note.—As o^V could project to two different points on the circle, it is necessary to specify "front surface" or "rear surface" to make it exact. If O were nearer the center there would be four possible locations, two on the inside surface.

Construction, assuming the *H*-projection, o^H.

1. Through o^H draw an arc of a circle, its radius equal to $o^H p^H$.

2. Project o_1 (where the arc crosses the meridian that is parallel to *V*) to the given meridian curve, at $o^V{}_1$.

3. Draw the trace of the horizontal plane through $o^V{}_1$.

4. Project o^H to o^V on this trace.

Note.—Two points are possible in this case, so "upper surface" and "lower surface" must be specified.

230. Tangent Planes.—As in other surfaces, a tangent plane to a double-curved surface is the locus of all tangent lines at any given point, each tangent to one of the infinite number of curves of the surface passing through that point.

The tangent plane to any surface of revolution is perpendicular to the meridian plane containing the point of tangency.

231. Problem 82.—To pass a plane tangent to a sphere at a point on the surface.

Analysis.—Pass a plane through the point perpendicular to the radius.

Let the student make the construction.

232. Problem 83.—To pass a plane tangent to any double-curved surface of revolution at a given point on the surface.

Let the given point be O on the surface of the torus, Fig. 157.

Analysis.—1. Draw a tangent to the right section through the point.

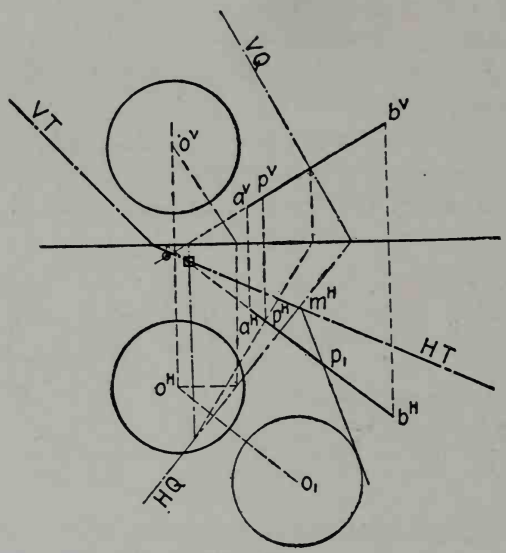


FIG. 158.

2. Draw a tangent to the meridian section at the point.

3. These two tangents will determine the plane.

Construction.—1. Draw the tangent OM to the circle through O, and locate its *V*-piercing point M.

2. Revolve O to $o_1o_1^V$, so that the meridian plane will be parallel to *V*.

3. Draw the tangent $o_1^V n_1^V$ to the meridian curve, in revolved position, and locate its *H*-piercing point N.

4. Counter-revolve N to its original position $n^V n^H$, into the trace of the meridian plane through O.

5. The two tangents will now determine the tangent plane, HT being parallel to $o^H m^H$, and VT being drawn through m^V.

233. Problem 84.—To pass a plane tangent to a sphere through a given straight line.

Analysis.—Regard the sphere as being enveloped by a cylinder **parallel to the given line,** and pass the plane tangent to the cylinder. The plane will be tangent also to the sphere.

Construction.—Let AB (Fig. 158) be the given line, and O be the center of the sphere.

1. Pass a plane, Q, through O perpendicular to AB.

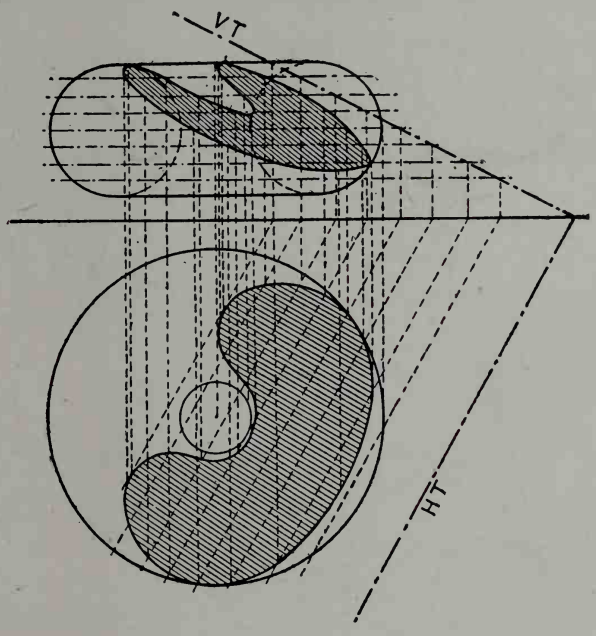


FIG. 159.

2. Find the point, P, in which AB pierces Q.

3. Revolve P and O about the *H*-trace of Q into *H* at p_1 and o_1, and draw a circle at o_1 of the same radius as that of the sphere.

4. Draw a tangent from p_1 to this circle.

5. Find its *H*-piercing point m^H (where it crosses HQ).

6. Draw the traces of T, the required tangent plane, through m^H and the piercing points of AB.

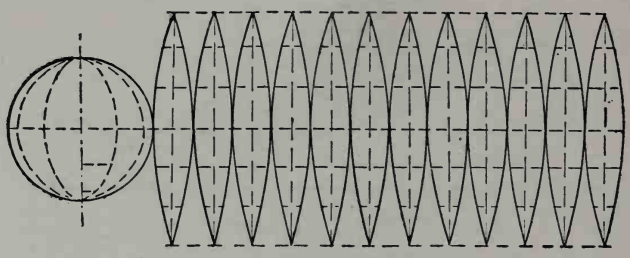
Note 1.—This problem usually has two solutions, but if the line $\widetilde{AB}$ pierces the sphere, the problem is impossible.

Note 2.—The projections of the cylinder were not drawn, as it is merely an imaginary assistance to the analysis. It is not even necessary to imagine

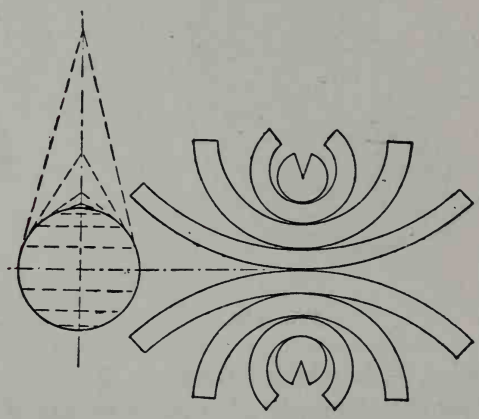
it, as the normal plane, Q, will have to pass through the center of the sphere, although it does not have to be perpendicular to AB. This is convenient, however, and would be necessary if the cylinder idea were carried out.

234. Problem 85.—To obtain the intersection of a plane with any surface of revolution.

Analysis.—The simplest curve that can be cut from a surface of revolution is the circle, which is cut by a plane perpendicular to the axis. Therefore a series of planes perpendicular to the axis will cut straight lines from the plane and circles from the



Meridian Method.



Zone Method.

FIG. 160.

given surface. The curve of intersection will pass through the various intersections of the straight lines and circles. Fig. 159 shows both projections of the line of intersection of the plane T with a torus.

The vertices (high and low points) of the curve may be exactly located by projecting the torus and plane on a plane perpendicular to the *H*-trace of the given plane. The vertices always lie in the meridian plane which is perpendicular to the given plane.

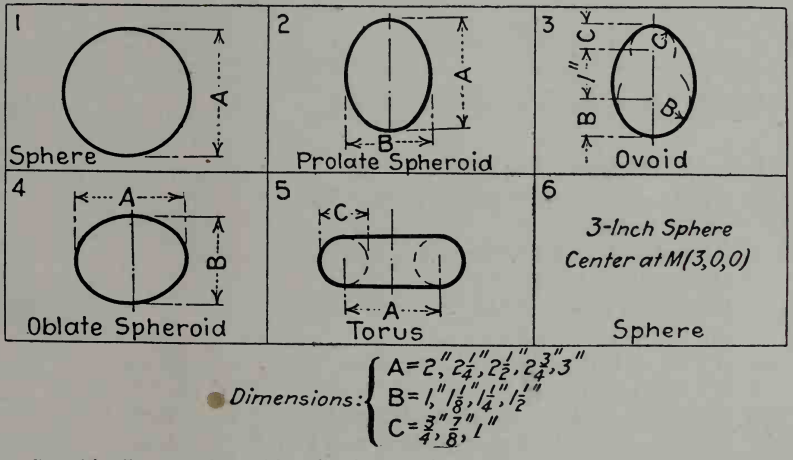
Note 1.—Any plane will cut a circle from a sphere, because any diameter of the sphere may be considered its axis.

Note 2.—The foregoing method is often useful in obtaining the intersections of planes with **ruled** surfaces of revolution, particularly the hyperboloid.

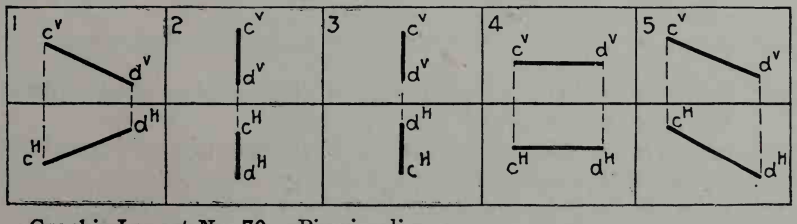
235. Development of Double-Curved Surfaces of Revolution.—Development in the strict sense is impossible, because there are no straight lines. However, in ornamental sheet metal work and the manufacture of globes, approximate surfaces are made from flat sheets.

This is done by two methods, the **"Meridian Method,"** or **"Gore Method,"** in which strips are cut along the meridian lines, and the **"Zone Method"** in which belts of the surface, zones, are regarded as conical surfaces. Both methods are shown in Fig. 160 and should require no explanation.

236. **EXERCISES**



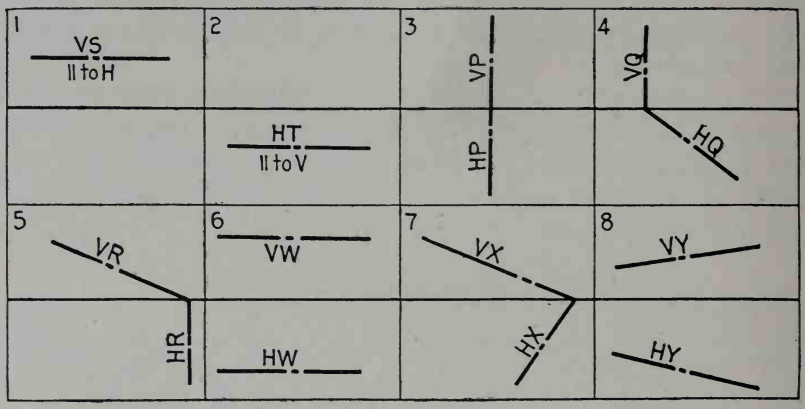
Graphic Layout No. 69.—Surfaces of revolution. Axis O(3, 0 − 1½) P(3 + 3 − 1½), all surfaces drawn ½ in. from *H*.



Graphic Layout No. 70.—Piercing lines.

Dimensioned Layout for Piercing Lines.

(1) A(2, 0 − 1¼) B(4 + 2 − 2).
(2) C(2, 0 − 2) D(4 + 2 − ½).
(3) E(3½ + 2 − ½) F(3½ + 1 − 2½).
(4) G(3¼ + 2 − 2) K(3¼ + ½ − 1).
(5) M(1 + 1 − 1½) N(4 + 1 − 1½).
(6) E(1 + 1 − ½) F(4 + 1 − 2½).



Graphic Layout No. 71.—Cutting planes.

Dimensioned Layout for Cutting Planes.

(1) S(1 + 1) (5 + 2) (1 − 1½) (5 − 3).
(2) T(− 1¾) ‖ to *V*.
(3) P(3½ + 3) 3½(3½ − 3).
(4) Q(1 + 3) 1(5 − 3).
(5) R(5 + 2) 1(1 − 3).
(6) W(+ 1) ∞ (− 3).
(7) X(+ 1½) ∞ (+ 3).
(8) Y(2+ 3) 6(5 − 3).

881. **Sphere** ——; A ——. Draw its projections, and locate C(2½ +1, x) on —— (front or rear) surface, and D(3½, y − 1) on —— (upper or lower) surface.
882. Sphere in Ex. 881. Find the points in which it is pierced by line ——.
883. Sphere in Ex. 881. Draw the projections and true size of the section made by plane ——.
884. Sphere in Ex. 881. Draw the traces of the plane tangent at —— (C or D).
885. Sphere in Ex. 881. Draw the traces of a tangent plane which contains the line X (3 + 3 − 1½) Y(5 + 1 − 2).
886. Sphere in Ex. 881. Draw the traces of a tangent plane which contains the line M(4 + 2 − 1) N(5 + 1 − 2).
887. Sphere in Ex. 881. Develop the surface by the Zone Method.
888. Sphere in Ex. 881. Develop the surface by the Meridian Method.

889. **Prolate Spheroid**; A ——, B ——. Draw its projections, and locate C(2⅝, x − 1) on the —— (upper or lower) surface, and D(3¼ + 1, y) on the —— (front or rear) surface.
890. Spheroid in Ex. 888. Find the points in which it is pierced by line ——.
891. Spheroid in Ex. 888. Draw the projection and true size of the curve cut by plane ——.
892. Spheroid in Ex. 888. Draw the traces of the tangent plane at —— (C or D).
893. Spheroid in Ex. 888. Develop the surface by the Zone Method.
894. Spheroid in Ex. 888. Develop the surface by the Meridian Method.
895. Spheroid in Ex. 888. Draw the traces of a tangent plane which contains M(3 + 3 − 1½) O(4 + 1 − 2).
896. Spheroid in Ex. 888. Draw the traces of the tangent plane which contains X(4 + 2 − 1) Y(5 + 1 − 2).
897. **Oblate Spheroid**; A ——, B ——. Draw its projections, and locate C(2, x − 1) on the —— (upper or lower) surface, and D(3¾ + ¾, y) on the —— (front or rear) surface.
899. Spheroid in Ex. 897. Draw the traces of the plane tangent at —— (C or D).
900. Spheroid in Ex. 897. Locate the points in which it is pierced by the line ——.
901. Spheroid in Ex. 897. Draw the projections and true size of the curve cut by the plane ——.
902. Spheroid in Ex. 897. Develop its surface by the Zone Method.
903. Spheroid in Ex. 897. Develop its surface by the Meridian Method.
904. **Torus**; A ——, C ——. Draw its projections, and locate C(2, x − 1) on the —— (upper or lower) and D(3½ + ⅞, y) on the —— (front or rear) surface.
905. Torus in Ex. 904. Draw the traces of the plane tangent at —— (C or D).
906. Torus in Ex. 904. Locate the points in which it is pierced by the line ——.
907. Torus in Ex. 904. Draw the projections and true size of the curve cut by the plane ——.
908. Torus in Ex. 904. Develop the fragment of the surface —— (below or above) the cutting plane —— (by Zone Method).
909. Torus in Ex. 904. Develop the fragment of the surface —— (below or above) the cutting plane —— (by Meridian Method).
910. **Ovoid**; B ——, C ——. Draw the projections of the surface, and locate C(2 + 1, x) on the —— (front or rear) and D(3¾, y − 1) on the —— (upper or lower) surface.
911. Ovoid in Ex. 910. Draw the traces of the plane tangent at —— (C or D).
912. Ovoid in Ex. 910. Draw the traces of a tangent plane which contains P(3 + 3 − 1½) X(5, 0 − 2).
913. Ovoid in Ex. 911. Draw the traces of a tangent plane which contains Y(3 + 2, 0) Z(5, 0 − 1¾).

914. Ovoid in Ex. 911. Locate the points in which the surface is pierced by the line ——.

915. Ovoid in Ex. 911. Draw the projections and true size of the curve cut by the plane ——.

916. Ovoid in Ex. 911. Develop the fragment of its surface —— (above or below) the cutting plane ——, by —— (Zone or Meridian) Method.

917. Find the plane tangent to three spheres, all resting on H, whose respective centers are A(2 + 1 − 1), B(3 + ½ − 1½), and C(3½ + ⅝ − ½).

918. Develop the surface of the ornamental sheet metal surface of revolution, shown in Fig. 161. Any convenient scale.

Note.—One strip, or meridian, will be sufficient.

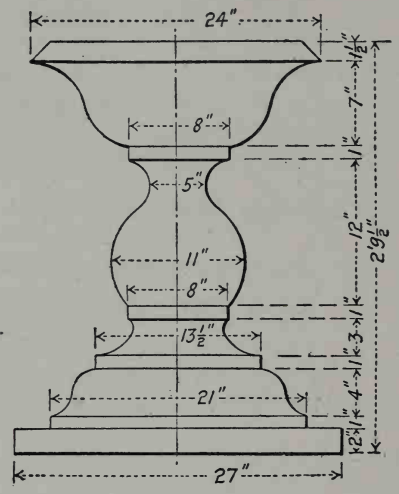


FIG. 161.

CHAPTER IX

INTERSECTIONS AND DEVELOPMENTS OF ALL SURFACES

237. Foreword.—In order to determine the line (or lines) of intersection between two curved surfaces, it is necessary to find a number of lines in each surface that will intersect those of the other. Thus a number of points, common to both surfaces, will be obtained.

To obtain these intersecting lines it is necessary to pass a series of auxiliary surfaces through the given surfaces. It is evident that the most desirable lines to be cut from the surfaces are **straight lines** and **circles,** wherever they are possible. Since **planes** can cut straight lines or circles from most surfaces, it is evident then that planes are chiefly used for this auxiliary work. The only other surface used for this work, in the writer's experience, is that of the **sphere,** which will cut circles from any **surface of revolution,** whose axis passes through the center of the sphere. It is conceivable that other surfaces (a cylinder or cone, for example) might be employed, but it is doubtful if it is ever necessary, or even desirable. Hence, the following rule will serve for general purposes:

238. Rule.—To obtain the line of intersection between two curved surfaces, pass a series of planes through the two surfaces, so that their intersections with the given surfaces will be the simplest possible lines. The line determined by the intersections of these lines, taken in proper order, will be the required line of intersection.

Important.—Since lines that do not lie on the **same surface** cannot intersect, it is obvious that no lines should be laid out on either of the intersecting surfaces until the cutting planes are drawn. Many students make the error of drawing elements of each surface, and expecting them to yield the line of intersection, when it is more than probable that very few of the elements thus drawn will intersect. **Follow the Rule.**

239. The Simplest Lines.—It has been noted that the simplest lines are the straight line and the circle. A straight line is cut

from a cone by passing a plane through the apex, and from a cylinder by passing a plane parallel to the axis. In the case of a cone intersecting a cylinder, then, a natural procedure would be to pass a series of planes through the apex of the cone and parallel to the axis of the cylinder. This is one example. The accompanying table may be of use in deciding what sort of plane to use in the various contingencies that arise.

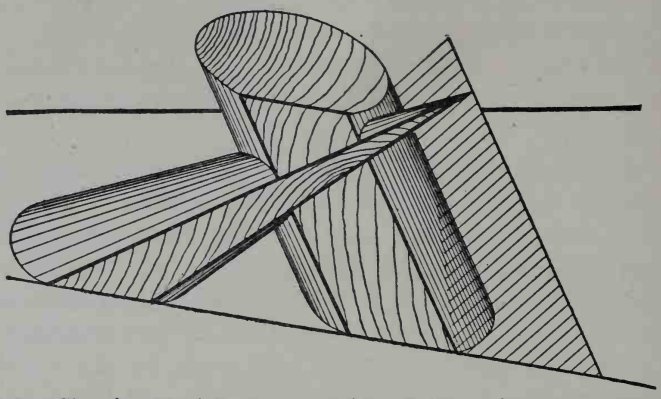


FIG. 162.—Showing straight lines cut from intersecting cone and cylinder by passing a plane through the cone apex parallel to the cylinder axis.

Surface	*Intersection*	*Kind of Plane*
Cone	Straight line	Through the apex.
Cone	Circle	Parallel to circular base.
Cylinder	Straight line	Parallel to the axis.
Cylinder	Circle	Parallel to a circular base.
Convolute	Straight line	Tangent to helical cylinder.
Convolute	Involute	Perpendicular to the axis.
Helicoid	Straight line	Through the axis.
Warped surface	Straight line	According to its generation.
Surface of revolution	Circle	Perpendicular to the axis.
Surface of revolution	Meridian curve	Through the axis.

It may be noted that planes cutting straight lines from warped surfaces are seldom easy to use because of the variety of their directions. There are cases of surfaces having a plane director where it is convenient to get straight line intersections.

Frequently it is not possible to use the planes in the foregoing list for **both** of the intersecting surfaces, but it is always possible to use them for one. In such cases the second surface may be cut in somewhat complicated curves. It is such cases as these that make it impossible to lay down rules for all cases.

The draftsman must use his judgment and instinct for the right auxiliary surfaces.

CONCENTRIC SPHERES METHOD

240. When two surfaces of revolution have intersecting axes the easiest method of obtaining their intersection is to use **concentric spheres.** By employing a sphere, as an auxiliary surface, with its center at the intersection of the axes, circles are cut from the two surfaces, because the intersections of two surfaces of revolution having a common axis will be circles. A typical construction will be shown in Problem 89.

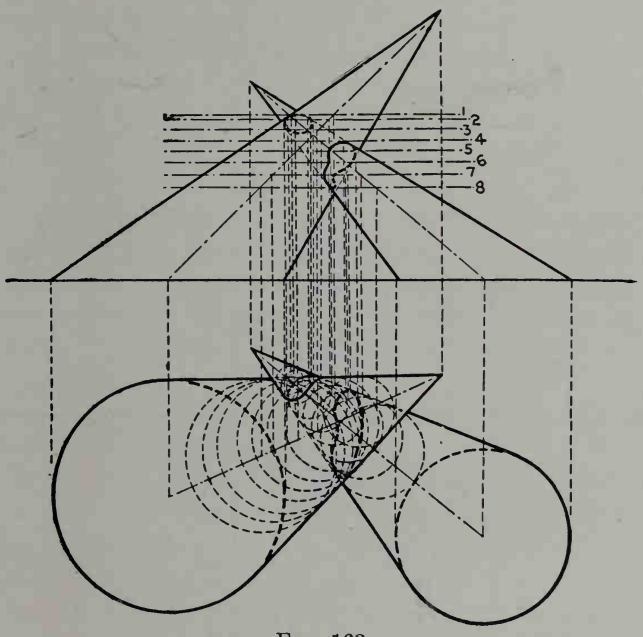


FIG. 163.

PROBLEMS IN INTERSECTIONS

A few representative problems of the many that might be given are selected for illustration.

241. Problem 86.—To find the intersection between two cones. Even in so specific a problem as this, the position and character of the two cones have much to do with the choice of auxiliary planes to be used. We give two representative cases, and both could be successfully worked by using another system of auxiliary planes.

First Case.—When the cones are oblique and their bases are circles in the same plane.

Analysis.—Pass a series of planes parallel to the bases, intersecting the cones in circles. Study Fig. 163 for the solution and construction. Let the student also work this by passing planes through the apices of the cones. The advantage of the latter method is that it gives definitely the outside elements of the cones that are involved in the intersection.

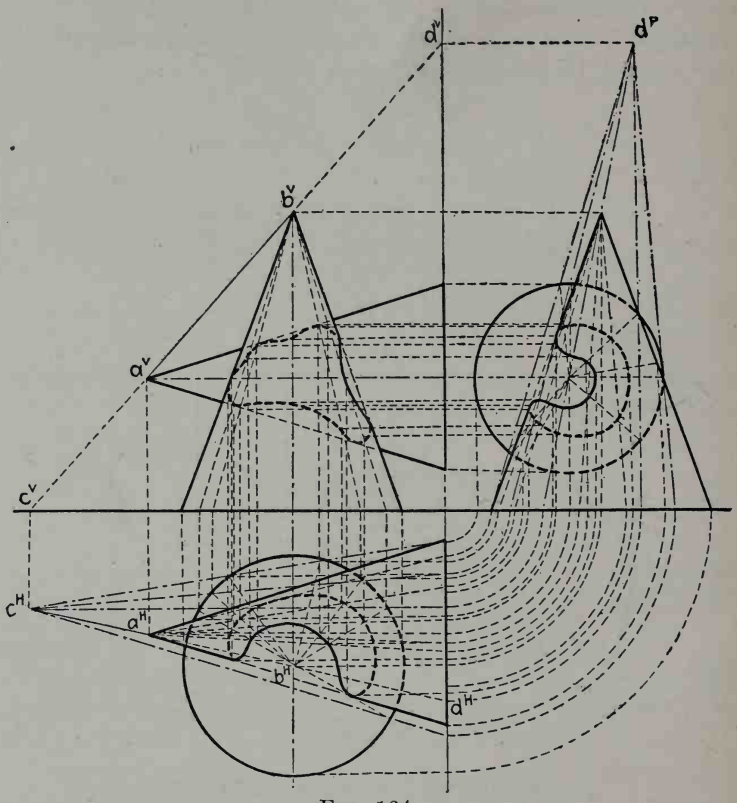


FIG. 164.

242. Second Case.—When the cones are right circular, and their bases are in different planes.

Analysis.—1. Pass a series of planes through the apices of the cones, cutting elements from each.

2. Connect the intersections of the elements in order.

Construction.—To find the intersection between the two right circular cones, whose apices are A and B, and whose bases are in *P* and *H* respectively (Fig. 164).

1. Connect A and B with a line. This line will be contained in all the planes passing through the apices, and its piercing points will lie in all the respective traces of these planes.

2. Find the *H*-piercing point, C, of the line AB, and the *P*-piercing point, D, of the same line.

3. Through c^H draw any number of traces of auxiliary planes, and draw the elements cut by them from the cone B.

4. Draw the profile traces of the same auxiliary planes, and draw the elements cut by them from the cone A.

5. Locate the intersections of all the elements with the elements cut by them from the cone A.

5. Locate the intersections of all the elements with the elements lying in their respective planes, obtaining the *H*-, *V*- and *P*-projections of each.

6. Draw the line of intersection through them in order, and determine the visible and invisible parts of each projection of the curve.

243. Problem 87.—To find the intersection of two cylinders. This problem, like those preceding, may have more than one means of solution, according to the relative situations and characteristics of the cylinders.

In the case of the cylinders in Fig. 165, two methods are available, as follows:

(A) Pass a series of planes parallel to both axes, cutting elements from both surfaces.

(B) Pass a series of planes parallel to both circular bases, cutting circles from the surfaces. Method B can only be employed when the circular bases are in the same or parallel planes, as they are in this figure.

In other cases, it may be necessary to pass a series of planes parallel to one of the planes of projection, or parallel to one axis and to the circular base of the other cylinder.

In the specimen solution, here given, planes are passed parallel to both axes.

Construction.—Let it be required to find the intersection between the cylinders in Fig. 164, whose axes are AB and CD, both having circular bases in *H*.

1. Draw HQ, passed through AB parallel to CD.

2. This trace intersects the base circles at m^H, n^H, o^H, and p^H, with *V*-projections at m^V, n^V, o^V, and p^V in *GL*.

3. Through M, N, O, and P draw elements parallel to the respective axes.

4. These four elements lie in the same plane, and will intersect each other in four points on the curve. Locate these points.

5. Draw a series of traces parallel to HQ, and repeat operations (2), (3), (4) with each.

6. Draw a smooth curve through all of the points, taking them in order.

Note.—Do not divide either circle into any number of equal arcs. Let the planes determine the elements.

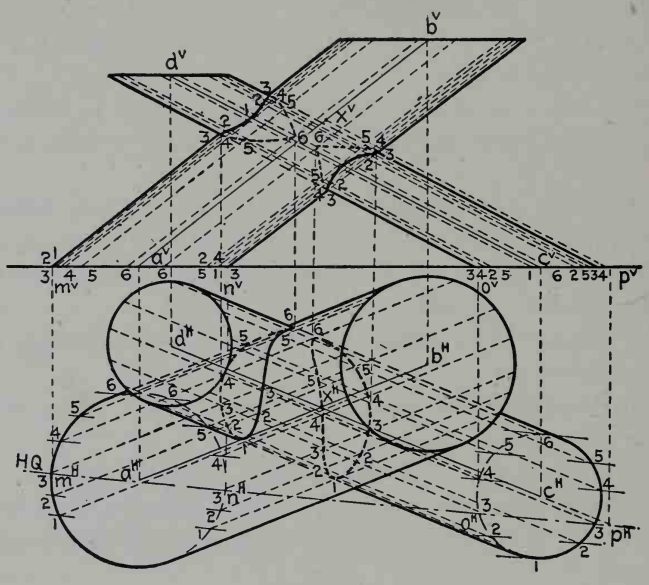


FIG. 165.

244. Problem 88.—To find the intersection of a cone and cylinder.

The following methods of passing planes may be employed:

1. Through the apex of the cone, parallel to the axis of the cylinder.

2. Parallel to the bases in any plane. (This method is very convenient if **both** surfaces have circular bases in *H*, *V*, or *P*.)

3. Parallel to the elements of the cylinder and perpendicular to the axis of the cone, if it be a right circular cone, if the conditions are right.

Concentric Sphere Method.—If both are surfaces of revolution with axes intersecting, the process in Problem 89 is usually the best.

Auxiliary Plane Method.—There are certain conditions when the easiest and neatest solution is made by projecting both sur-

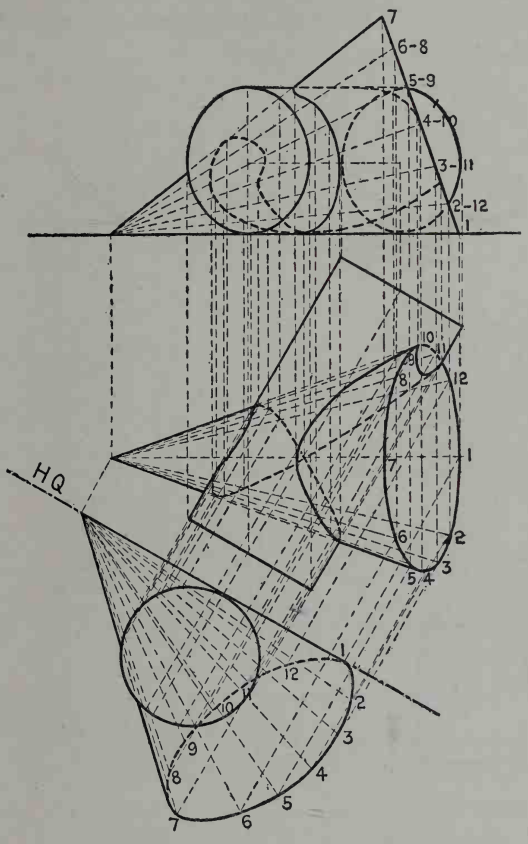


Fig. 166.

faces on a plane perpendicular to the axis of the cylinder. The specimen solution given here is for such a condition.

Construction.—See Fig. 166.

1. Project the cone and cylinder on Q, perpendicular to the axis of the cylinder.

2. The various elements, 1, 2, etc., of the cone will intersect the circle in their piercing points with the surface of the cylinder.

3. Project these points back to the H- and V-projections of the elements, and draw the curve through them.

Note.—Draw elements of the cone only.

Hint.—To avoid confusion, number your points and elements, because the following out of the correct order of these points is often very difficult.

245. Problem 89.—To find the intersection of any two surfaces of revolution whose axes intersect.

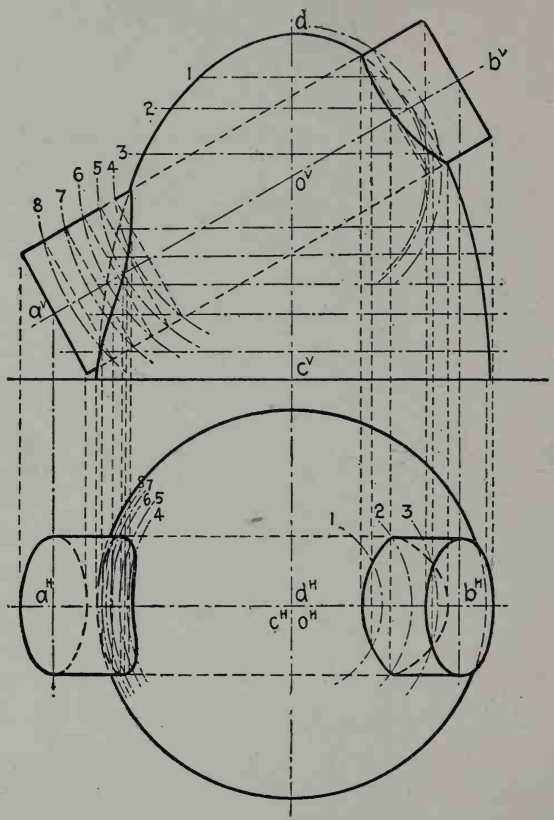


Fig. 167.

Analysis.—1. At the intersection of the axes as a center draw the projections of a number of concentric spheres.

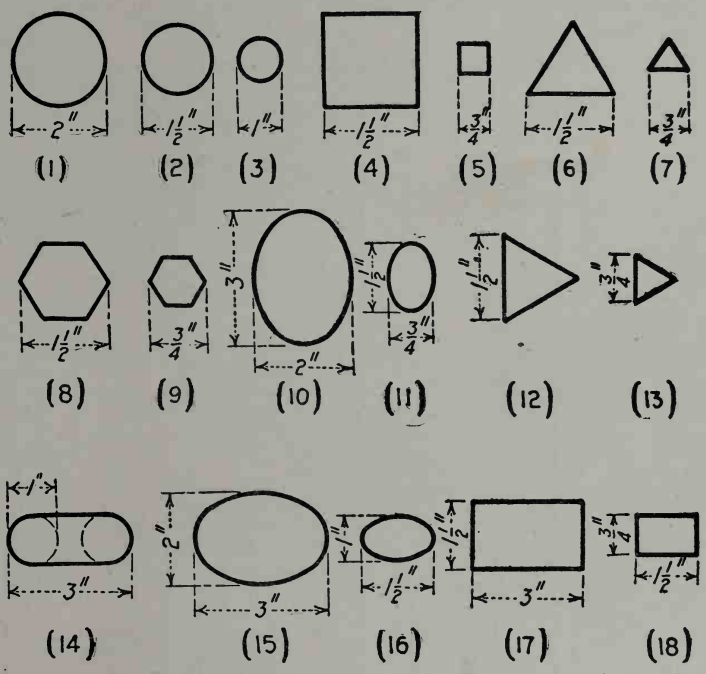
2. These spheres will cut circles from the two surfaces.

3. The intersections of the proper circles will yield points on the required intersection.

Construction.—Let the circular cylinder, whose axis is AB (Fig. 167), and the ellipsoid, whose axis is CD, be the two surfaces of revolution, with axes intersecting at O.

1. With o^v as a center, draw the V-projections of several spheres. The V-projections of the circles cut by these spheres will be straight lines. Why?

2. The circles cut from the ellipsoid will project on H as circles. Draw these projections.



Graphic Layout No. 72.—Bases, or Generators, of Surfaces, Spheres, Cones, Cylinders, Cubes, Prisms, Pyramids, Tori, Prolate Spheroids, Oblate Spheroids.

3. From the V-projections of the intersections of the proper circles project to the circles 1, 2, 3, etc. To do this it will not be necessary to draw the H-projections of the cylinder circles. Why?

4. Draw the two projections of the lines of intersection through these points in order.

Note 1.—The elements of the cylinder are not necessary in this solution.

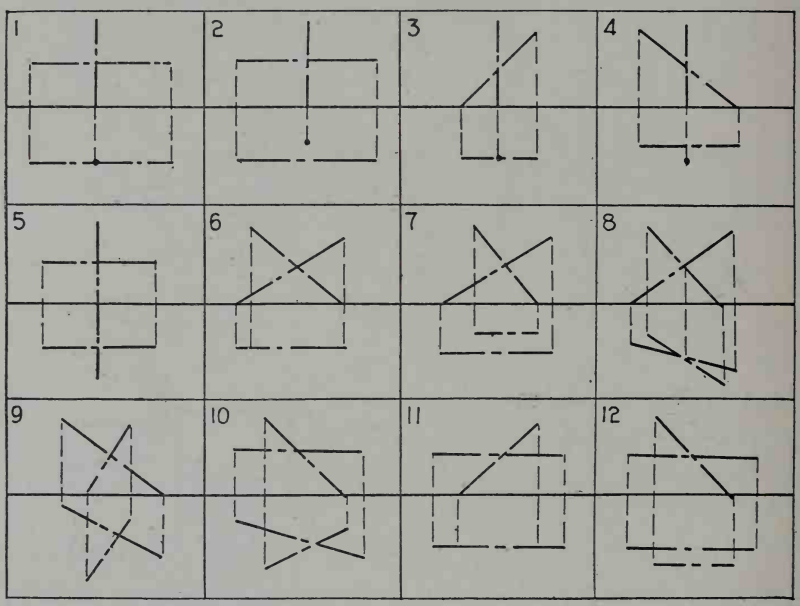
Note 2.—If the position of the surfaces is not as advantageous as in this figure, project them on an auxiliary plane parallel to both axes. However,

nearly all such problems are given in this position, and the difficult positions will hardly ever be encountered.

246. EXERCISES

Directions.—In making up the data for the exercises from the following layouts:

1. State what each surface is to be, whether cone, sphere, prism, torus, etc.
2. Give one of the combinations of axes.
3. Give the base or generator of each surface. (Graphic Layout No. 72.)
4. State whether the base is oblique or perpendicular to the axis, if necessary.
5. Give the altitude, if necessary, 2 in., 2½ in., 3 in.



Graphic Layout No. 73.—Axes for intersecting solids. (May be located in *I* or *III*.)

Data	Specimen Exercise	
1. Surfaces A——, B——.	A-Prolate Spheroid, B-Oblique Cone.	
2. Axes——.	No. 3.	
3. Base or Generator,——, ——.	No. 10.	No. 2.
4. Oblique or Perpendicular——,	Perpendicular.	Base in *H*.
5. Altitude——, ——.	——	2½ in.

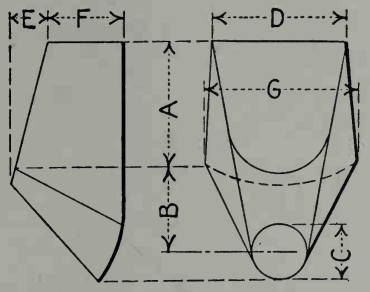
919. Draw the line of intersection between solids A and B, as specified in the data.

920. Develop the surface of solid A, as given in Ex. 919.

921. Develop the surface of solid B, as given in Ex. 919.

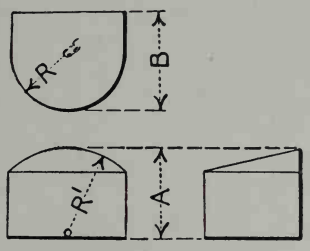
PRACTICAL EXERCISES IN VARIOUS SURFACES

922. Draw the line of intersection between the 3-in. sphere, center at O(3 + 1½ − 2) and a 1½-in. circular cylinder, axis A(1 + 1½ − 2) B(4 + 1½ − ½).



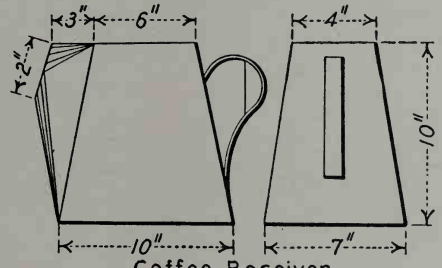
	A	B	C	D	E-F	G
1	20	20	8	12	8	16
2	30	28	9	14	9	18
3	40	36	10	16	12	21
4	50	40	12	20	14	28

Loading Skip for Mixer

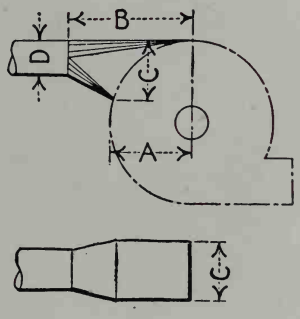


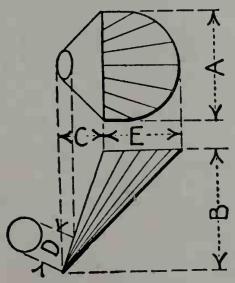
	A	B	R	R'
1	9	9	4½	8
2	12	12	6	10
3	16	16	8	14
4	20	20	10	18

Cylindroidal Hood



Coffee Receiver



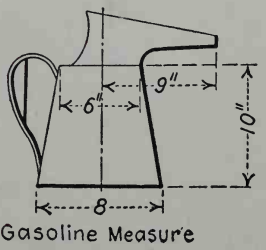


	A	B	C	D	E
1	18	40	10	10	16
2	20	40	10	10	16
3	24	50	12	12	20
4	30	60	15	16	26

Concrete Hopper

	A	B	C	D
1	20	13	14	6
2	30	50	20	8
3	40	64	30	9
4	48	72	40	10
5	60	96	44	14

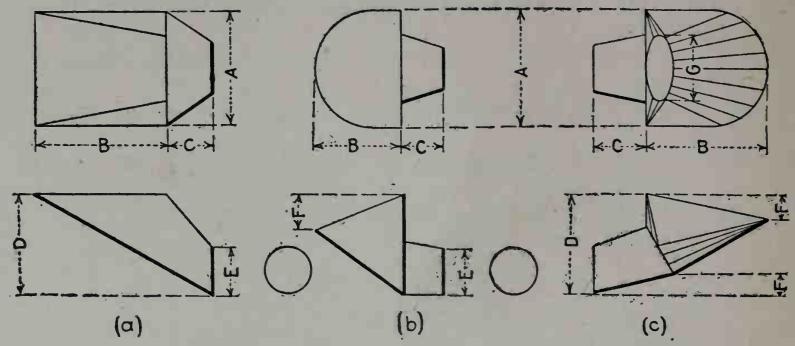
Fan Transition Piece



Gasoline Measure

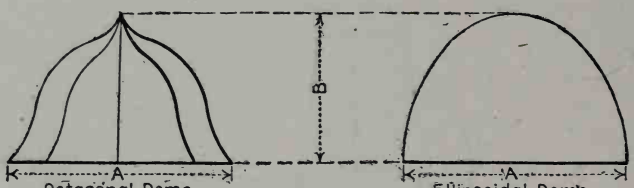
FIG. 168.—Exercises in various surfaces.

923. Lay out the patterns for the Loading Skip for Mixer, No.——, Fig. 168.



	A	B	C	D	E	F	G
1	20	24	10	36	8	12	11
2	30	30	16	45	10	12	15
3	40	44	20	60	12	12	18
4	60	56	30	96	16	16	20
5	72	80	30	108	20	18	28

Concrete Mixer Hopper



A= 10 Ft, 12 Ft, 16 Ft, 20 Ft, 24 Ft, 40 Ft, 50 Ft, 60 Ft.
B= 8 Ft, 10 Ft, 12 Ft, 14 Ft, 16 Ft, 20 Ft, 24 Ft, 30 Ft, 36 Ft.

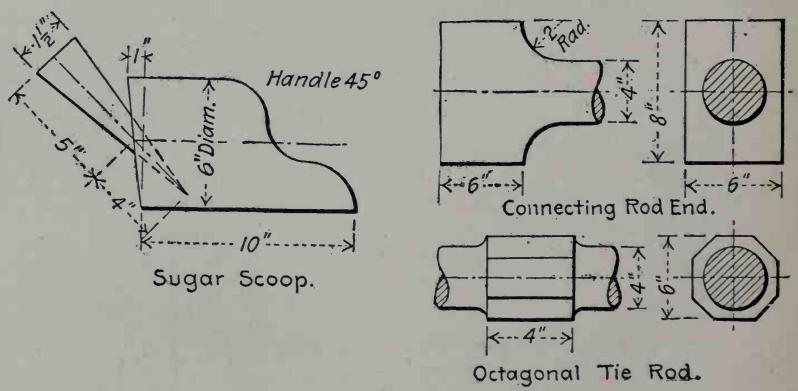


FIG. 169.—Practical exercises.

924. Lay out the patterns for the Cylindroidal Hood, No.——, Fig. 168.
925. Lay out the patterns for the Receiver for Coffee Grinder, Fig. 168.

926. Lay out the patterns for the Fan Transition Piece, Fig. 168.
927. Lay out the patterns for the Concrete Hopper——, Fig. 168.

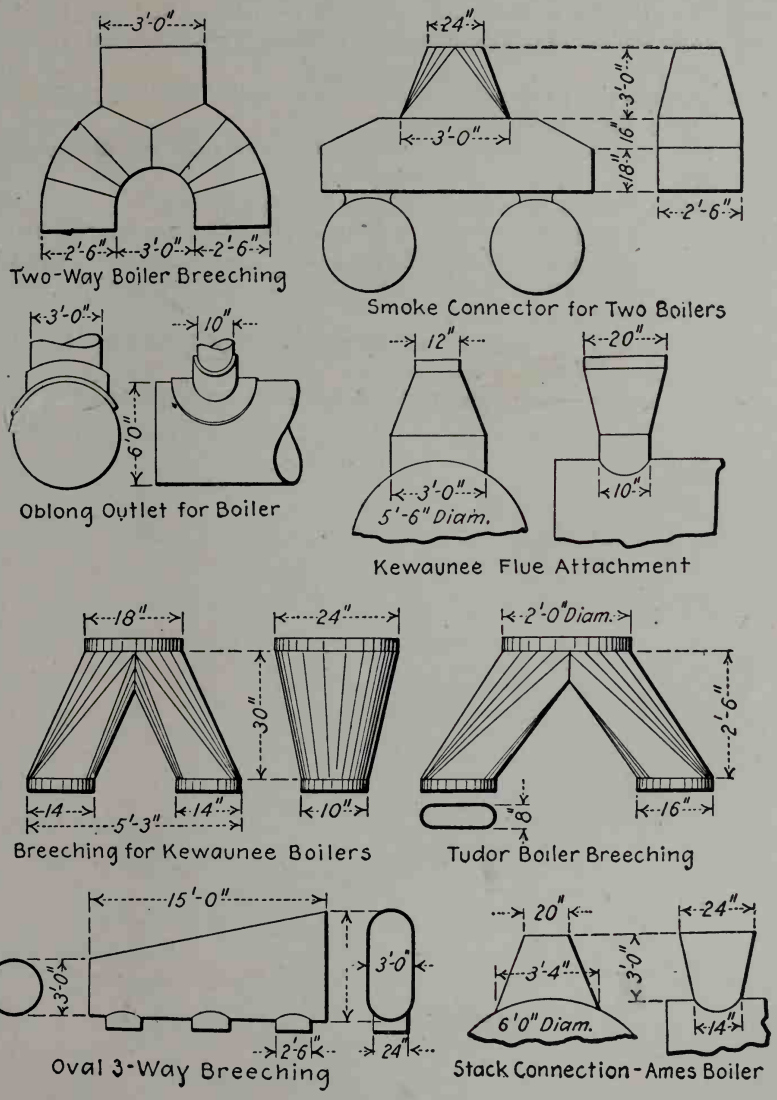


FIG. 170.—Boiler problems.

928. Lay out the patterns for the Concrete Mixer Hopper——, Fig. 169. Dimensions given in inches in the table.
929. Develop the surface of the Octagonal Dome, Fig. 169. Let A = ——, and B = ——. One strip is sufficient.

930. Develop the surface of the Ellipsoidal Dome, Fig. 169. Let A = ——, and B = ——.
931. Lay out the patterns of the Sugar Scoop, Fig. 169.
932. Draw the intersection of surfaces in the Connecting Rod End, Fig. 169.
933. Draw the line of intersection of the surfaces in the Octagonal Tie Rod, Fig. 169.
934. Lay out the patterns for the Two-way Boiler Breeching, Fig. 170. Supply the necessary dimensions.
935. Lay out the patterns for the Smoke Connector for Two Boilers, Fig. 170.
936. Lay out the patterns for the Oblong Outlet, Fig. 170.
937. Lay out the patterns for the Kewaunee Flue Attachment, Fig. 170.
938. Lay out the patterns for the Breeching for Kewaunee Boilers, Fig. 170.
939. Lay out the patterns for the Tudor Boiler Breeching, Fig. 170. Supply the necessary dimensions.
940. Lay out the patterns for the Oval Three-way Breeching, Fig. 170. Supply the necessary dimensions.
941. Lay out the patterns for the Ames Boiler Stack Connection, Fig. 170.

CHAPTER X

PICTORIAL PROJECTION
PERSPECTIVE

247. One of the most important and interesting applications of Descriptive Geometry is the art of **Perspective,** or, as it is sometimes called, **Conical Projection.** As was pointed out in the beginning of the work, Perspective Projection differs from Orthographic in that it represents the object by an image which is practically identical with that formed on the retina of the eye of the observer. The title, Conical Projection, is a correct one, since the picture is made by the projection of rays that converge in a point.

Numerous methods are given by various writers of text-books, and educators make use of all of them. The method presented here is offered, for two reasons: (1) its extreme simplicity and reasonableness; and (2) because it is a pure application of Descriptive Geometry, and, being such, it can be given to the student already grounded in the fundamentals without much detailed illustration and complicated directions.

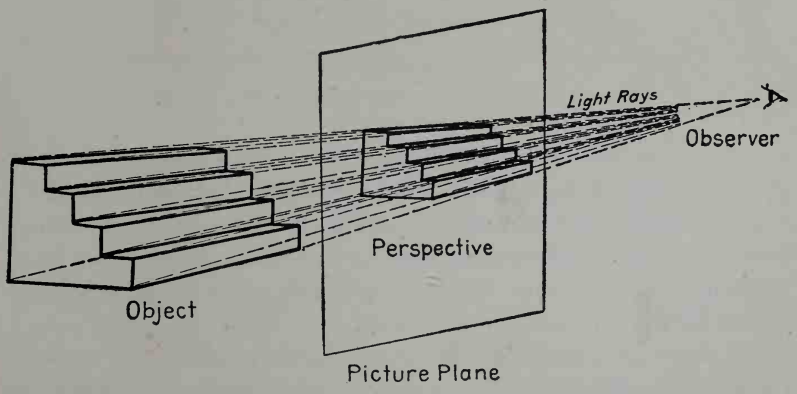


FIG. 171.—How the pictorial image is made.

248 CONICAL PROJECTION

If a plane, called the **Picture Plane,** be so placed as to intersect the rays coming from an object to the eye, the piercing points of

the rays will yield a perfect visual image of the object. This is the exact process of the camera, so that a photograph is an approximately perfect perspective.

249. The Construction of a Perspective from a Working Drawing.

In Fig. 172 a set of steps is shown in third angle projection, somewhat below the horizon.

Note.—The horizon should not be confused with *GL*, as it has not the same function.

The point of view for the perspective may be taken from any convenient position, but it is customary to view the object "from

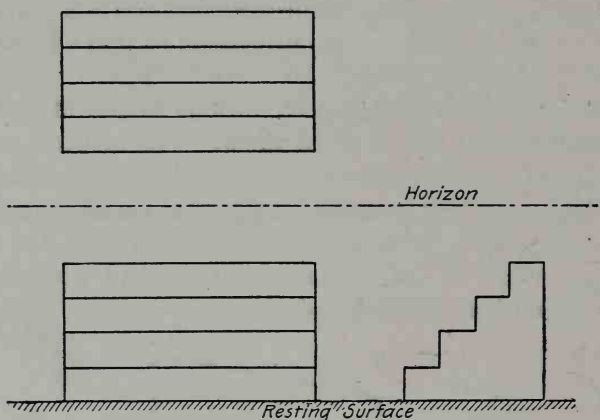


FIG. 172.—Object in orthographic projection.

an angle," so as to show two sides. This takes away the uncompromising appearance of the front elevation, as it appears in the working drawings, and offers opportunity to the draftsman of taking the most interesting view. In making interior views of buildings, and in stage scenery, etc., the point of sight is often taken directly in front of the scene.

To draw the picture of the steps, place the plan and elevation (or side view, if preferred) in a convenient position on the drawing, with the horizon at its proper elevation; see Fig. 173. Locate the **point of sight,** s^H, s^V, at a suitable position, where both front and side are nicely visible. Note that s^V is **always on the horizon.** The picture plane, HP, is placed so as to give the desired scale, and should be **about** perpendicular to the central line of sight. The scale is determined by the ratio of the distances of the object

and the picture plane from the eye. In this figure, the ratio is about 3 : 4, and the picture is therefore about three-quarter scale. If an enlargement is desired, place HP **beyond** the plan of the object.

For convenience, the picture plane is taken as a **vertical** plane (even though the central line of sight may not be horizontal); therefore HP, its *H*-trace, contains the *H*-projections of all its points. Thus the piercing points of the rays are easily determined, since they are projected on the *H*-trace.

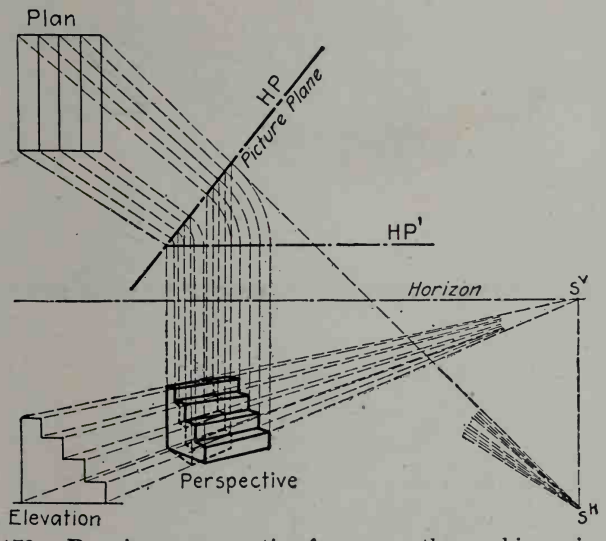


FIG. 173.—Drawing a perspective from an orthographic projection.

By connecting s^V with all points in the elevation, and s^H with all points in the plan, the *H*- and *V*-projections of the **projecting cone** are drawn. The section of this cone made by the picture plane is the picture. Since the picture plane is not parallel to *V*, the picture is not shown in true size, unless revolved parallel to *V*.

In Fig. 173, the *V*-projection of the conic section made by the picture plane is shown in dotted lines. By revolving HP to HP′ (parallel to *V*) the picture is shown in true size in solid lines.

250. To Construct a Perspective of an Object and its Shadow. The process in this case differs in no way from that of the preceding problem. The intersection of the cone by the picture plane is made, and then revolved parallel to *V*.

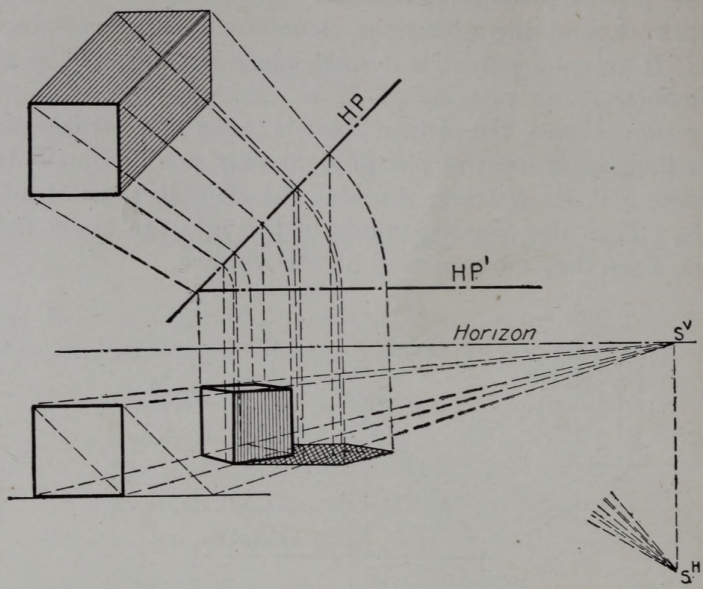
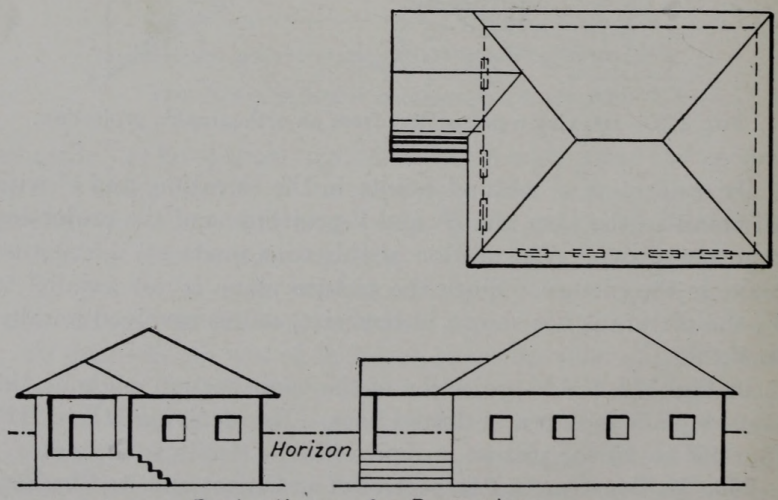


FIG. 174.—Perspective of an object, showing its shade and shadow.



Projections of a Bungalow

FIG. 175.

As an illustration of the application to more complicated objects, a bungalow is shown in Fig. 176, drawn in perspective by

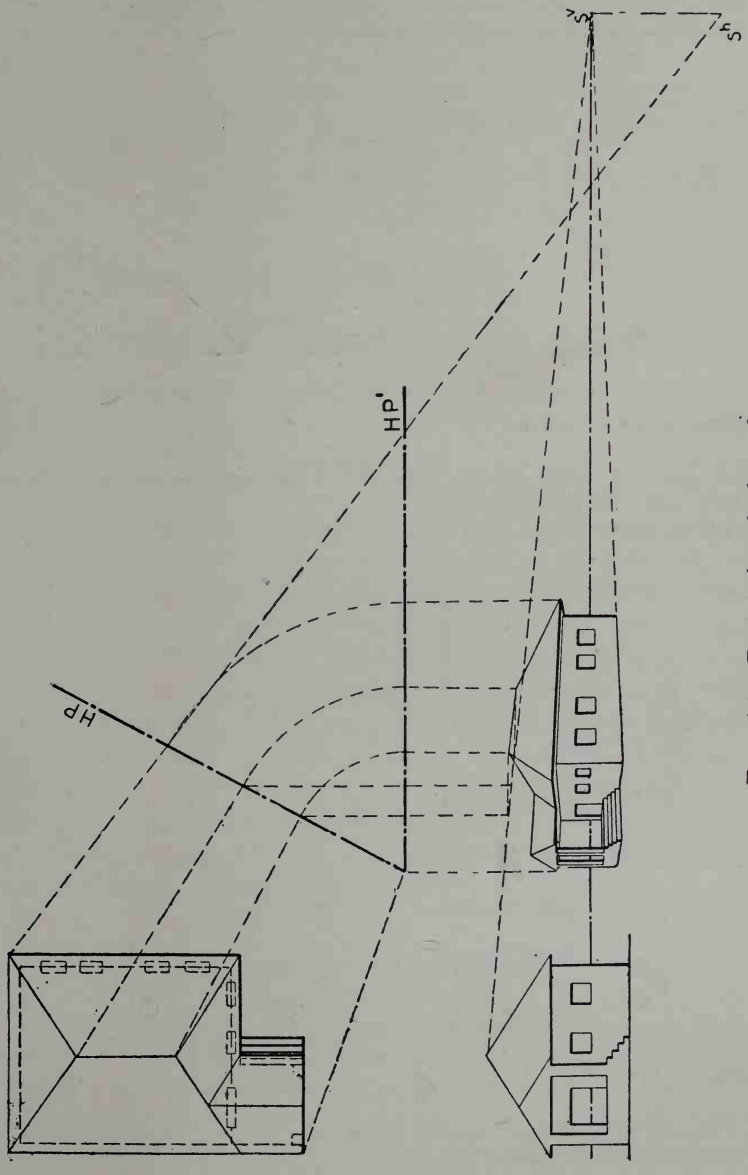
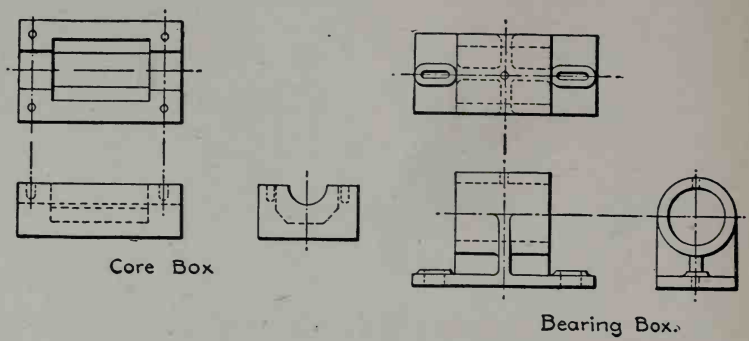


Fig. 176.—Perspective of a bungalow.

conical projection from the working drawing as given in Fig. 175. In Fig. 176 only a few construction lines are shown.

251. EXERCISES

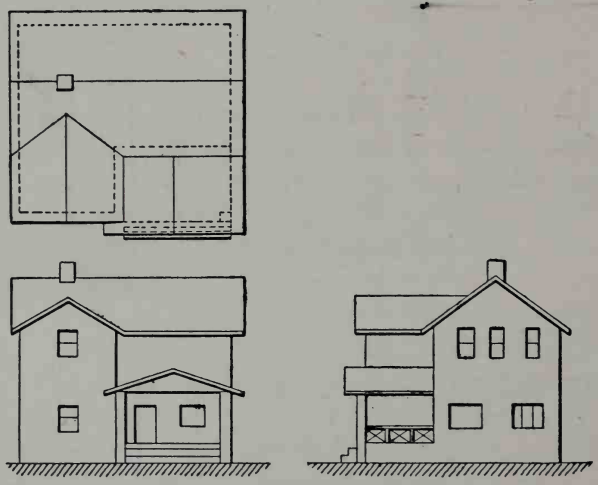
Draw the perspective of the following objects, to the scale designated by the instructor, one-half, three-quarters, full, or double the scale of the working drawings. Make the working drawings as large as the space will allow



Graphic Layout No. 74.

to give plenty of room for the perspective and its construction. Place all small objects below the horizon.

942. 1½-in. cube. Scale ——.



Graphic Layout No. 75.

943. Frustum of a square pyramid, lower base 1½ in., upper base ¾ in., altitude (base to base) 1 in. Scale ——.

944. Wooden box 20 in. long, 12 in. wide, 6 in. deep, walls 1 in. thick, without top. Scale ——.

945. Place a 1¼-in. equilateral triangular prism, 1¼ in. long, on a 1-in.

cube (resembling a house with gable roof). Draw the shades and shadows, and their perspective. Scale ——.

946. Place a 1¼-in. square pyramid, ½-in. altitude, on a 1-in. cube. Draw the shades, shadows, and perspective. Scale ——.

947. Core box for octagonal core, Graphic Layout No. 74. Scale ——.

948. Bearing box, Graphic Layout No. 74. Scale ——.

949. House, Graphic Layout No. 75. Scale ——.

950. House, Graphic Layout No. 75. Shades, shadows, and perspective. Scale——.

951. Draw the perspective and the shadow, of the object, or objects, in Ex. —— (selected from 714–744, Chap. IV).

Pseudo-perspectives

ISOMETRIC, CAVALIER, AND CABINET PROJECTIONS

252. For certain purposes mechanical variants on perspective are preferred to conical projection. These variants are collectively called **Single Plane Projection.** The system of outstanding preference of these is called **Isometric Projection,** which means "Equal Measure" projection. It is orthographic projection on an oblique plane, as will be shown. Isometric has its advantages and objections compared with perspective. Its advantages are: (1) ease and rapidity of execution, its principal lines being laid off with the 30° triangle; (2) measurements of principal lines are to scale, which facilitates drawing and makes it more valuable for shop purposes; (3) there are engineering advantages in isometric layouts of wiring and piping jobs. It has been called "Practical Perspective" by some writers. Its chief and only serious objection is that it does not take distance into account, and therefore there is a distortion, which gives an unpleasant appearance when the object is long in recession.

ISOMETRIC PROJECTION

Construction.—Place a cube so that its base diagonal is parallel to V; see Fig. 177. Pass a V-projecting plane T through A and B, and project the plane on R drawn parallel to T. Its R-projection will be isometric; that is, the several edges will all be of equal length, as they are in reality, and the projection will show three faces in one view. The isometric projections of these edges are $\frac{81,647}{100,000}$ of their true length. The edges of the cube in this

projection run in three directions at equal angles (120°). Thus, in **Isometric,** the 90° angles of the three systems of parallels of the cube (called the **isometric axes**) radiate at 120° from each other.

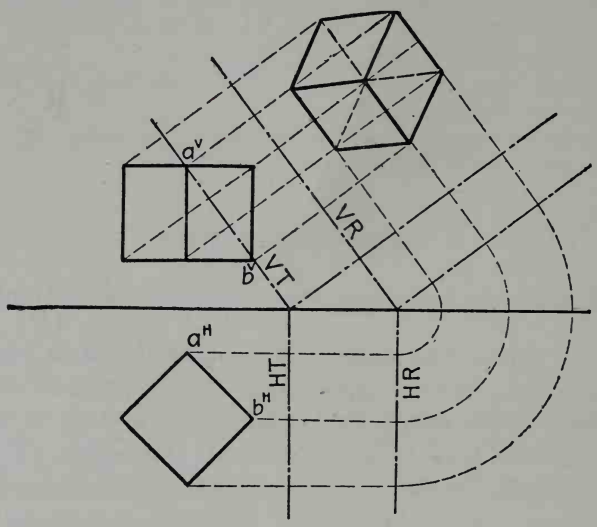


FIG. 177.

ISOMETRIC DRAWING

253. Isometric Drawing differs from isometric projection only in that the latter would require all "isometric lines" to be drawn to a reduced $\left(\frac{81}{100}\right)$ scale. Since most working drawings are themselves drawn to a reduced scale, and the isometric is chiefly for pictorial purposes, this distinction is of no importance, and the isometric lines may be drawn the full length of the working drawing.

ISOMETRIC AXES

Lay off AO, BO, and CO, as shown in Fig. 178, radiating at 120°, AO and CO being drawn at 30° from the horizontal. These three lines represent three of the adjacent edges of the cube, BO a vertical and AO and CO horizontal lines at right angles to each other. In isometric all objects are drawn so that the rectangular lines run in these directions. Other lines (non-isometric) are laid off by joining points on the isometric lines, or by erecting ordinates from known points.

254. Isometric drawing of a cube.

Construction.—Given the cube in Fig. 179.

1. Lay off the isometric axes, as in Fig. 178.

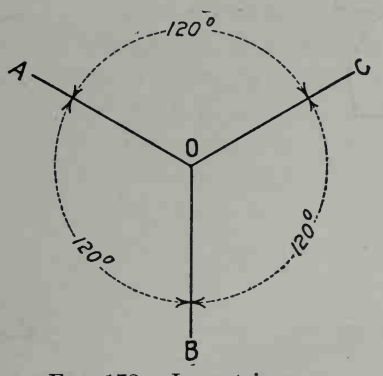


FIG. 178.—Isometric axes.

2. Make each edge, AO, BO, CO, equal to the true length of the cube edges.

3. Complete the three parallelograms on these edges.

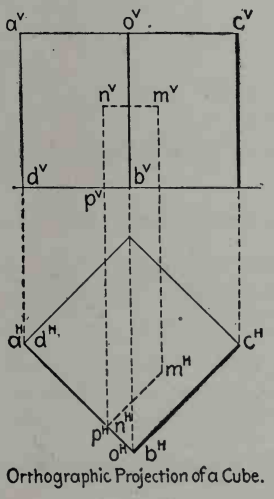


FIG. 179.

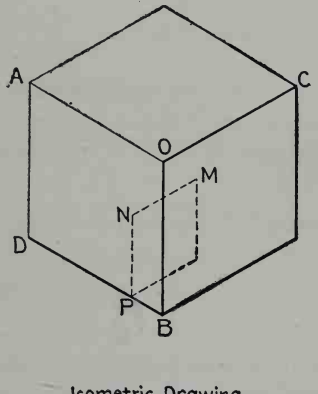


FIG. 180.

255. To locate a point isometrically.

Refer to Figs. 179 and 180. Let M, inside the cube, be the required point, given by its projections m^H, m^V.

1. Draw isometric ordinates, as follows: (*a*) a perpendicular MN to the face AOBD; (*b*) a perpendicular NP to the edge DB.

2. Lay off these ordinates in their true length on the proper isometric lines in Fig. 180, thus locating M in its proper place within the cube.

255. To draw an exact circle.

Construction.—Refer to Figs. 181 and 181*a*. On the left face of the cube (Fig. 181) is inscribed a circle, with squares in each corner touching the circle at E, G, *H*, and K.

1. Draw each of these squares in isometric, Fig. 181*a*.

2. Draw an ellipse in the face of the cube, Fig. 181*a*, touching E′, G′, *H′*, and K′, and the center point of each of the four edges.

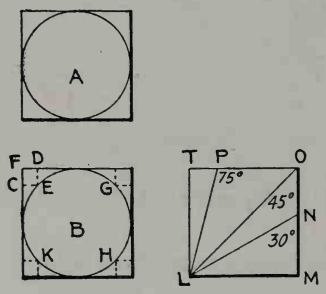


Fig. 181.

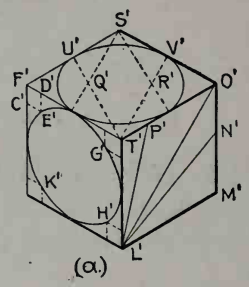


Fig. 181*a*.

256. To draw an approximate circle.

Construction.—Refer to Fig. 181*a*.

1. Connect S′ and T′ with the middle points of the opposite edges.

2. Using T′U′ as a radius, draw arcs from T′ and S′ as centers, from middle point to middle point.

3. Using Q′ and R′ as centers, and Q′U′ as radius, complete the oval.

Note.—This system of drawing the circle isometrically is almost exclusively employed in commercial drafting, since it can be done with the compasses and is clean cut in appearance.

257. To draw lines making any desired angle with any given line.

Construction.—Required to draw lines at 30°, 45°, and 75° to LM, Fig. 181.

1. Draw LN, LO, and LP in Fig. 181 at their true angles to LM.

2. From M′ (Fig. 181*a*) lay off the ordinates M′N′ and M′O′ equal to MN and MO.

3. From T′ lay off T′P′ equal to TP. The lines L′N′, L′O′, L′T′ will be the required lines in isometric.

Note.—If the angles were laid off at M, instead of L, they would work out differently. The corners at M and L are each 90°, but in isometric one is 60° and the other 120°, both representing the same angle.

258. To draw the frustum of a hexagonal pyramid.

Construction.—Exactly as in previous exercises. Fig. 182 is the working drawing and Fig. 182*a* the isometric. Construction lines are shown.

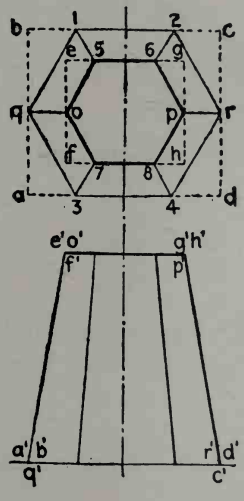


Fig. 182.

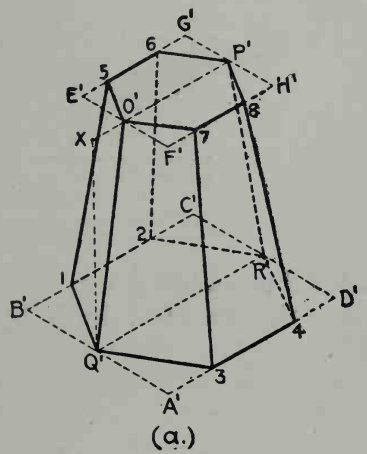


Fig. 182*a*.

Note.—Do not draw in the invisible lines. They are drawn here to make the figure complete, but in practical work, where isometric drawings are used, the dotted lines are seldom drawn.

259. Other Methods of Pictorial Projection.

Under certain conditions, a closer approximation of a perspective than that shown by the isometric is obtained by the use of other oblique projections, called **Cavalier and Cabinet Projections.**

Cavalier Projection.—The object is placed with one face parallel to the picture plane, and the projectors are considered inclined 45° to the picture plane. Thus one face is the same as the orthographic view would be, and all lines perpendicular to the picture plane are projected in their true length. It thus becomes a form of isometric projection, with less distortion,

because one face is the same as othographic projection would make it.

Cabinet Projection.—To further reduce the distortion, the receding lines are shortened one-half. Otherwise the drawing is made like cavalier projection.

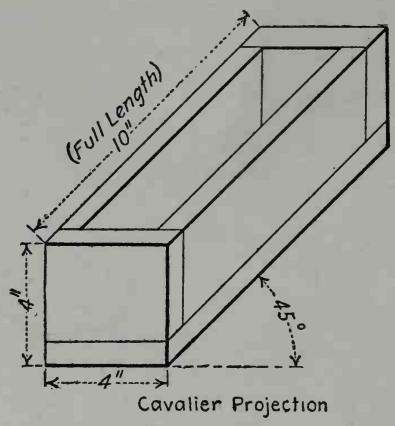


FIG. 183.

Fig. 183 shows a box drawn by the cavalier method, and Fig. 184 shows the same box drawn by the cabinet and isometric methods.

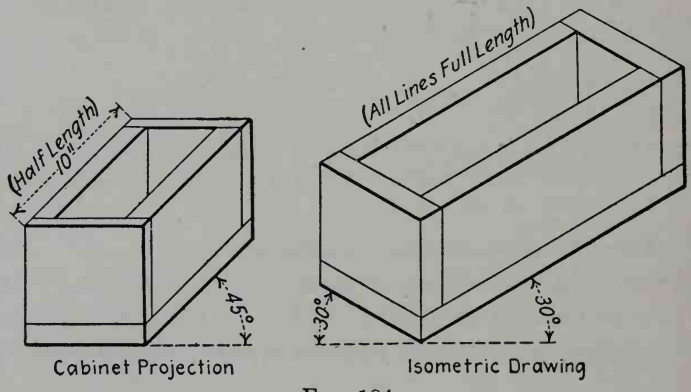


FIG. 184.

EXERCISES IN SINGLE PLANE PROJECTION

260. Make——(isometric, cavalier, cabinet) drawings of the following object, making them to reduced scale, if necessary.

952. Wooden box, without cover, 10 in. long, 6 in. wide, 4 in. deep, made of ¾-in. boards. Stand on end, open to the front.

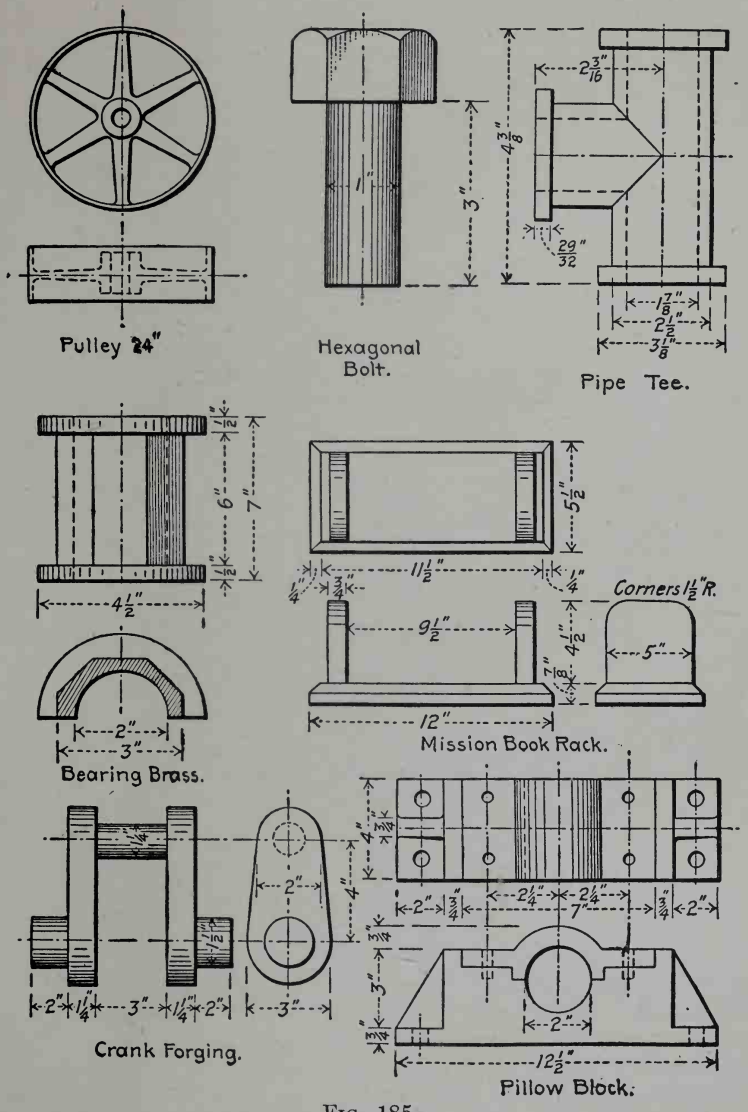

Pulley 24"
Hexagonal Bolt.
Pipe Tee.
Bearing Brass.
Mission Book Rack.
Corners 1½"R.
Crank Forging.
Pillow Block.

FIG. 185.

953. Same box, in natural position, with a cover open 120°.
954. Pentagonal plinth, 2-in. diameter, ½ in. thick, surmounted by a 1-in. cylinder, 1½ in. high.
955. Bolt shown in Fig. 185.
956. Pulley, Fig. 185.
957. Book rack, Fig. 185.
958. 1½-in. pipe tee, Fig. 185.

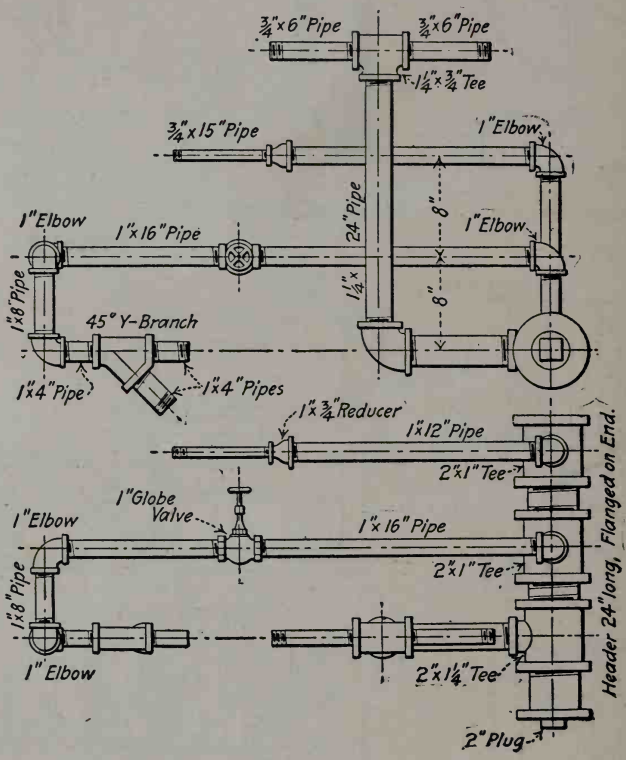


FIG. 186.

959. Bearing brass, Fig. 185.
960. Crank forging, Fig. 185.
961. Pillow block, Fig. 185.
962. Frustum of pyramid, given in Ex. 943.
963. Saw horse, Fig. 86, Chap. III.
964. Stand, Fig. 89, Chap. III.
965. Piping layout, Fig. 186. This is intended for a large sheet, and makes a fine drawing if traced.

Note.—First make the layout of the center lines, making necessary allowances for all connections.

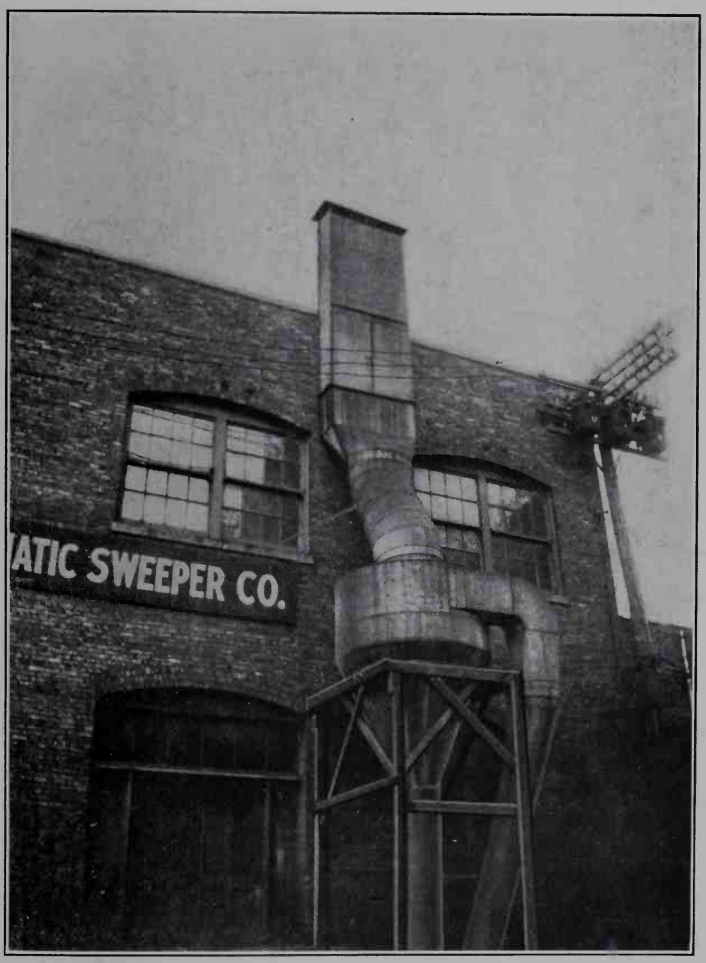


FIG. 187.—A problem in sheet metal work, involving nearly every problem in Descriptive Geometry, including the intersection and development of plane, single-curved, and warped surfaces. The picture itself is a problem in shades, shadows, and perspective.